Vectors - Lesson 1

Vectors - Addition, Subtraction, Equality and Scalar Multiplication

LI

- Know what a vector is.
- Sketch vectors.
- Add and subtract vectors.
- Scalar multiply vectors.

SC

- Arithmetic.
- Count squares.

Vector - quantity with magnitude (aka size) and direction.

Scalar - quantity with magnitude only. (these are just numbers)

Examples of Vectors

Displacement, velocity, force, electric field.

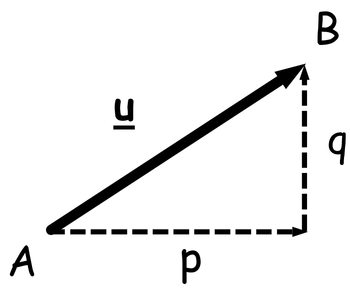
Examples of Scalars

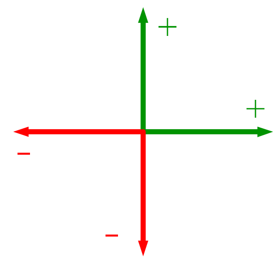
Distance, speed, frequency, temperature, mass, volume.

Vectors exist in n dimensions, but we will only study 2D and 3D vectors

Basic Vectors Terminology and Notation

2D Vectors





Components of $\underline{u}$

$$\underline{u} = \overrightarrow{AB} = \begin{pmatrix} p \\ q \end{pmatrix}$$

p is the x - component of $\underline{u}$

q is the y - component of $\underline{u}$

3D Vectors

Components of $\underline{v}$

$$\underline{v} = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$$

p is the x - component of $\underline{v}$

q is the y - component of $\underline{v}$

r is the z - component of $\underline{v}$

Equality of Vectors

Vectors are equal if all their respective components are equal :

$$\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \Rightarrow p = a \text{ and } q = b$$

Similar for 3D vectors

Addition and Subtraction of Vectors

The sum (aka resultant) of vectors $\underline{u}$ and $\underline{w}$ (denoted $\underline{u} + \underline{w}$) is obtained by adding their respective components :

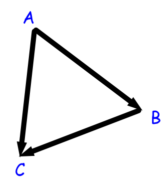
$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} a + p \\ b + q \\ c + r \end{pmatrix}$$

Similar for 2D vectors

Head to Tail Rule

Resultant of $\overrightarrow{AB}$ and $\overrightarrow{BC}$:

$$\boxed{\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}}$$



The difference of vectors $\underline{u}$ and $\underline{w}$ (denoted $\underline{u} - \underline{w}$) is the vector obtained by subtracting the components of $\underline{w}$ from the components of $\underline{u}$

Scalar Multiplication of a Vector (by a Number)

The scalar multiple of $\underline{u}$ by a number k is the vector obtained by multiplying each component of $\underline{u}$ by k :

$$\underline{u} = \begin{pmatrix} p \\ q \end{pmatrix} \Rightarrow k\,\underline{u} = \begin{pmatrix} kp \\ kq \end{pmatrix}$$

Similar for 3D vectors

Negative of a Vector

The negative of $\underline{u}$ is obtained by negativing all the components of $\underline{u}$:

$$\underline{u} = \begin{pmatrix} p \\ q \end{pmatrix} \Rightarrow -\underline{u} = \begin{pmatrix} -p \\ -q \end{pmatrix}$$

Similar for 3D vectors

The Zero Vector

The zero vector $\underline{0}$ is the vector with all components equal to 0 :

$$\underline{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Similar for 2D vectors

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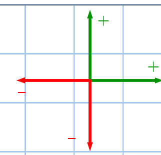
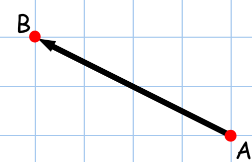


Example 1

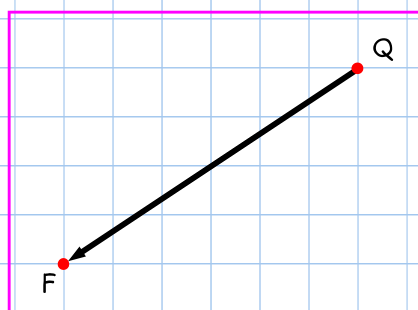
State the components of $\overrightarrow{AB}$.

