Solving Trigonometric Equations - Lesson 1

Solving Trigonometric Equations
(No Rearranging : 0° to 360°)

LI

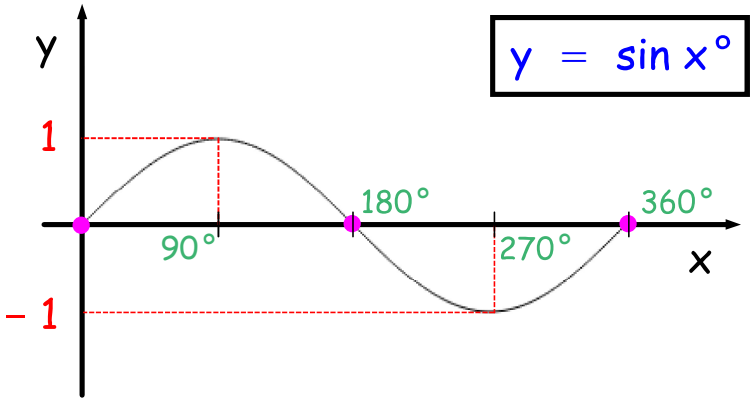
- Solve trigonometric equations of the form :

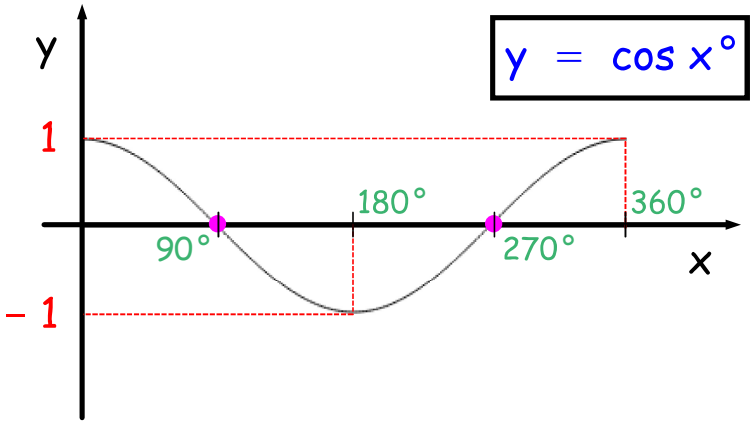
$$\sin x^\circ = a$$
$$\cos x^\circ = a \qquad \text{between } 0^\circ \text{ and } 360^\circ$$
$$\tan x^\circ = a$$

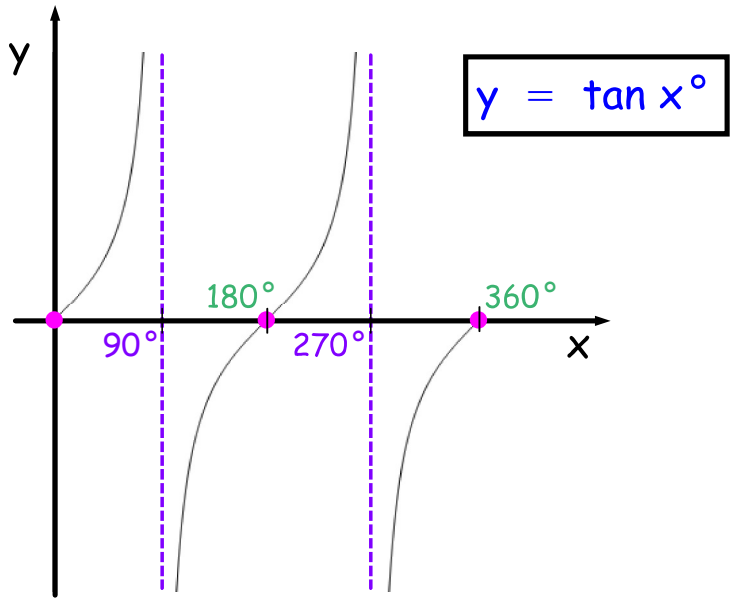
SC

- Graphs of $y = \sin x^\circ$, $y = \cos x^\circ$ and $y = \tan x^\circ$ between 0° and 360°.
- Use the ASTC Diagram .

