Similarity - Lesson 1

Similar Shapes (2D) and Similar Triangles

LI

- Work out missing lengths in complicated diagrams involving 2D shapes, especially similar triangles.

SC

- Work out Scale Factor.
- Draw out the 2 similar triangles separately.

Intuitively, two shapes are similar if they have :

- Same shape
- Different size

A scale factor (SF) describes how big (or small) one shape is compared to another

Enlargement - SF bigger than 1

Reduction - SF smaller than 1

The scale factor (aka length scale factor - LSF) is often written using the letter k

2 shapes are congruent if they have :

- Same shape
- Same size

$k = 1$

Two 2D shapes are similar if the ratios of all corresponding sides are equal

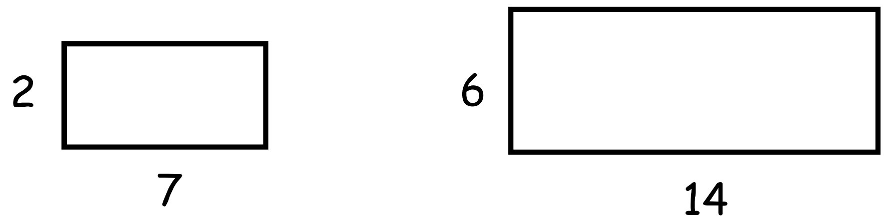
Given two similar shapes, identify which shape (smaller or bigger) has the missing side and work out k. Then the missing length (L_{NEW}) is worked out using the corresponding length on the other shape (L_{OLD}) using the formula :

$$L_{NEW} = k \times L_{OLD}$$

Aug 23-07:47

Example 1

Are these two rectangles similar ? Justify your answer.

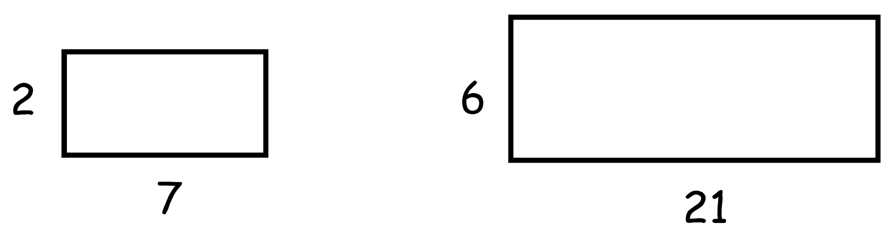


$$\frac{6}{2} = 3 \qquad \frac{14}{7} = 2$$

As corresponding ratios are unequal ($2 \neq 3$), not similar

Example 2

Are these two rectangles similar ? Justify your answer.



$$\frac{6}{2} = 3 \qquad \frac{21}{7} = 3$$

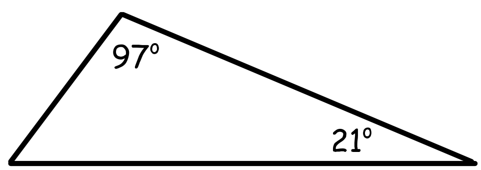
As all corresponding ratios are equal, similar

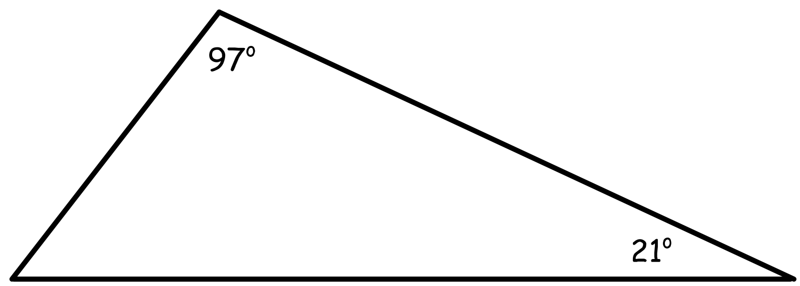
For triangles only, there is another way to describe similarity.