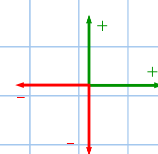


$$\boxed{\overrightarrow{AB} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}}$$

Example 2

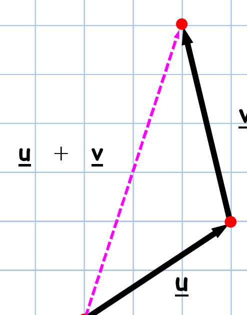
Sketch the vector $\overrightarrow{QF} = \begin{pmatrix} -6 \\ -4 \end{pmatrix}$.

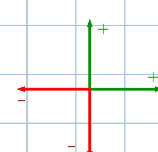




Example 3

If $\underline{u} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\underline{v} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$, draw a vector diagram to obtain the components of the resultant vector $\underline{u} + \underline{v}$.





$$\boxed{\underline{u} + \underline{v} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}}$$

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Example 4

If $\underline{a} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$, $\underline{b} = \begin{pmatrix} -4 \\ 11 \\ -7 \end{pmatrix}$ and $\underline{c} = \begin{pmatrix} -1 \\ 7 \\ -4 \end{pmatrix}$,

find :

(a) $-5\underline{c}$.

(b) $2\underline{a} + 3\underline{b}$.

(c) $\underline{a} + \underline{b} - 2\underline{c}$.

(a)

$$-5\underline{c} = -5\begin{pmatrix} -1 \\ 7 \\ -4 \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} 5 \\ -35 \\ 20 \end{pmatrix}}$$

(b)

$$2\underline{a} + 3\underline{b} = 2\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + 3\begin{pmatrix} -4 \\ 11 \\ -7 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ 6 \\ -2 \end{pmatrix} + \begin{pmatrix} -12 \\ 33 \\ -21 \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} -8 \\ 39 \\ -23 \end{pmatrix}}$$

(c)

$$\underline{a} + \underline{b} - 2\underline{c} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \begin{pmatrix} -4 \\ 11 \\ -7 \end{pmatrix} - 2\begin{pmatrix} -1 \\ 7 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ 14 \\ -8 \end{pmatrix} - \begin{pmatrix} -2 \\ 14 \\ -8 \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}}$$

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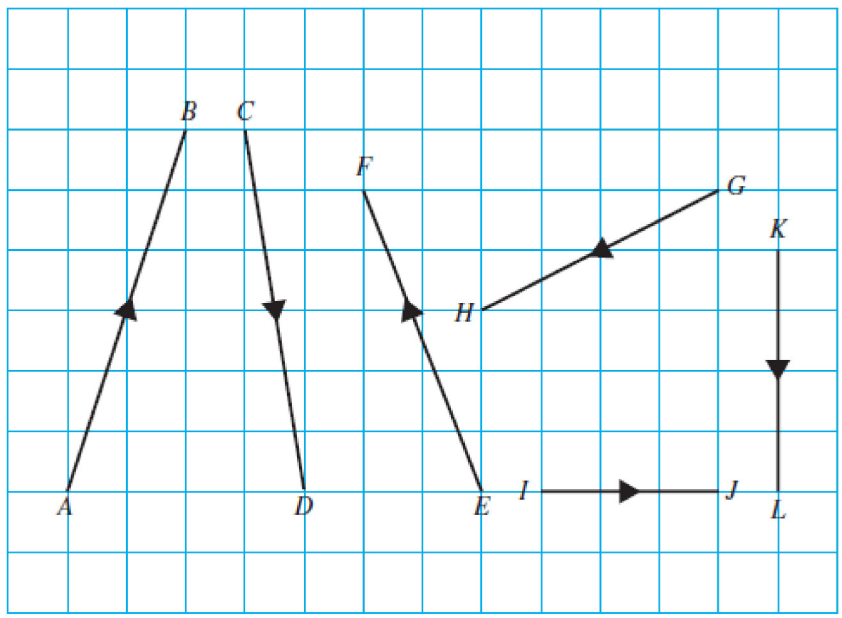
Questions

