## Using Pythagoras' Theorem and Trigonometry in a Circle

## LI

- Use Pythagoras' Theorem and trigonometry to find missing lengths in a circle.
SC
- Property of a chord bisecting a radius.


If a chord and a radius meet at $90^{\circ}$, the chord is bisected

If a radius bisects a chord, then the diameter and chord are at $90^{\circ}$ to each other

## Example 1

Calculate p to 1 d.p..


$$
\begin{aligned}
& \mathrm{p}^{2}=13^{2}+20^{2} \\
& \mathrm{p}^{2}=169+400 \\
& \mathrm{p}^{2}=569 \\
& \mathrm{p}=\sqrt{569} \\
& \mathrm{p}=23.853 \ldots \\
& p=23.9 \mathrm{~cm}(1 \text { d.p. })
\end{aligned}
$$

## Example 2

Calculate $x$ to 1 d.p..

SOH CAH TOA

$$
\begin{aligned}
\sin \theta & =\frac{O}{H} \\
\sin 37^{\circ} & =\frac{w}{8} \\
w & =8 \times \sin 37^{\circ} \\
w & =4.81 \ldots \\
x & =8-w \\
x & =8-4.81 \ldots \\
x & =3.18 \ldots \\
x & =3.2 \mathrm{~cm}(1 \text { d.p. })
\end{aligned}
$$

## Questions

1 Calculate the value of $x$. Give your answers to 2 decimal places where necessary.
a

b

C

d

e

f


2 Find the value of $x$.

b

C

d


$$
\begin{array}{rlrl} 
& & \text { Answers } \\
\mathbf{1} & \mathbf{a} & x & =19.94 \mathrm{~cm} \\
& \mathbf{b} & x & =5 \mathrm{~cm} \\
& \text { c } & x & =1 \mathrm{~cm} \\
& \text { d } & x & =19.93 \mathrm{~cm} \\
& \text { e } & x & =5.37 \mathrm{~cm} \\
& \mathbf{f} & x & =13.42 \mathrm{~m} \\
\mathbf{2} & \text { a } & x & =6.53 \mathrm{~cm} \\
& \mathbf{b} & x & =6.58 \mathrm{~m} \\
& \text { c } & x & =13.16 \mathrm{~cm} \\
& \text { d } & x & =23.05 \mathrm{~m}
\end{array}
$$

## Questions

1 A cylindrical water tank with diameter 140 cm is partly filled with water. The depth of the water is 30 cm . Calculate the width of $A B$.


2 A set of park gates form part of a circle with radius 4 m . The width of the gates is 6 m . Calculate the height $h$ of the gates.


3 A radar on ship A covers a radius of 60 km . Ship $B$ travels on a straight course through the points $P$ and $Q$. The length of $P Q$ is 100 km . Ship $A$ has a warning alarm which is activated if another ship passes within 30 km of it. Will the alarm be activated as ship $B$ passes?


5 The cross-section of a lampshade is part of a circle with radius 15 cm . The width of the top is 9 cm and the height is 22 cm . What is the width $w$ of the bottom?


4 A circular table with diameter 2 m has foldable sides as shown in the diagram. When the sides are folded, the table sides are 1.8 m in length. What is its width $w$ ?


6 A fruit bowl is part of the cross-section of a circle with radius 20 cm . The width of the top is 36 cm and the width of the base is 12 cm . Calculate the height $h$ of the bowl.


7 A doorway is made from a rectangle and part of a circle with radius 70 cm .
Calculate its width $w$.


9 A badge for employees of Beta Corp is shown. The badge is made from an equilateral triangle, with sides of length 3 cm , which touches the circumference of the circle at $P, Q$ and $R$.
a Calculate the size of angle $O P Q$.
b Calculate the length of the radius of the circle $O P$.

10 A factory security light positioned at point $O$ covers a distance of 40 m in all directions. The length of the factory wall $A B$ is 60 m . Calculate the size of angle $A O B$.

A drinking trough is part of the crosssection of a circle with diameter 70 cm . The width of the top is 46 cm . What is the depth $d$ of the water when the trough is full?


## Answers

$1 A B=114.9 \mathrm{~cm}$
$2 h=1.35 \mathrm{~m}$
3 Ship B passes within 33.17 km , so alarm will not be activated.
$4 w=0.872 \mathrm{~m}$
$5 \quad w=25.76 \mathrm{~cm}$
$6 h=27.80 \mathrm{~cm}$
$7 w=114.89 \mathrm{~cm}$
$8 d=8.62 \mathrm{~cm}$
9 a $O P Q=30^{\circ}$
b $O P=1.732 \mathrm{~cm}$
$10 A O B=97.18^{\circ}$

