Vectors - Lesson 2

Vectors - Magnitude

LI

- Calculate the magnitude of a 2D or 3D vector.

SC

- Magnitude formula.
- Simplifying surds.

Jan 10-12:11

If $\underline{u} = \overrightarrow{AB} = \begin{pmatrix} x \\ y \end{pmatrix}$, the magnitude of $\underline{u}$ is the scalar :

$$|\underline{u}| = \sqrt{x^2 + y^2}$$

If $\underline{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, the magnitude of $\underline{v}$ is the scalar :

$$|\underline{v}| = \sqrt{x^2 + y^2 + z^2}$$

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Example 1 (Non-calculator)

A vector $\underline{\mathbf{m}}$ has components $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Find its magnitude.

$$|\underline{\mathbf{m}}| = \sqrt{x^2 + y^2}$$

$$|\underline{\mathbf{m}}| = \sqrt{3^2 + 4^2}$$

$$|\underline{\mathbf{m}}| = \sqrt{9 + 16}$$

$$|\underline{\mathbf{m}}| = \sqrt{25}$$

$$\boxed{|\underline{\mathbf{m}}| = 5}$$

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Example 2 (Calculator)

Calculate the magnitude of $3\underline{a} - 4\underline{b}$ if $\underline{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\underline{b} = \begin{pmatrix} -6 \\ -2 \end{pmatrix}$. Write the answer as a surd in simplest form.

$$3\underline{a} - 4\underline{b} = 3\begin{pmatrix} 2 \\ -1 \end{pmatrix} - 4\begin{pmatrix} -6 \\ -2 \end{pmatrix}$$

$$3\underline{a} - 4\underline{b} = \begin{pmatrix} 6 \\ -3 \end{pmatrix} - \begin{pmatrix} -24 \\ -8 \end{pmatrix}$$

$$3\underline{a} - 4\underline{b} = \begin{pmatrix} 30 \\ 5 \end{pmatrix}$$

$$|3\underline{a} - 4\underline{b}| = \sqrt{x^2 + y^2}$$

$$|3\underline{a} - 4\underline{b}| = \sqrt{30^2 + 5^2}$$

$$|3\underline{a} - 4\underline{b}| = \sqrt{900 + 25}$$

$$|3\underline{a} - 4\underline{b}| = \sqrt{925}$$

$$|3\underline{a} - 4\underline{b}| = \sqrt{25}\sqrt{37}$$

$$|3\underline{a} - 4\underline{b}| = 5\sqrt{37}$$

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Example 3 (Non-calculator)

Calculate the magnitude of $\underline{u} + \underline{v}$ if $\underline{u} = \begin{pmatrix} -10 \\ -7 \\ 30 \end{pmatrix}$ and $\underline{v} = \begin{pmatrix} 4 \\ 7 \\ -33 \end{pmatrix}$ as a surd in simplest form.

$$\underline{u} + \underline{v} = \begin{pmatrix} -10 \\ -7 \\ 30 \end{pmatrix} + \begin{pmatrix} 4 \\ 7 \\ -33 \end{pmatrix}$$

$$\underline{u} + \underline{v} = \begin{pmatrix} -6 \\ 0 \\ -3 \end{pmatrix}$$

$$|\underline{u} + \underline{v}| = \sqrt{x^2 + y^2 + z^2}$$

$$|\underline{u} + \underline{v}| = \sqrt{(-6)^2 + 0^2 + (-3)^2}$$

$$|\underline{u} + \underline{v}| = \sqrt{36 + 0 + 9}$$

$$|\underline{u} + \underline{v}| = \sqrt{45}$$

$$|\underline{u} + \underline{v}| = \sqrt{9}\sqrt{5}$$

$$|\underline{u} + \underline{v}| = 3\sqrt{5}$$

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Questions

1 Calculate the magnitude of each vector leaving your answer as a surd in simplest form where appropriate.

a $\mathbf{m} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ **b** $\mathbf{n} = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$ **c** $\mathbf{o} = \begin{pmatrix} 24 \\ -7 \end{pmatrix}$ **d** $\mathbf{p} = \begin{pmatrix} -7 \\ -12 \end{pmatrix}$

e $\mathbf{q} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ **f** $\mathbf{r} = \begin{pmatrix} -2 \\ 7 \\ 11 \end{pmatrix}$ **g** $\mathbf{s} = \begin{pmatrix} 8 \\ -1 \\ 0 \end{pmatrix}$ **h** $\mathbf{t} = \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix}$