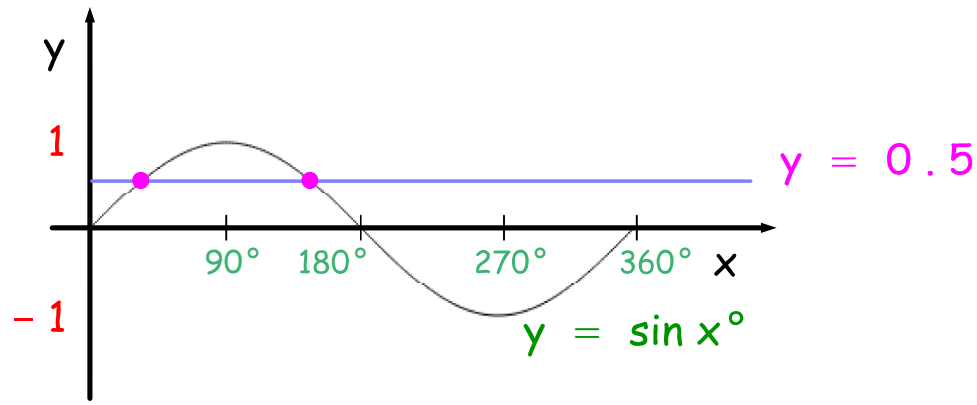
Trigonometric Graphs Between 0° and 360°







What does it mean to solve the equation $\sin x° = 0.5$?

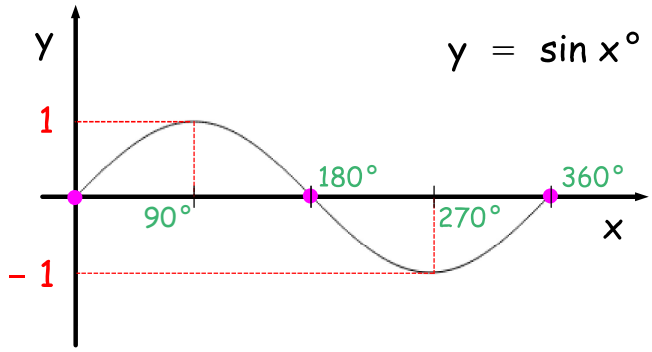


It means, where (i.e. for which x - value(s)) does the graph of $y = \sin x°$ cross (aka intersect or meet) the graph of $y = 0.5$?

The ASTC Diagram

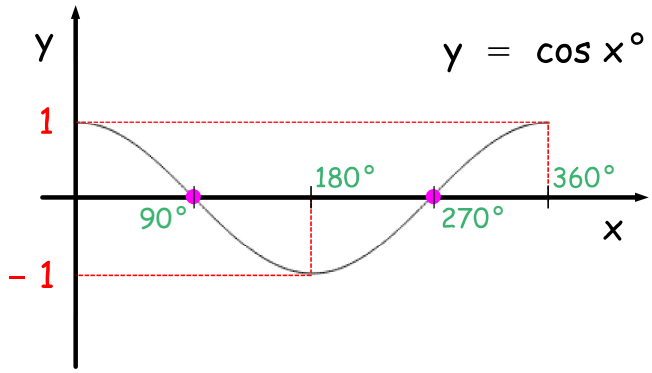
. . . tells us the range of x - values for which sine, cosine and tangent (i.e. their y - values) are positive or negative and helps us solve for x.