Triangles are similar if any 2 pairs of corresponding angles are the same

Example 3

Are these two triangles similar ? Justify your answer.

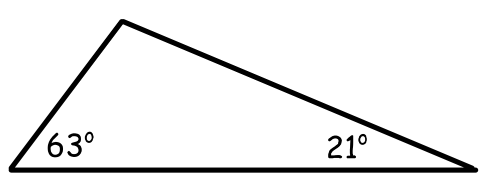


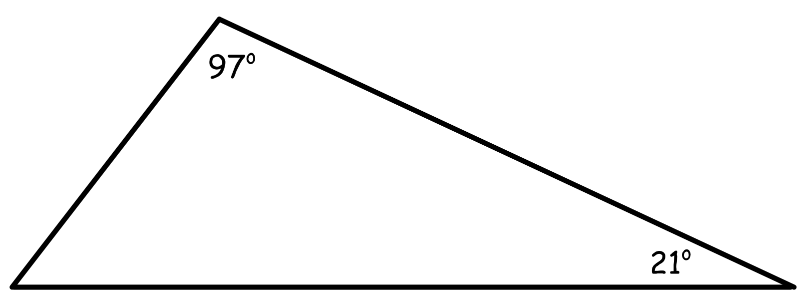


As two pairs of corresponding angles are equal, they are similar

Example 4

Are these two triangles similar ? Justify your answer.



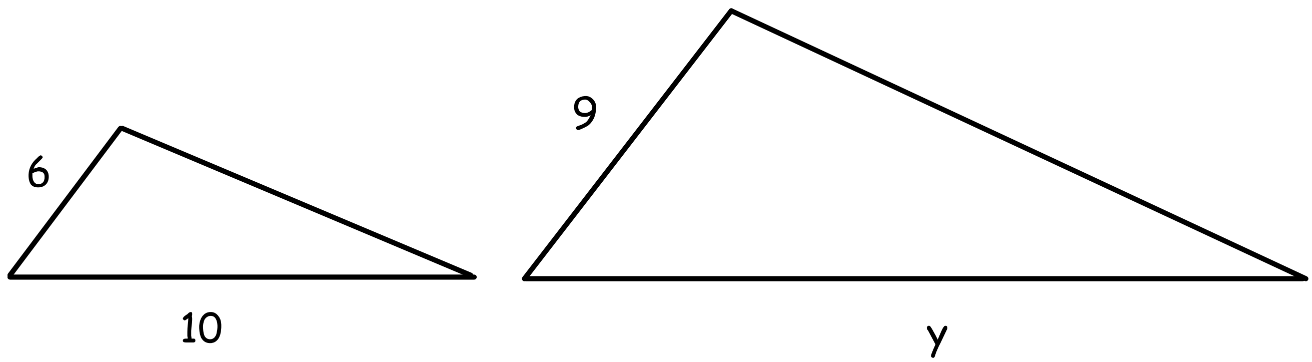


The smaller triangle has missing angle 96° and the bigger triangle has missing angle 62°.

Not similar, as no two pairs of corresponding angles are equal

Example 5

Calculate y if these are similar triangles :