2 Calculate the magnitude of the resultant vectors leaving your answer as a surd in simplest form where appropriate.

a $\begin{pmatrix} 4 \\ 5 \end{pmatrix} + \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ **b** $\begin{pmatrix} 2 \\ 9 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ **c** $\begin{pmatrix} 4 \\ -6 \end{pmatrix} + \begin{pmatrix} -5 \\ 7 \end{pmatrix}$

d $\begin{pmatrix} 3 \\ 5 \end{pmatrix} + \begin{pmatrix} -12 \\ -7 \end{pmatrix}$ **e** $\begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ 1 \\ 1 \end{pmatrix}$ **f** $\begin{pmatrix} 2 \\ 7 \\ -1 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \\ 4 \end{pmatrix}$

g $\begin{pmatrix} -2 \\ -1 \\ -6 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1.5 \\ 3 \end{pmatrix}$ **h** $\begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ **i** $\begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

j $\begin{pmatrix} 5 \\ -2 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ **k** $\begin{pmatrix} -1 \\ -5 \end{pmatrix} - \begin{pmatrix} 6 \\ -3 \end{pmatrix}$ **l** $\begin{pmatrix} 5 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$

m $\begin{pmatrix} 5 \\ 1 \\ 4 \end{pmatrix} - \begin{pmatrix} 12 \\ 2 \\ 2 \end{pmatrix}$ **n** $\begin{pmatrix} 21 \\ 10 \\ 8 \end{pmatrix} - \begin{pmatrix} 9 \\ 2 \\ 4 \end{pmatrix} - \begin{pmatrix} 5 \\ 5 \\ 5 \end{pmatrix}$

3 Calculate the length of side AC of triangle ABC where $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$.

4 If vector $\mathbf{v} = \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$ and $\mathbf{u} = \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix}$, calculate the magnitude of:

a $\mathbf{v}$ **b** $\mathbf{u}$ **c** $\mathbf{v} + \mathbf{u}$ **d** $2\mathbf{v}$

e $3\mathbf{u}$ **f** $2\mathbf{v} + 3\mathbf{u}$ **g** $3\mathbf{v} - 2\mathbf{u}$ **h** $5\mathbf{v} - \mathbf{u}$

5 A ship's journey can be represented by the vectors $\mathbf{a} = \begin{pmatrix} 12 \\ 0 \end{pmatrix}$ then $\mathbf{b} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$.

Calculate the displacement of the ship assuming distances are in kilometres.

Feb 4-12:58

Answers

1	a	5	2	a	$\sqrt{193}$	3	$2\sqrt{13}$
	b	$\sqrt{65}$		b	$\sqrt{122}$	4	a $2\sqrt{6}$
	c	25		c	$\sqrt{2}$		b $\sqrt{35}$
	d	$\sqrt{193}$		d	$\sqrt{85}$		c $\sqrt{59}$
	e	$\sqrt{29}$		e	$\sqrt{106}$		d $4\sqrt{6}$
	f	$\sqrt{174}$		f	$\sqrt{130}$		e $3\sqrt{35}$
	g	$\sqrt{65}$		g	$\sqrt{11.25}$		f $\sqrt{411}$
	h	$5\sqrt{2}$		h	$\sqrt{10}$		g $2\sqrt{89}$
				i	$\sqrt{2}$		h $\sqrt{635}$
				j	$\sqrt{5}$	5	13 km
				k	$\sqrt{53}$		
				l	$\sqrt{6}$		
				m	$3\sqrt{6}$		
				n	$\sqrt{59}$		

Dec 18-14:20