Transposition of Formulae - Lesson 5

## Transposition of Formulae <br> (All Types)

## LI

- Transpose formulae.

SC
-,,$+- x$ and $\div$.

- Rearranging equations with fractions
- Powers and roots.


## Transpose means to change position

In practice, 'transpose' (aka change the subject) means 'solve'.

Transposing a formula means you have less formulae to remember

## Example 1

Transpose $x+b=c$ for $x$.

$$
\begin{aligned}
x+b & =c \\
-b & -b
\end{aligned}
$$

## Example 2

Transpose $y=5 x$ for $x$.

$$
\begin{gathered}
y=5 x \\
5 x=y \\
\div 5 \quad \div 5 \\
x=\frac{y}{5}
\end{gathered}
$$

## Example 3

Transpose $C=2 \pi r$ for $r$.

$$
\begin{aligned}
C & =2 \pi r \\
2 \pi r & =C \\
\div 2 \pi & \div 2 \pi \\
r & =\frac{C}{2 \pi}
\end{aligned}
$$

## Example 4

Transpose $y=3 x-7$ for $x$.

$$
\begin{aligned}
y & =3 x-7 \\
3 x-7 & =y \\
+7 & +7 \\
3 x & =y+7 \\
\div 3 & \div 3
\end{aligned}
$$

## Example 5

Transpose $y=2-6 x$ for $x$.

$$
\begin{aligned}
& y=2-6 x \\
& +6 x \quad+6 x \\
& y+6 x=2 \\
& -y \quad-y
\end{aligned}
$$

$$
\begin{aligned}
& x=\frac{2-y}{6}
\end{aligned}
$$

## Example 6

Transpose $y=\frac{x}{3}$ for $x$.

$$
\begin{gathered}
y=\frac{x}{3} \\
\frac{x}{3}=y \\
\times 3 \quad x^{3} \\
x=3 y
\end{gathered}
$$

## Example 7

Transpose $y=\frac{5}{11} \times$ for $x$.

$$
\begin{aligned}
& y=\frac{5}{11} x \\
& \frac{5}{11} x=y \\
& x+11 \quad x 11 \\
& 5 x=11 y \\
& \div 5 \quad \div 5
\end{aligned} \quad \begin{aligned}
& \\
& x=\frac{11}{5} y
\end{aligned}
$$

## Example 8

Transpose $f=\frac{4-p}{9}$ for $p$.

$$
\begin{aligned}
& f=\frac{4-p}{9} \\
& \times 9 \quad \times 9 \\
& f g=4-p \\
& +p \quad+p \\
& f g+p=4 \\
& -f g \quad-f g \\
& p=4-f g
\end{aligned}
$$

## Example 9

Transpose $Q=\frac{k}{2}-\frac{p}{3}$ for $k$.

$$
Q=\frac{\mathrm{k}}{2}-\frac{\mathrm{p}}{3}
$$

$$
Q=\frac{3 k-2 p}{6}
$$

$$
\begin{gathered}
6 Q=3 k-2 p \\
+2 p \quad+2 p
\end{gathered}
$$

$$
6 Q+2 p=3 k
$$

$$
\underset{\div 3}{3 k}=\underset{\div 3}{6}+2 p
$$

$$
k=\frac{6 Q+2 p}{3}
$$

## Example 10

Transpose $y=\frac{7}{w}$ for $w$.

$$
\begin{gathered}
y=\frac{7}{w} \\
x w \quad x w \\
w y=7 \\
\div y \quad \div y \\
w=\frac{7}{y}
\end{gathered}
$$

## Example 11

Transpose $y=4+\frac{3}{x}$ for $x$.

$$
\begin{aligned}
y & =4+\frac{3}{x} \\
-4 & -4 \\
y-4 & =\frac{3}{x} \\
x x \quad & x x \\
x(y-4) & =3 \\
\div(y-4) & \div(y-4) \\
x & =\frac{3}{y-4}
\end{aligned}
$$

## Example 12

Transpose $T=5-\frac{L}{G}$ for $G$.