1 Write each vector in component form.



2 Sketch each of these vectors.

a $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ b $\overrightarrow{CD} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ c $\overrightarrow{EF} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ d $\overrightarrow{GH} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

e $\overrightarrow{IJ} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ f $\overrightarrow{KL} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ g $\overrightarrow{MN} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}$ h $\overrightarrow{PQ} = \begin{pmatrix} -3 \\ -7 \end{pmatrix}$

Questions

1 Draw a vector diagram for **m** and **n** and the resultant vector **m** + **n**. State the components of **m** + **n**.

a $\mathbf{m} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ **b** $\mathbf{m} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ **c** $\mathbf{m} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$

2 Calculate the resultant vector **x** – **y**.

a $\mathbf{x} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \mathbf{y} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ **b** $\mathbf{x} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}, \mathbf{y} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ **c** $\mathbf{x} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}, \mathbf{y} = \begin{pmatrix} -4 \\ -5 \end{pmatrix}$

3 Write the negative vector and sketch a diagram of the positive and the negative vector.

a $\mathbf{p} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ **b** $\mathbf{q} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$ **c** $\mathbf{r} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ **d** $\mathbf{s} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

e $\mathbf{t} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ **f** $\mathbf{u} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$ **g** $\mathbf{v} = \begin{pmatrix} -1 \\ -6 \end{pmatrix}$ **h** $\mathbf{w} = \begin{pmatrix} -5 \\ 0 \end{pmatrix}$

4 Calculate the resultant vectors.

$\mathbf{v} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}, \mathbf{w} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}, \mathbf{x} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}, \mathbf{y} = \begin{pmatrix} -3 \\ -7 \end{pmatrix}$

a $\mathbf{v} + \mathbf{w}$ **b** $\mathbf{x} - \mathbf{w}$ **c** $\mathbf{v} + \mathbf{w} + \mathbf{x}$ **d** $\mathbf{x} - \mathbf{y} - \mathbf{v}$

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5 **a** If $\mathbf{a} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, write in component form the vector $3\mathbf{a}$.

b If $\mathbf{v} = \begin{pmatrix} 4 \\ 10 \end{pmatrix}$, write in component form the vector $\frac{1}{2}\mathbf{v}$.

c If $\mathbf{d} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$, write in component form the vector $4\mathbf{d}$.

d If $\mathbf{x} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$, write in component form the vector $-2\mathbf{x}$.

e If $\mathbf{y} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$, write in component form the vector $-4\mathbf{y}$.

6 For vectors **u**, **v**, and **w**: $\mathbf{u} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$, $\mathbf{w} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$, calculate the resultant vector in component form.

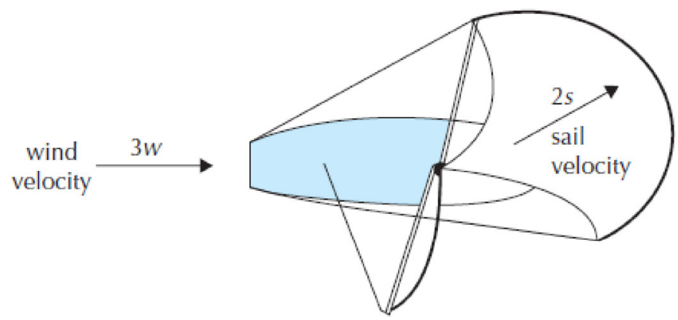
a $\mathbf{u} + \mathbf{v} + \mathbf{w}$ **b** $2\mathbf{u} + 3\mathbf{v}$ **c** $3\mathbf{u} - 2\mathbf{v}$ **d** $5\mathbf{u} + \mathbf{v} - \mathbf{w}$

