

Symmetry Axis and Turning Point - Lesson 1

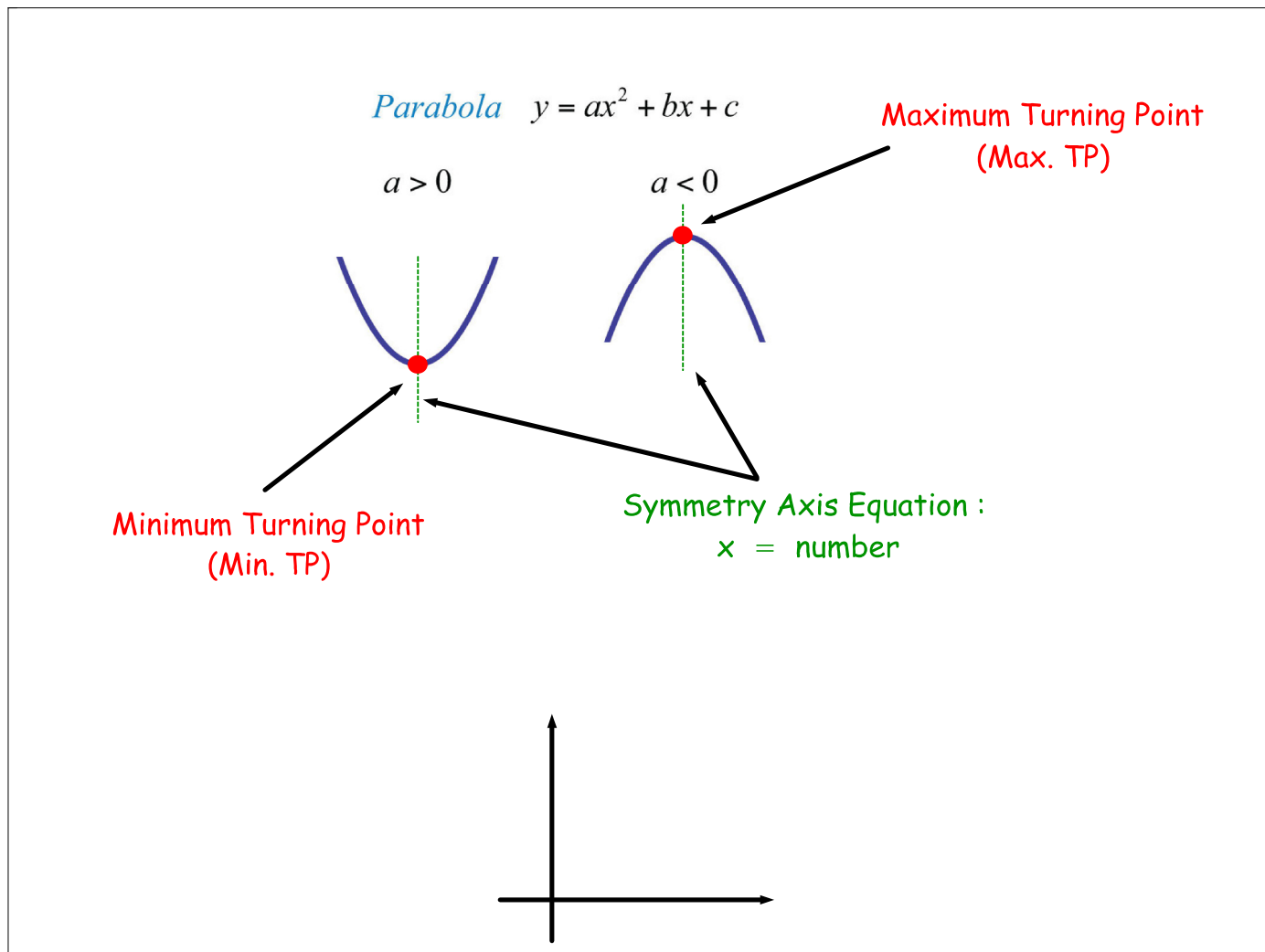
Finding the Symmetry Axis Equation and Maximum
or Minimum Turning Point Coordinates Using
Completing the Square

LI

- Find the symmetry axis equation and TP of any quadratic function.

SC

- Completing the Square.



Any quadratic expression $a x^2 + b x + c$ can be written in the form $p (x + q)^2 + r$. This is called **Completing the Square**.

Completing the square is used to find (for a parabola) the :

- Equation of the symmetry axis : $x = -q$.
- Maximum (or minimum) turning point coordinates : $(-q, r)$.