We can figure this out from the graphs



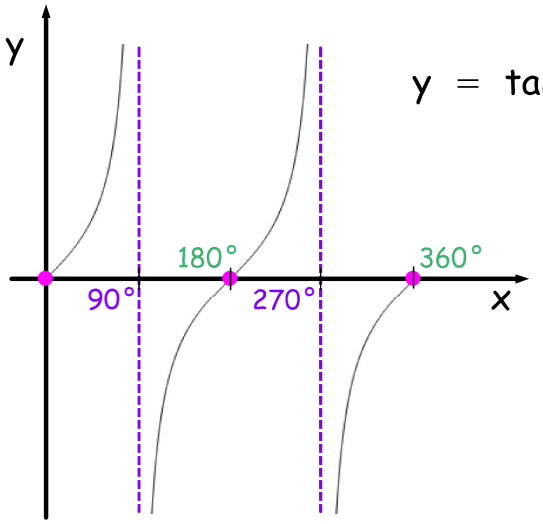
Sine + ve when 0° < x < 180°.

Sine – ve when 180° < x < 360°.



Cosine + ve when 0° < x < 90° and 270° < x < 360°.

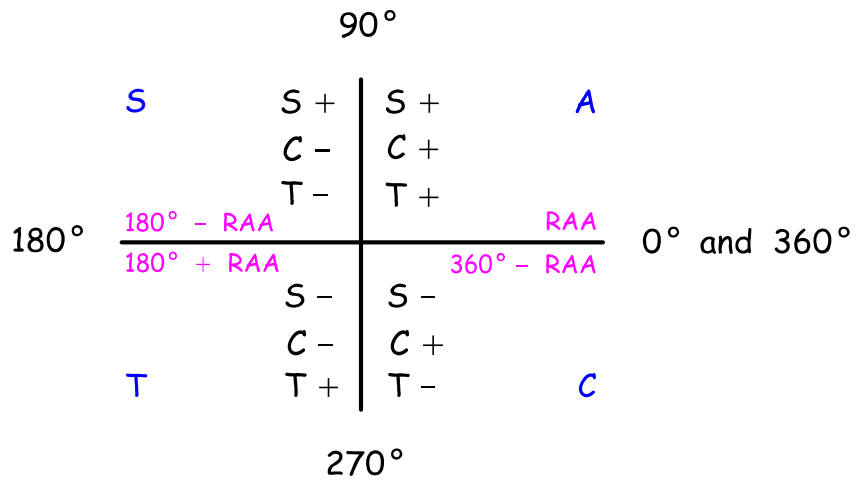
Cosine – ve when 90° < x < 270°.



Tangent + ve when 0° < x < 90° and 180° < x < 270°.

Tangent – ve when 90° < x < 180° and 270° < x < 360°.

The information regarding positivity and negativity of sine, cosine and tangent in the boxes to the right of the graphs on the previous page can be summarised in the following diagram, variously known as the 'ASTC diagram', the 'CAST diagram' etc. :



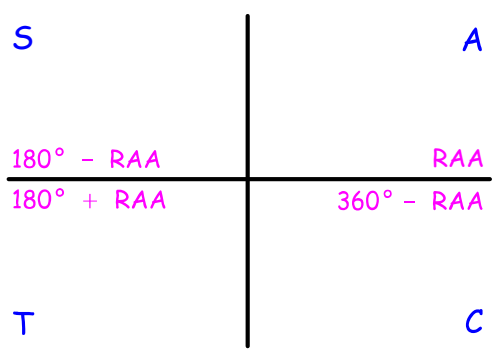
Some things to note :

- The blue letters tell us which of S(ine), C(osine), T(angent) or A(ll of them) only are positive in the given quadrant.

- The pink calculations show us how to work out solutions for x if we can find a special acute angle, called the Related Acute Angle (RAA); this comes from the symmetry of the graphs.

- Angles measured anti-clockwise from $0°$ are positive, whilst those measured clockwise from $0°$ are negative (so, for example, $\sin(-17°)$ will be negative).