e $3\mathbf{v} + 2\mathbf{w} - \mathbf{u}$ **f** $4\mathbf{w} - 2\mathbf{u} + \mathbf{v}$ **g** $\frac{1}{2}\mathbf{v} + 2\mathbf{u} + 3\mathbf{w}$ **h** $-\mathbf{v} - 2\mathbf{u} - 3\mathbf{w}$

7 A sailing dinghy is shown.

Vector $\mathbf{w} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ represents the normal wind velocity and vector $\mathbf{s} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ represents the sail velocity.

Both forces act together to create a resultant vector force. Calculate the resultant vector.

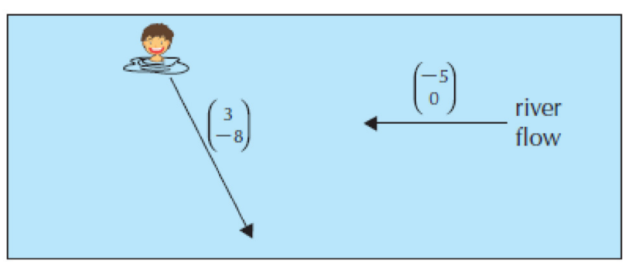


8 A white snooker ball is hit in a direction represented by the vector $\mathbf{w}_1 = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$. The white ball then hits a red ball. After it hits the red ball, the movement of the white ball is described by the vector $\mathbf{w}_2 = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$.

a What is the resultant vector that represents the movement of the white ball?

b The same shot is repeated but the white ball is hit with three times the force and reflects off the red ball with double the original force. What is the resultant vector in this shot?

9 A swimmer is swimming across a river in the direction described by the vector. The river flow has the direction of the vector as shown.



a What is the resultant vector of the swimmer?

b What would be the effect if the river flow doubled?

Questions

1 Find the resultant of each set of vectors.

a $\begin{pmatrix}4\\5\end{pmatrix}+\begin{pmatrix}3\\7\end{pmatrix}$ **b** $\begin{pmatrix}2\\9\end{pmatrix}+\begin{pmatrix}-1\\2\end{pmatrix}$ **c** $\begin{pmatrix}4\\2\\3\end{pmatrix}+\begin{pmatrix}5\\1\\1\end{pmatrix}$

d $\begin{pmatrix}2\\7\\-1\end{pmatrix}+\begin{pmatrix}-2\\4\\4\end{pmatrix}$ **e** $\begin{pmatrix}-2\\-1\\-6\end{pmatrix}+\begin{pmatrix}-1\\2\\2\end{pmatrix}+\begin{pmatrix}1\\1.5\\3\end{pmatrix}$ **f** $\begin{pmatrix}6\\0\\3\end{pmatrix}+\begin{pmatrix}-3\\-2\\1\end{pmatrix}+\begin{pmatrix}-3\\0\\-4\end{pmatrix}$