Enlargement

$$k = \frac{9}{6} = \frac{3}{2}$$

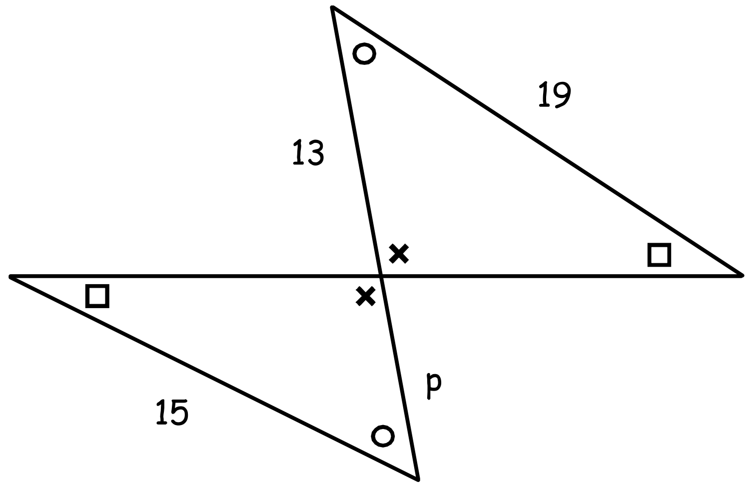
$$L_{NEW} = k \times L_{OLD}$$

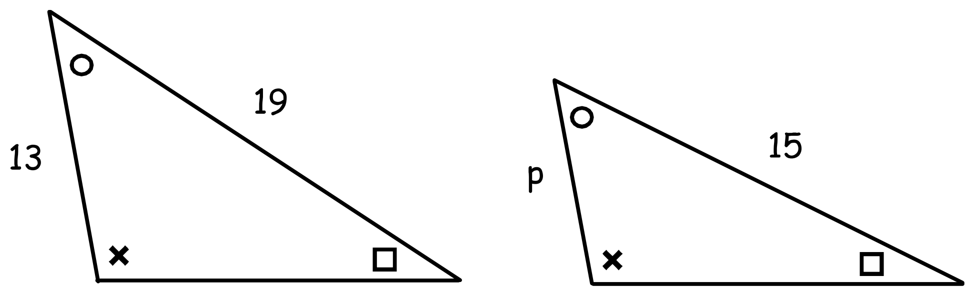
$$y = \frac{3}{2} \times 10$$

$$\boxed{y = 15}$$

Example 6

Calculate p (to 2 d.p.) :





Triangles are similar as all corresponding angles are the same.

Reduction

$$k = \frac{15}{19}$$

$$L_{NEW} = k \times L_{OLD}$$

$$p = \frac{15}{19} \times 13$$

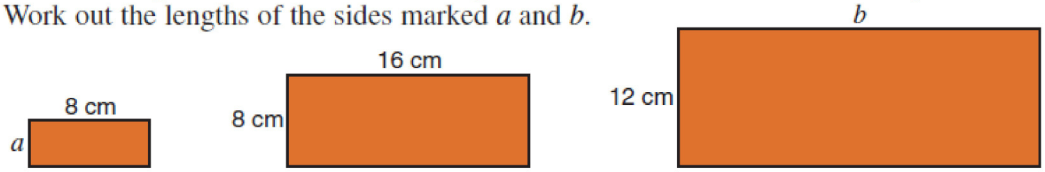
$$p = 10.263\ldots$$

$$\therefore \quad \boxed{p = 10.26 \text{ (2 d.p.)}}$$

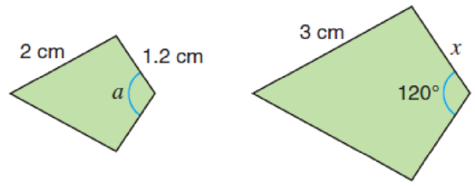
Mar 15-09:09

Questions

1) These rectangles are all similar. The diagrams have not been drawn accurately.
Work out the lengths of the sides marked *a* and *b*.



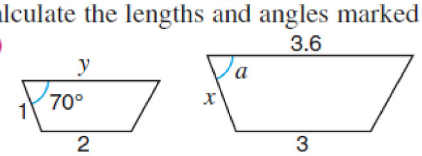
2) These two kites are similar.
(a) What is the scale factor of their lengths?
(b) Find the length of the side marked *x*.
(c) What is the size of angle *a*?



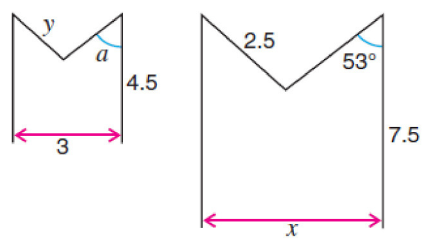
3) A shape has width 8 cm and length 24 cm.
It is enlarged to give a new shape with width 10 cm.
Calculate the length of the new shape.