In practice, a much simpler form of the diagram is used :



Strategy for Solving Trigonometric Equations

STEP 1

- Sketch graphs (to see how many times the graphs cross).

STEP 2

- Calculate Related Acute Angle (use calc.).

STEP 3

- Look to main equation and see if sin, cos or tan is $+$ ve or $-$ ve.

STEP 4

- Use ASTC Diagram to get answers.

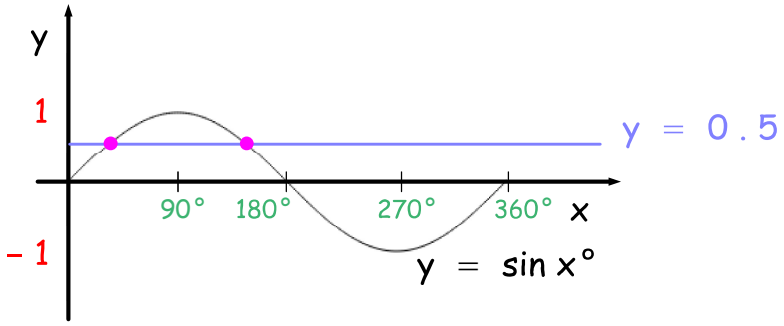
Once answers for x are obtained, it should be checked that all answers are in the correct range of x - values stipulated in the question and that there are no other answers; this is easily done by looking at the graphs in the first step.

Example 1

Solve $\sin x^\circ = 0.5$ for $0^\circ \le x^\circ \le 360^\circ$.

main equation

STEP 1



(Expecting 2 answers)

STEP 2

$$RAA = \sin^{-1}(0.5)$$

$$\Rightarrow \quad \underline{RAA = 30^\circ}$$

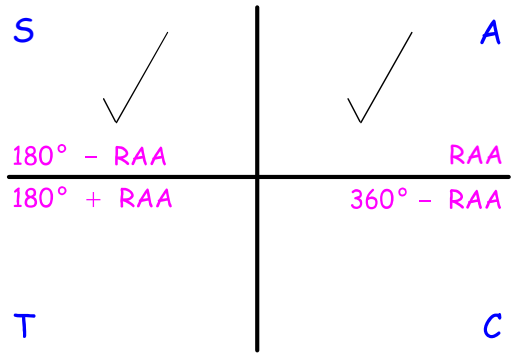
STEP 3

main equation

$$\sin x^\circ = 0.5$$

$\boxed{\text{sin is + ve}}$

STEP 4



$$x^\circ = RAA, 180^\circ - RAA$$

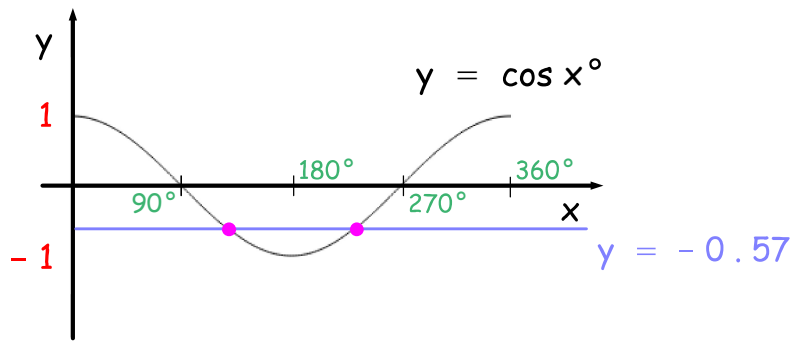
$$\therefore \quad x^\circ = 30^\circ, 180^\circ - 30^\circ$$

$$\Rightarrow \quad \boxed{x^\circ = 30^\circ, 150^\circ}$$

Example 2

Solve $\overset{\text{main equation}}{\cos x^\circ = -0.57}$ for $0^\circ \le x^\circ \le 360^\circ$ (1 d.p.).