2 Find the resultant of each set of vectors.

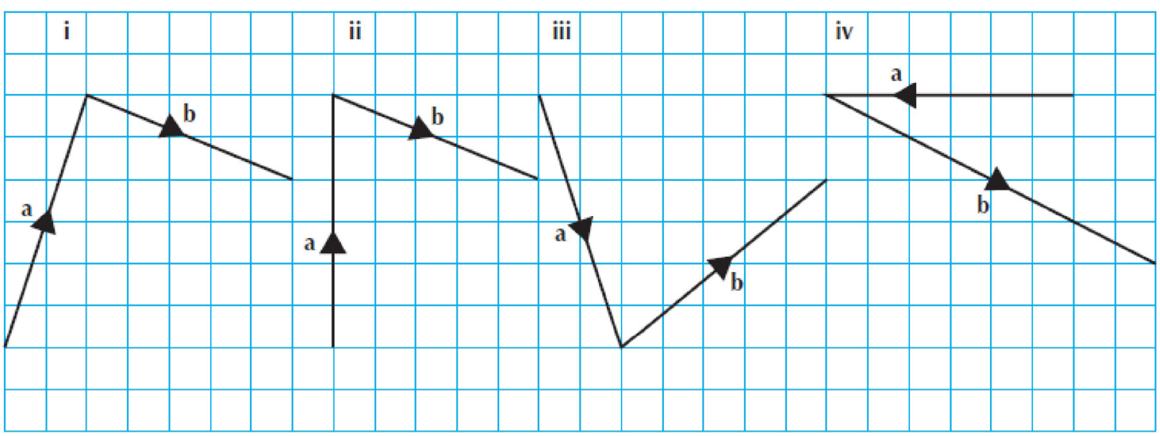
a $\begin{pmatrix}3\\2\end{pmatrix}-\begin{pmatrix}4\\1\end{pmatrix}$ **b** $\begin{pmatrix}5\\-2\end{pmatrix}-\begin{pmatrix}3\\-1\end{pmatrix}$ **c** $\begin{pmatrix}-1\\-5\end{pmatrix}-\begin{pmatrix}6\\-3\end{pmatrix}$

d $\begin{pmatrix}5\\4\\2\end{pmatrix}-\begin{pmatrix}3\\3\\3\end{pmatrix}$ **e** $\begin{pmatrix}5\\1\\4\end{pmatrix}-\begin{pmatrix}12\\2\\2\end{pmatrix}$ **f** $\begin{pmatrix}21\\10\\8\end{pmatrix}-\begin{pmatrix}9\\2\\4\end{pmatrix}-\begin{pmatrix}5\\5\\5\end{pmatrix}$

3 For each vector:

a write down the vector in component form

b calculate the resultant vector **a** + **b** in component form.



4 For each of the vectors **a** and **b** in Question 3, calculate the resultant vector:

a 2**a** + 4**b** **b** 3**a** – 2**b** **c** –**a** – 3**b** **d** 5**a** – 5**b**

5 Calculate the missing values.

a $\begin{pmatrix}4\\2\\a\end{pmatrix}-\begin{pmatrix}3\\5\\1\end{pmatrix}=\begin{pmatrix}1\\-3\\5\end{pmatrix}$ **b** $\begin{pmatrix}x\\y\\z\end{pmatrix}+\begin{pmatrix}3\\4\\2\end{pmatrix}=\begin{pmatrix}5\\-2\\2\end{pmatrix}$ **c** $\begin{pmatrix}-6\\1\\4\end{pmatrix}+\begin{pmatrix}a\\3\\2b\end{pmatrix}=\begin{pmatrix}-b\\4\\2a\end{pmatrix}$