4) In each part, the two figures are similar. Lengths are in centimetres.
Calculate the lengths and angles marked with letters.

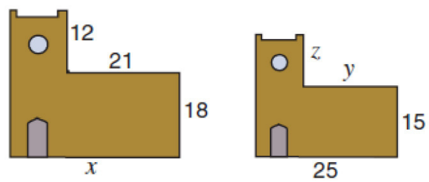
(a)



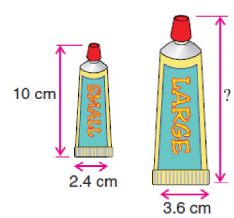
(b)



(c)



5) These two tubes are similar.
The width of the small size is 2.4 cm and the height of the small size is 10 cm.
The width of the large size is 3.6 cm.
Calculate the height of the large size.



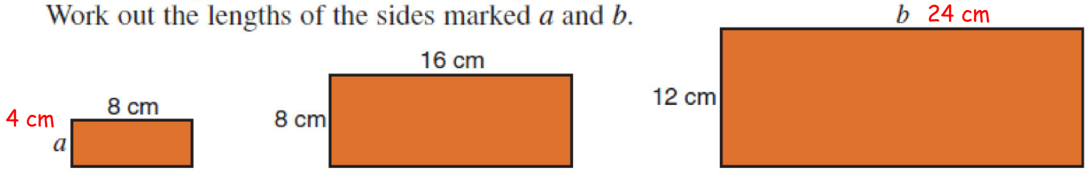
6) A motor car is 4.2 m long and 1.4 m high.
A scale model of the car is 8.4 cm long.
What is the height of the model?

7) The smallest angle in triangle T is 18°.
Triangle T is enlarged by a scale factor of 2.
How big is the smallest angle in the enlarged triangle?

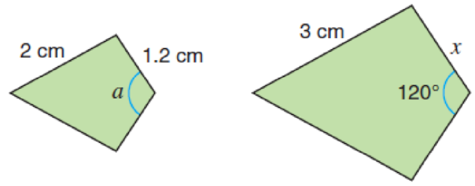
8) A castle has height 30 m.
The height of the castle wall is 6 m.
A scale model of the castle has height 25 cm.
Calculate the height of the castle wall in the scale model.

Answers

1) These rectangles are all similar. The diagrams have not been drawn accurately.
Work out the lengths of the sides marked a and b.



2) These two kites are similar.
(a) What is the scale factor of their lengths? 3/2
(b) Find the length of the side marked x. 1.8 cm
(c) What is the size of angle a? 120°

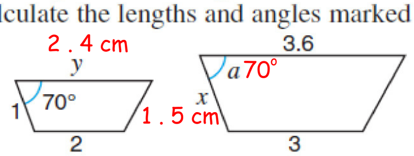


3) A shape has width 8 cm and length 24 cm.
It is enlarged to give a new shape with width 10 cm.
Calculate the length of the new shape.

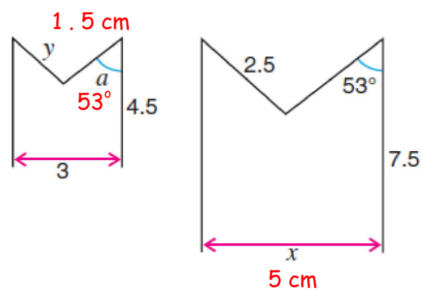
30 cm

4) In each part, the two figures are similar. Lengths are in centimetres.
Calculate the lengths and angles marked with letters.

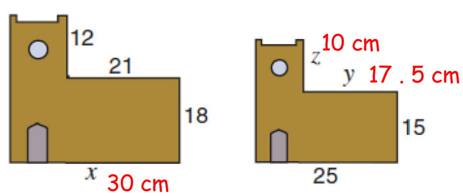
(a)



(b)



(c)



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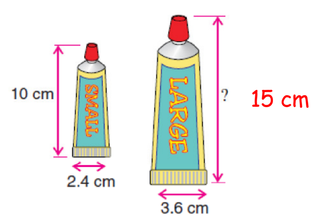
The width of the large size is 3.6 cm.
Calculate the height of the large size.