If $p > 0$, then Min. TP.

If $p < 0$, then Max. TP.

Example 1

Find the equation of the symmetry axis and the coordinates of the turning point for the quadratic $x^2 + 6x + 13$.

$$\begin{aligned} & x^2 + 6x + 13 \\ & \text{half the x-coefficient} \swarrow \\ & = (x + 3)^2 - 9 + 13 \\ & \quad \quad \quad \swarrow \\ & \quad \quad \quad \text{square and subtract it to} \\ & \quad \quad \quad \text{keep expression the} \\ & \quad \quad \quad \text{same} \\ & \quad \quad \quad \text{(always subtract here)} \end{aligned}$$

$$= \underline{(x + 3)^2 + 4}$$

Symmetry axis equation : $x = -3$; TP : $(-3, 4)$

Example 2

Find the equation of the symmetry axis and the coordinates of the turning point for the quadratic $x^2 - 8x + 1$.

$$\begin{aligned} & x^2 - 8x + 1 \\ &= (x - 4)^2 - 16 + 1 \\ &= \underline{(x - 4)^2 - 15} \end{aligned}$$

Symmetry axis equation : $x = 4$; TP : $(4, -15)$

Example 3

Find the equation of the symmetry axis and the coordinates of the turning point for the quadratic $x^2 + 3x + 5$.

$$\begin{aligned}
 & x^2 + 3x + 5 \\
 & \text{half the x-coefficient} \swarrow \\
 & = (x + 3/2)^2 - 9/4 + 5 \\
 & \quad \quad \quad \searrow \text{square and subtract it to keep expression the same (always subtract here)}
 \end{aligned}$$

$$= (x + 3/2)^2 - 9/4 + 20/4$$

$$= \underline{(x + 3/2)^2 + 11/4}$$

Symmetry axis equation : $x = -3/2$; TP : $(-3/2, 11/4)$

Example 4

Find the equation of the symmetry axis and the coordinates of the turning point for the quadratic $x^2 - 7x - 2$.

$$\begin{aligned}
 & x^2 - 7x - 2 \\
 &= (x - 7/2)^2 - 49/4 - 2 \\
 &= (x - 7/2)^2 - 49/4 - 8/4 \\
 &= \underline{(x - 7/2)^2 - 57/4}
 \end{aligned}$$

Symmetry axis equation : $x = 7/2$; TP : $(7/2, -57/4)$

Questions

Find the symmetry axis equation and the TP coordinates of :

1) $u^2 - 8u + 18$	11) $c^2 - 3c + 17$
2) $g^2 + 10g + 24$	12) $z^2 + 5z + 34$
3) $z^2 - 4z - 1$	13) $i^2 + 9i + 32$
4) $p^2 - 6p + 14$	14) $s^2 - 7s + 19$
5) $e^2 - 12e + 8$	15) $k^2 + 11k + 122$
6) $a^2 - 6a - 1$	16) $x^2 + 3x - 55$
7) $u^2 + 6u + 20$	17) $f^2 - 13f + 65$
8) $h^2 - 10h - 4$	18) $b^2 + 7b + 77$
9) $i^2 + 12i - 7$	19) $k^2 - 15k + 87$
10) $s^2 - 8s + 4$	20) $a^2 + 21a - 4$

Answers

- | | |
|------------------------|----------------------------------|
| 1) $u = 4; (4, 2)$ | 11) $c = 3/2; (3/2, 59/4)$ |
| 2) $g = -5; (-5, -1)$ | 12) $z = -5/2; (-5/2, 111/4)$ |
| 3) $z = 2; (2, -5)$ | 13) $i = -9/2; (-9/2, 47/4)$ |
| 4) $p = 3; (3, 5)$ | 14) $s = 7/2; (7/2, 27/4)$ |
| 5) $e = 6; (6, -28)$ | 15) $k = -11/2; (-11/2, 367/4)$ |
| 6) $a = 3; (3, -10)$ | 16) $x = -3/2; (-3/2, -229/4)$ |
| 7) $u = -3; (-3, 11)$ | 17) $f = 13/2; (13/2, 91/4)$ |
| 8) $h = 5; (5, -29)$ | 18) $b = -7/2; (-7/2, 259/4)$ |
| 9) $i = -6; (-6, -43)$ | 19) $k = 15/2; (15/2, 123/4)$ |
| 10) $s = 4; (4, -12)$ | 20) $a = -21/2; (-21/2, -457/4)$ |