Answers

1 **a** $\begin{pmatrix} 2 \\ 6 \end{pmatrix}$ AB

b $\begin{pmatrix} 1 \\ -6 \end{pmatrix}$ CD

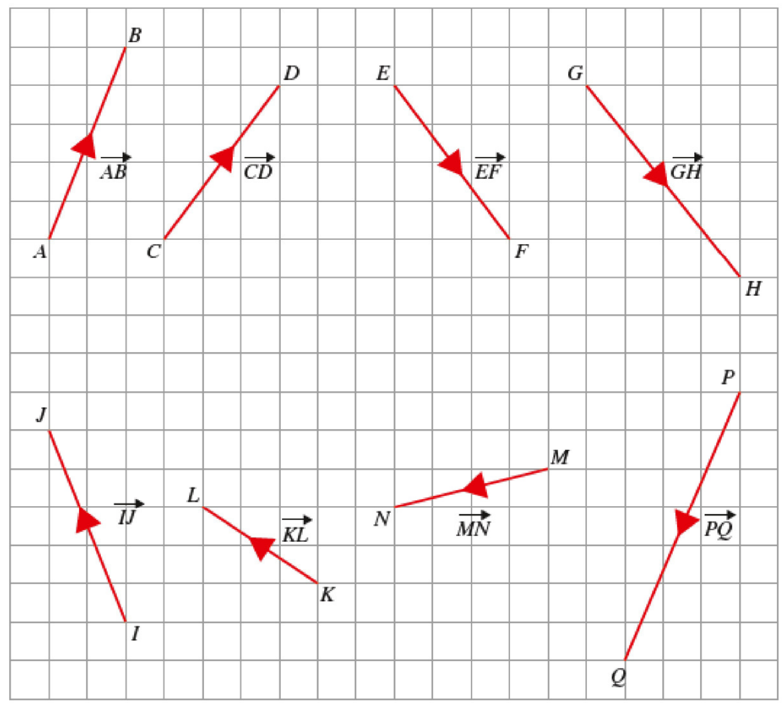
c $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$ EF

d $\begin{pmatrix} -4 \\ -2 \end{pmatrix}$ GH

e $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ IJ

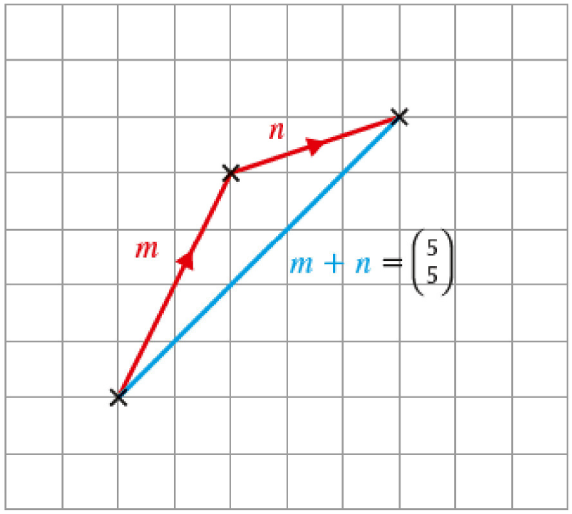
f $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$ KL