6) A motor car is 4.2 m long and 1.4 m high.
A scale model of the car is 8.4 cm long.
What is the height of the model?

2.8 cm

7) The smallest angle in triangle T is 18°.
Triangle T is enlarged by a scale factor of 2.
How big is the smallest angle in the enlarged triangle?

18°

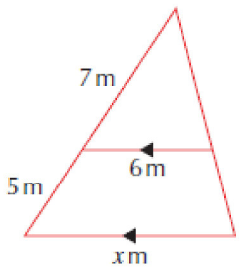
8) A castle has height 30 m.
The height of the castle wall is 6 m.
A scale model of the castle has height 25 cm.
Calculate the height of the castle wall in the scale model.

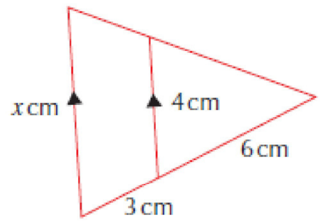
5 cm

Dec 27-13:50

Questions

1 Calculate x. Give your answers to 2 decimal places.

a


b


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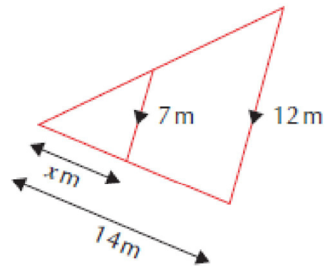
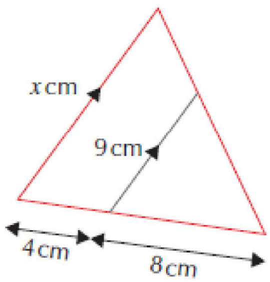


d


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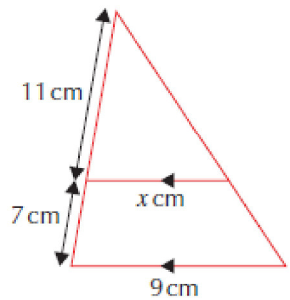

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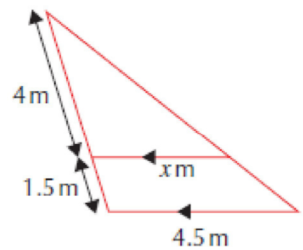
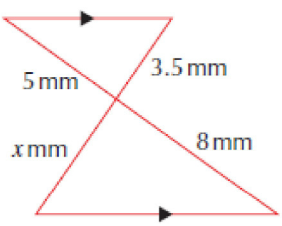


2 Calculate x. Give your answers to 2 decimal places.

a


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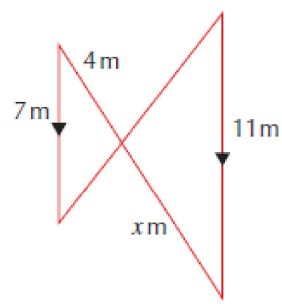

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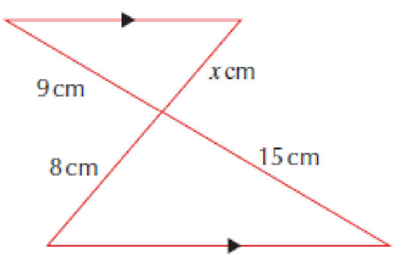

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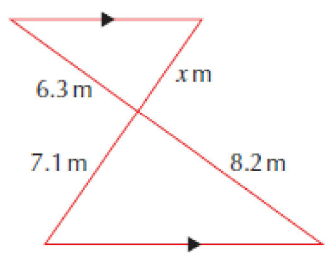
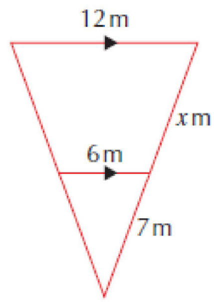