STEP 1



(Expecting 2 answers)

STEP 2

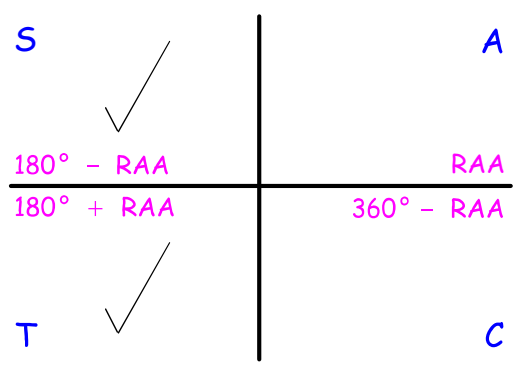
$$RAA = \cos^{-1}(0.57)$$

$$\Rightarrow \quad \underline{RAA = 55.24\ldots^\circ}$$

STEP 3

$$\overset{\text{main equation}}{\cos x^\circ = -0.57} \qquad \boxed{\cos \text{ is } -\text{ve}}$$

STEP 4



$$x^\circ = 180^\circ - RAA,\ 180^\circ + RAA$$

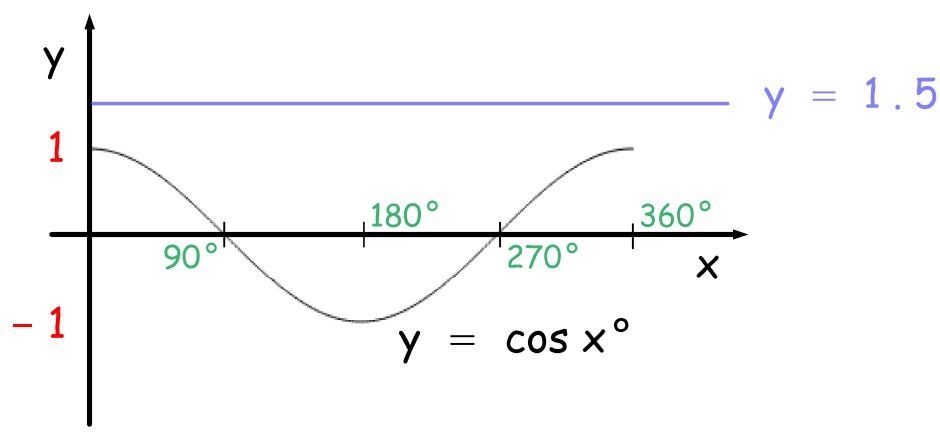
$$\therefore \quad x^\circ = 180^\circ - 55.24^\circ,\ 180^\circ + 55.24^\circ$$

$$\Rightarrow \quad \boxed{x^\circ = 124.8^\circ,\ 235.2^\circ}$$

Example 3

Solve $\cos x° = 1.5$ for $0° \leq x° \leq 360°$.

STEP 1



(Expecting 0 answers)

Graphs do not meet.

$\therefore$ $\boxed{\text{No solutions}}$

Questions

Solve these equations for for $0° \leq x° \leq 360°$ (1 d.p.) :

1) $\sin x° = -0.5$
2) $\cos x° = 0.5$
3) $\tan x° = 17$
4) $\cos x° = -0.6$
5) $\sin x° = 0.8$
6) $\cos x° = 0.1$
7) $\sin x° = -0.2$
8) $\cos x° = 0.4$
9) $\sin x° = -0.35$
10) $\cos x° = 0.66$
11) $\sin x° = 0.23$
12) $\tan x° = -0.88$
13) $\sin x° = 1$
14) $\cos x° = 1$
15) $\sin x° = -1$
16) $\cos x° = -2$