2

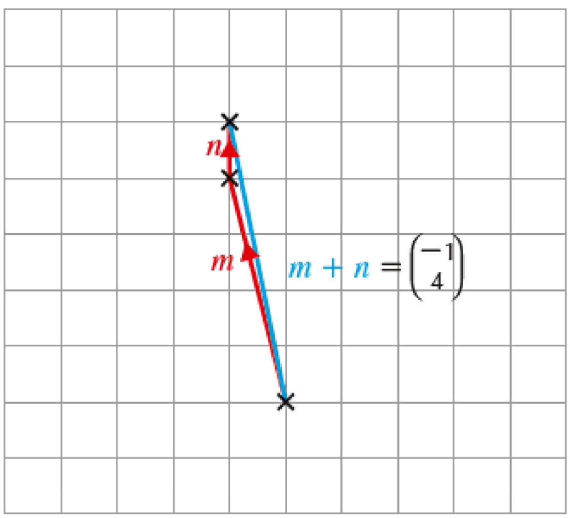


Answers

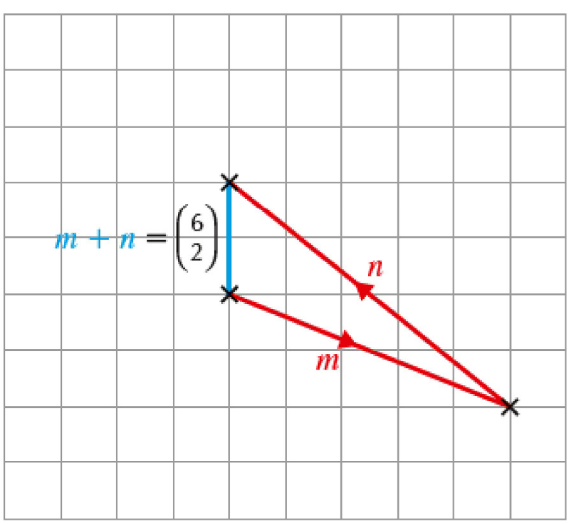
1 a



b



c

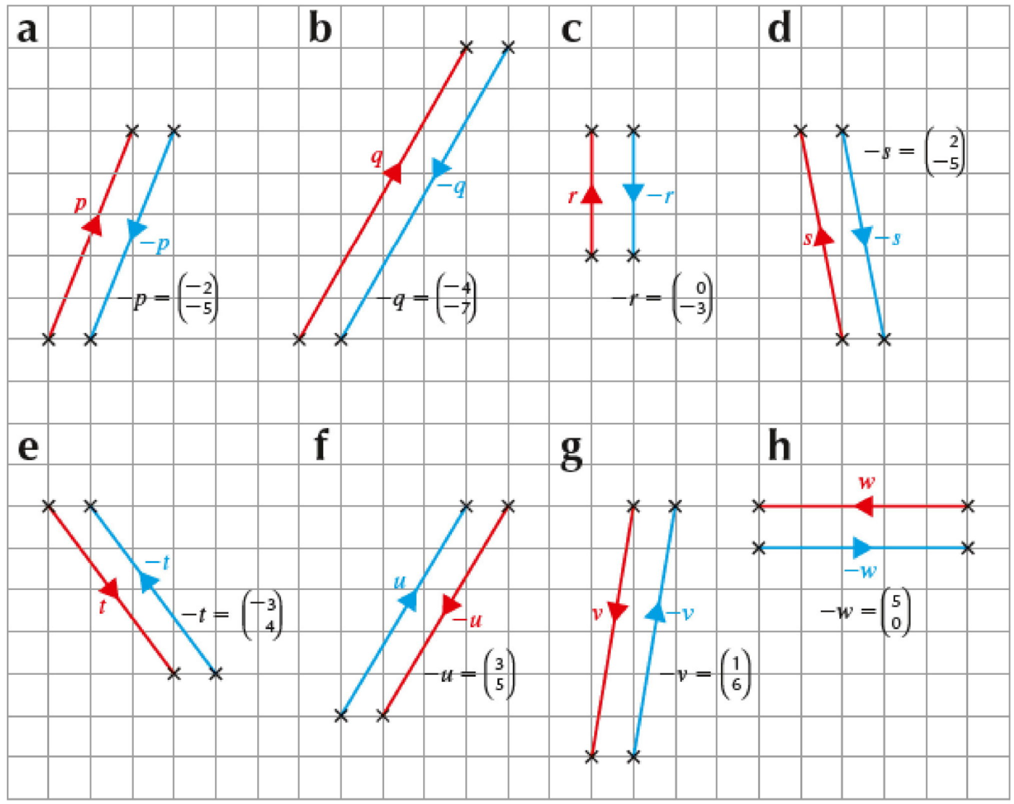


2 a $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

b $\begin{pmatrix} 3 \\ 7 \end{pmatrix}$

c $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$

3



4 a $\begin{pmatrix}-2\\7\end{pmatrix}$

b $\begin{pmatrix}6\\2\end{pmatrix}$

c $\begin{pmatrix}4\\11\end{pmatrix}$

d $\begin{pmatrix}11\\6\end{pmatrix}$

5 a $\begin{pmatrix}6\\15\end{pmatrix}$

b $\begin{pmatrix}2\\5\end{pmatrix}$

c $\begin{pmatrix}-12\\16\end{pmatrix}$

d $\begin{pmatrix}-8\\-10\end{pmatrix}$

e $\begin{pmatrix}-12\\20\end{pmatrix}$

6 a $\begin{pmatrix}3\\8\end{pmatrix}$

b $\begin{pmatrix}12\\20\end{pmatrix}$

c $\begin{pmatrix}5\\-9\end{pmatrix}$

d $\begin{pmatrix}19\\10\end{pmatrix}$

e $\begin{pmatrix}-1\\19\end{pmatrix}$

f $\begin{pmatrix}-12\\8\end{pmatrix}$

g $\begin{pmatrix}1\\8\end{pmatrix}$

h $\begin{pmatrix}-2\\-11\end{pmatrix}$

7 $\begin{pmatrix}22\\4\end{pmatrix}$

8 a $\begin{pmatrix}3\\11\end{pmatrix}$

b $\begin{pmatrix}6\\26\end{pmatrix}$

9 a $\begin{pmatrix}-2\\-8\end{pmatrix}$

b $\begin{pmatrix}-7\\-8\end{pmatrix}$

Answers

1 **a** $\begin{pmatrix} 7 \\ 12 \end{pmatrix}$
b $\begin{pmatrix} 1 \\ 11 \end{pmatrix}$
c $\begin{pmatrix} 9 \\ 3 \\ 4 \end{pmatrix}$
d $\begin{pmatrix} 0 \\ 11 \\ 3 \end{pmatrix}$
e $\begin{pmatrix} -2 \\ 2.5 \\ -1 \end{pmatrix}$
f $\begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix}$

2 **a** $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$