3 Calculate x. Give your answers to 2 decimal places.

a



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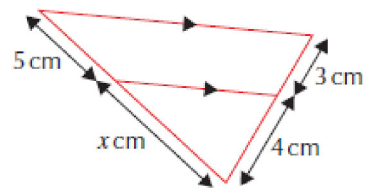

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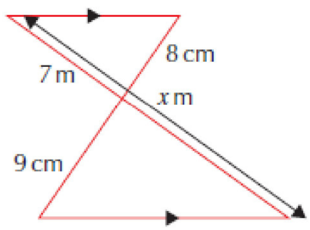

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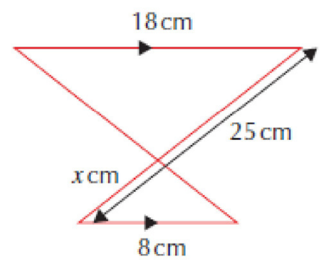
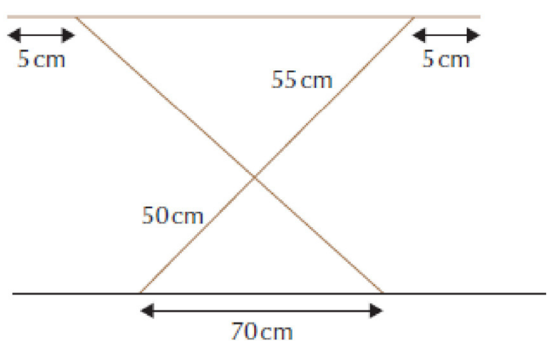
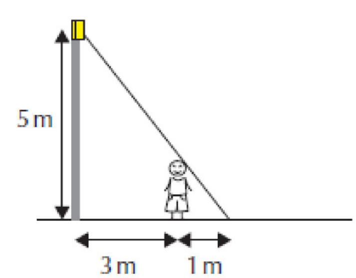


4 A side-on view of a table is shown. The table-top overlaps the legs by 5 cm on each side. The feet of the legs are 70 cm apart. Calculate the length of the table top. Give your answer to 3 significant figures.



5 Jake stands 3 m from the base of a street lamp which is 5 m tall. The length of his shadow is 1 m. How tall is Jake? Give your answer to 3 significant figures.

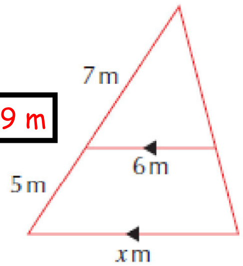


Answers

1 Calculate x. Give your answers to 2 decimal places.

a

10 . 29 m



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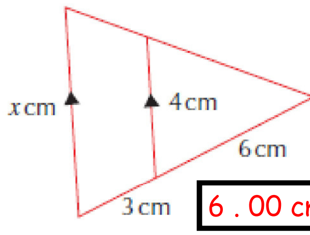
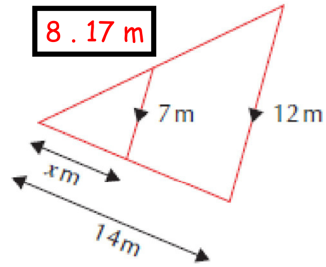


6 . 00 cm

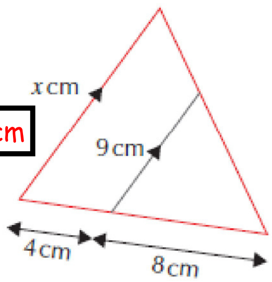
c

8 . 17 m



d

13 . 50 cm



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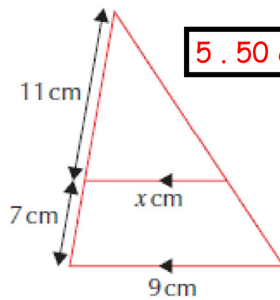
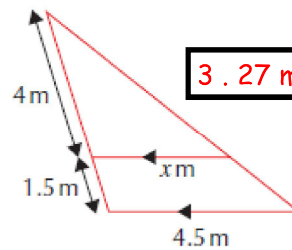