Answers

1) $\sin x° = -0.5$ 210°, 330°

2) $\cos x° = 0.5$ 60°, 300°

3) $\tan x° = 17$ 86.6°, 266.6°

4) $\cos x° = -0.6$ 126.9°, 233.1°

5) $\sin x° = 0.8$ 53.1°, 126.9°

6) $\cos x° = 0.1$ 84.3°, 275.7°

7) $\sin x° = -0.2$ 191.5°, 348.5°

8) $\cos x° = 0.4$ 66.4°, 293.6°

9) $\sin x° = -0.35$ 200.5°, 339.5°

10) $\cos x° = 0.66$ 48.7°, 311.3°

11) $\sin x° = 0.23$ 13.3°, 166.7°

12) $\tan x° = -0.88$ 138.7°, 318.7°

13) $\sin x° = 1$ 90°

14) $\cos x° = 1$ 0°, 360°

15) $\sin x° = -1$ 270°

16) $\cos x° = -2$ No solutions

CfE N5 Maths

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Questions

1 Solve the following equations for $0 \leqslant x \leqslant 360$, giving your answers to 1 decimal place.

a $\sin x° = 0.3$ **b** $\cos x° = 0.56$ **c** $\tan x° = 3$ **d** $\sin x° = 0.645$

e $\tan x° = 4.5$ **f** $\cos x° = 0.892$ **g** $\tan x° = 0.23$ **h** $\cos x° = 0.54$

2 Solve the following equations for $0 \leqslant x \leqslant 360$, giving your answers to 1 decimal place.

a $\cos x° = -0.34$ **b** $\tan x° = -4$ **c** $\sin x° = -0.75$ **d** $\tan x° = -6.5$

e $\sin x° = -0.456$ **f** $\cos x° = -0.23$ **g** $\sin x° = -0.324$ **h** $\tan x° = -1.2$

3 Solve the following equations for $0 \leqslant p \leqslant 360$. Round your answers to 2 decimal places.

a $\sin p° = 0.5$ **b** $\cos p° = -0.2$ **c** $\tan p° = -3.4$ **d** $\cos p° = 0.443$

e $\sin p° = -0.17$ **f** $\tan p° = 12$ **g** $\sin p° = 0.9$ **h** $\cos p° = -0.205$

Answers

1	a	$\theta = 17.5^\circ$, $\theta = 162.5^\circ$	2	a	$x = 109.9^\circ$, $x = 250.1^\circ$	3	a	$p = 30.00^\circ$, $p = 150.00^\circ$
	b	$\theta = 55.9^\circ$, $\theta = 304.1^\circ$		b	$x = 104.0^\circ$, $x = 284.0^\circ$		b	$p = 101.54^\circ$, $p = 258.46^\circ$
	c	$\theta = 71.6^\circ$, $\theta = 251.6^\circ$		c	$x = 228.6^\circ$, $x = 311.4^\circ$		c	$p = 106.39^\circ$, $p = 286.39^\circ$
	d	$\theta = 40.2^\circ$, $\theta = 139.8^\circ$		d	$x = 98.7^\circ$, $x = 278.7^\circ$		d	$p = 63.70^\circ$, $p = 296.30^\circ$
	e	$\theta = 77.5^\circ$, $\theta = 257.5^\circ$		e	$x = 207.1^\circ$, $x = 332.9^\circ$		e	$p = 189.79^\circ$, $p = 350.21^\circ$
	f	$\theta = 26.9^\circ$, $\theta = 333.1^\circ$		f	$x = 103.3^\circ$, $x = 256.7^\circ$		f	$p = 85.24^\circ$, $p = 265.24^\circ$
	g	$\theta = 13.0^\circ$, $\theta = 193.0^\circ$		g	$x = 198.9^\circ$, $x = 341.1^\circ$		g	$p = 64.16^\circ$, $p = 115.84^\circ$
	h	$\theta = 57.3^\circ$, $\theta = 302.7^\circ$		h	$x = 129.8^\circ$, $x = 309.8^\circ$		h	$p = 101.83^\circ$, $p = 258.17^\circ$