$$
\begin{aligned}
& T=5-\frac{L}{G} \\
&-5-5 \\
& T-5=-\frac{L}{G} \\
& \times(-1) \times(-1) \\
& 5-T=\frac{L}{G} \\
& \times G \times G \\
& G(5-T)=\begin{array}{l}
L \\
\div(5-T)
\end{array} \\
& \div(5-T) \\
& G=\frac{L}{5-T}
\end{aligned}
$$

## Example 13

Transpose $V=\frac{m}{A-4}$ for $A$.

$$
\begin{array}{r}
V=\frac{m}{A-4} \\
\times(A-4) \quad \times(A-4)
\end{array}
$$

$$
V(A-4)=m
$$

$$
\div V \quad \div V
$$

$$
\begin{aligned}
A-4= & \frac{m}{V} \\
+4 & +4
\end{aligned}
$$

$$
A=\frac{m}{V}+4
$$

## Example 14

Transpose $T=\sqrt{w}$ for $w$.

$$
T=\sqrt{w}
$$

$$
\sqrt{\mathbf{w}}=\mathrm{T}
$$

()$^{2} \quad()^{2}$

$$
w=T^{2}
$$

## Example 15

Transpose $y=x^{2}$ for $x$.

$$
\begin{gathered}
y=x^{2} \\
x^{2}=y \\
\sqrt{ } \quad \sqrt{-} \\
x=\sqrt{y}
\end{gathered}
$$

## Example 16

Transpose $A=\pi r^{2}$ for $r$.

$$
\begin{aligned}
A & =\pi r^{2} \\
\pi r^{2} & =A \\
\div \pi \quad & \div \pi
\end{aligned}
$$

$$
r^{2}=\frac{A}{\pi}
$$



$$
r=\sqrt{\frac{A}{\pi}}
$$

## Example 17

Transpose $S=\frac{\sqrt{W H}}{20}$ for $H$.

$$
S=\frac{\sqrt{W H}}{20}
$$

$$
\frac{\sqrt{W H}}{\substack{20 \\ \times 20}}=S
$$

$$
\sqrt{W H}=20 \mathrm{~s}
$$

$$
()^{2} \quad()^{2}
$$

$$
W H=400 S^{2}
$$

$$
\div W \quad \div W
$$

$$
H=\frac{400 \mathrm{~S}^{2}}{\mathrm{~W}}
$$

## Questions

1 Make $x$ the subject of the following formulae.
a $\quad x+2=p$
b $\quad x+t=w$
C $x-5=q$
d $10+x=a$
e $f-x=4$
f $m n-x=k$

2 Make $x$ the subject of the following formulae.
a $y=2 x$
b $y=-4 x$
c $y=3 x+2$
d $y+5 x=4$
e $y-7 x=3$
f $y=-5 x+1$
g $a+b x=c$
h $1-p q r x=m$
i $m+3 n x=7 p$

3 For each of the following maths and science formulae, change the subject of the formula to the letter in brackets.
a $d=v t \quad(t)$
b $\quad W=m g \quad(m)$
c $\quad V=I R$
(R)
d $V=l b h \quad(b)$
e $F=m a$
(a)
f $E=m g h \quad(m)$
g $Q=I t \quad(I)$
h $\quad l=N h f$
(h)
i $v=u+a t$
(t)

## Answers

| $\begin{array}{rlrl} 1 & \mathbf{a} & x & =p-2 \\ \mathbf{b} & x & =w-t \\ \mathbf{c} & x & =q+5 \\ \mathbf{d} & x & =a-10 \\ & \mathbf{e} & x & =f-4 \\ & \mathbf{f} & x & =m n-k \end{array}$ | $\begin{array}{cll} \hline 2 & \mathbf{a} & x=\frac{y}{2} \\ & \mathbf{b} & x=-\frac{y}{4} \\ & \mathbf{c} & x=\frac{y-2}{3} \\ & \mathbf{d} & x=\frac{4-y}{5} \\ & \mathbf{e} & x=\frac{y-3}{7} \\ & \mathbf{f} & x=\frac{1-y}{5} \\ & \mathbf{g} & x=\frac{c-a}{b} \\ & \text { h } & x=\frac{1-m}{p r} \\ & \mathbf{i} & x=\frac{7 p-m}{3 n} \end{array}$ | $\begin{array}{cll} \hline 3 & \mathbf{a} & t=\frac{d}{v} \\ & \mathbf{b} & m=\frac{w}{g} \\ & \mathbf{c} & R=\frac{V}{l} \\ & \mathbf{d} & b=\frac{V}{l h} \\ & \mathbf{e} & a=\frac{F}{m} \\ & \mathbf{f} & m=\frac{E}{g h} \\ \mathbf{g} & I=\frac{Q}{t} \\ & \mathbf{h} & h=\frac{l}{N f} \\ & \mathbf{i} & t=\frac{v-u}{a} \end{array}$ |
| :---: | :---: | :---: |