5 . 50 cm

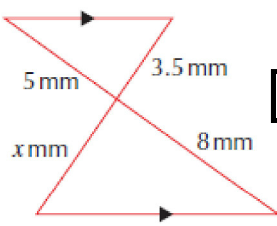
f



3 . 27 m

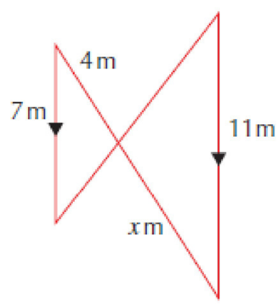
2 Calculate x. Give your answers to 2 decimal places.

a



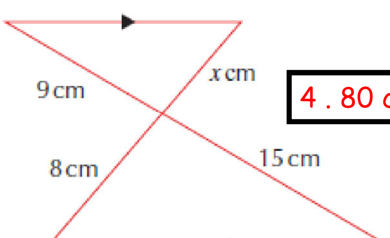
5 . 60 mm

b



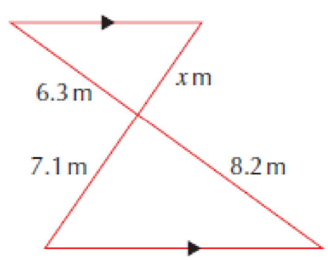
6 . 29 m

c



4 . 80 cm

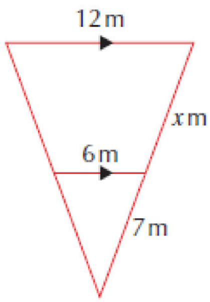
d



5 . 45 m

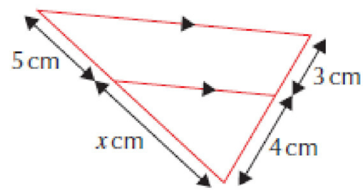
3 Calculate x. Give your answers to 2 decimal places.

a



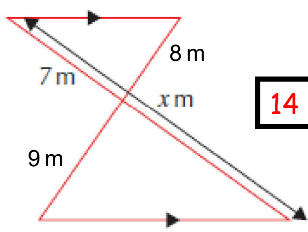
7 . 00 m

b



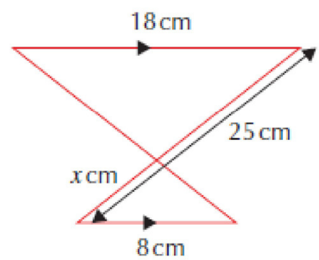
6 . 67 cm

c



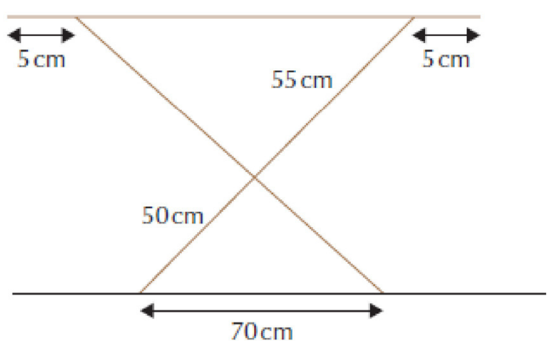
14 . 88 m

d



7 . 69 cm

4 A side-on view of a table is shown. The table-top overlaps the legs by 5 cm on each side. The feet of the legs are 70 cm apart. Calculate the length of the table top. Give your answer to 3 significant figures.



87 . 0 cm

5 Jake stands 3 m from the base of a street lamp which is 5 m tall. The length of his shadow is 1 m. How tall is Jake? Give your answer to 3 significant figures.

1 . 25 m