b $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$
c $\begin{pmatrix} -7 \\ -2 \end{pmatrix}$
d $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$
e $\begin{pmatrix} -7 \\ -1 \\ 2 \end{pmatrix}$
f $\begin{pmatrix} 7 \\ 3 \\ -1 \end{pmatrix}$

3 **i** $\begin{pmatrix} 2 \\ 6 \end{pmatrix}, \begin{pmatrix} 5 \\ -2 \end{pmatrix}; \begin{pmatrix} 7 \\ 4 \end{pmatrix}$
ii $\begin{pmatrix} 0 \\ 6 \end{pmatrix}, \begin{pmatrix} 5 \\ -3 \end{pmatrix}; \begin{pmatrix} 5 \\ 3 \end{pmatrix}$
iii $\begin{pmatrix} 2 \\ -6 \end{pmatrix}, \begin{pmatrix} 5 \\ 4 \end{pmatrix}; \begin{pmatrix} 7 \\ -2 \end{pmatrix}$
iv $\begin{pmatrix} -6 \\ 0 \end{pmatrix}, \begin{pmatrix} 8 \\ -4 \end{pmatrix}; \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

4 **a** **i** $\begin{pmatrix} 24 \\ 4 \end{pmatrix}$; **ii** $\begin{pmatrix} -4 \\ 22 \end{pmatrix}$; **iii** $\begin{pmatrix} -17 \\ 0 \end{pmatrix}$; **iv** $\begin{pmatrix} -15 \\ 40 \end{pmatrix}$
b **i** $\begin{pmatrix} 20 \\ 4 \end{pmatrix}$; **ii** $\begin{pmatrix} -10 \\ 22 \end{pmatrix}$; **iii** $\begin{pmatrix} -15 \\ 0 \end{pmatrix}$; **iv** $\begin{pmatrix} -25 \\ 40 \end{pmatrix}$
c **i** $\begin{pmatrix} 24 \\ 4 \end{pmatrix}$; **ii** $\begin{pmatrix} -4 \\ 26 \end{pmatrix}$; **iii** $\begin{pmatrix} -17 \\ -6 \end{pmatrix}$; **iv** $\begin{pmatrix} -15 \\ -50 \end{pmatrix}$
d **i** $\begin{pmatrix} 20 \\ -16 \end{pmatrix}$; **ii** $\begin{pmatrix} -34 \\ 8 \end{pmatrix}$; **iii** $\begin{pmatrix} -18 \\ 12 \end{pmatrix}$; **iv** $\begin{pmatrix} -70 \\ 20 \end{pmatrix}$

5 **a** $a = 6$
b $x = 2,\ y = -6,\ z = 0$
c $a = 4,\ b = 2$