## Questions

Make $x$ the subject of the following formulae.
a $\quad y=\frac{x}{4}$
b $y=\frac{x}{5}$
c $\quad y=\frac{3 x}{7}$
d $y=\frac{2 x}{3}$
e $y=\frac{1}{6} x$
f $y=\frac{4}{5} x$
g $\quad y=\frac{x+3}{8}$
h $y=\frac{x+1}{4}$
i $y=\frac{x-3}{2}$
j $y=\frac{3 x+5}{4}$
k $y=\frac{5 x-1}{3}$
| $y=\frac{2-3 x}{5}$

## Answers

$$
\begin{array}{ll}
\mathbf{a} & x=4 y \\
\mathbf{b} & x=5 y \\
\mathbf{c} & x=\frac{7 y}{3} \\
\mathbf{d} & x=\frac{3 y}{2} \\
\mathbf{e} & x=6 y \\
\mathbf{f} & x=\frac{5 y}{4} \\
\mathbf{g} & x=8 y-3 \\
\mathbf{h} & x=4 y-1 \\
\mathbf{i} & x=2 y+3 \\
\mathbf{j} & x=\frac{4 y-5}{3} \\
\mathbf{k} & x=\frac{3 y+1}{5} \\
\mathbf{l} & x=\frac{2-5 y}{3}
\end{array}
$$

## Questions

Make $x$ the subject of the following formulae.
a $y=\frac{1}{x}$
b $y=\frac{8}{x}$
c $y=\frac{3}{x+5}$
d $y=\frac{2}{x-1}$
e $y=\frac{3}{2 x+5}$
f $y=\frac{7}{1-6 x}$

## Answers

a $\quad x=\frac{1}{y}$
b $\quad x=\frac{8}{y}$
c $\quad x=\frac{3}{y}-5$
d $\quad x=\frac{2}{y}+1$
e $\quad x=\frac{3}{2 y}-\frac{5}{2}$
f $\quad x=\frac{1}{6}-\frac{7}{6 y}$

## Questions

Make $x$ the subject of the following formulae.
a $y=x^{2}$
b $y=4 x^{2}$
c $y=\frac{2}{3} x^{2}$
d $\quad y=\frac{3}{5} x^{2}$
e $\quad y=\sqrt{x}$
f $y=3 \sqrt{x}$
g $y=\frac{1}{2} \sqrt{x}$
h $y=\sqrt{x+5}$
i $\quad y=5 x^{2}+2$
j $y=6-x^{2}$
k $y=\frac{3}{5} x^{2}+4$
I $y=2 \sqrt{x-3}$

## Answers

$$
\begin{array}{llrl}
\mathbf{a} & & x=\sqrt{y} \\
\text { b } & & x=\frac{\sqrt{y}}{2} \\
\text { c } & x & =\sqrt{\frac{3 y}{2}} \\
\text { d } & x & =\sqrt{\frac{5 y}{3}} \\
\text { e } & x & =y^{2} \\
\text { f } & & x=\frac{y^{2}}{9} \\
\text { g } & x & =4 y^{2} \\
\mathbf{h} & x & =y^{2}-5 \\
\mathbf{i} & x & =\sqrt{\frac{y-2}{5}} \\
\text { j } & & x=\sqrt{6-y} \\
\text { k } & & x=\sqrt{\frac{5 y-20}{3}} \\
\text { l } & & x=\frac{y^{2}}{4}+3
\end{array}
$$

