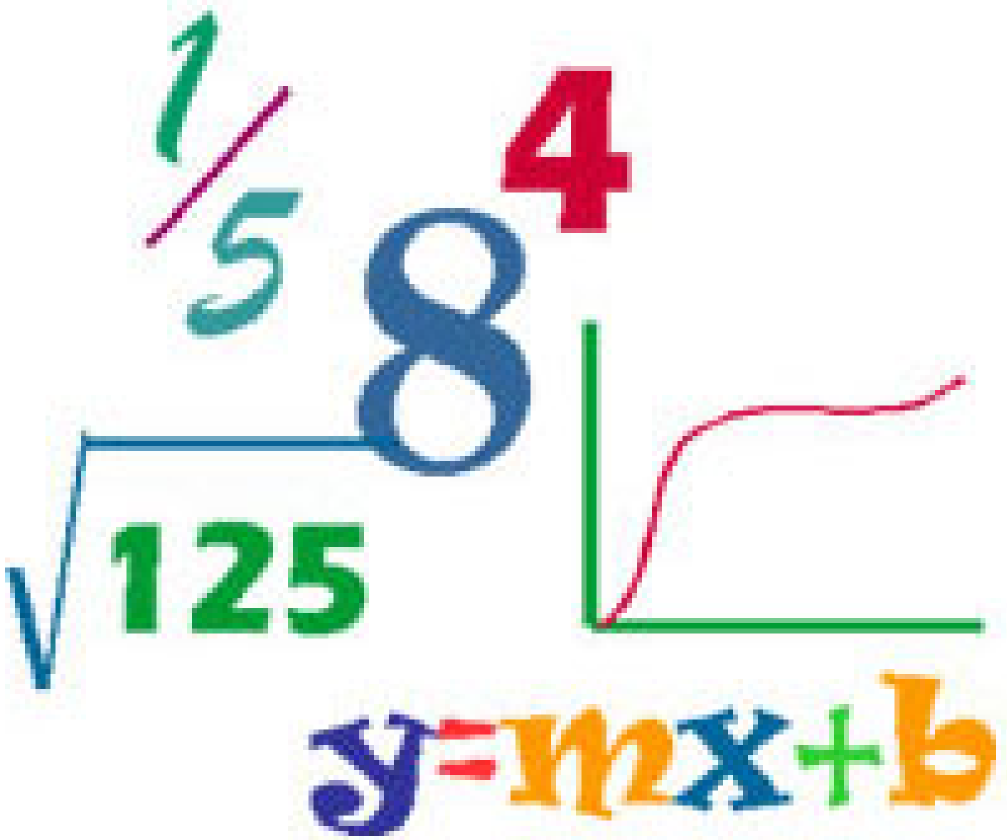
LINWOOD HIGH

S4 CREDIT NOTES

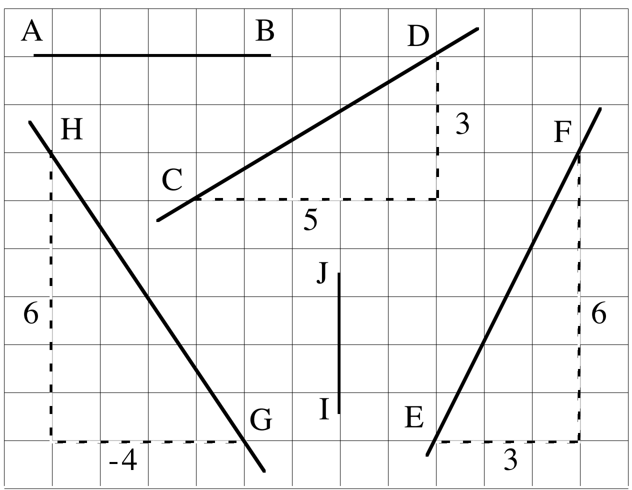


INDEX:

CHAPTER 1: STRAIGHT LINE

GRADIENT The slope of a line is given by the ratio: $m = \dfrac{\textit{vertical change}}{\textit{horizontal change}}$

For example,



$m_{AB} = 0$ horizontal

$m_{CD} = \dfrac{3}{5}$ positive m

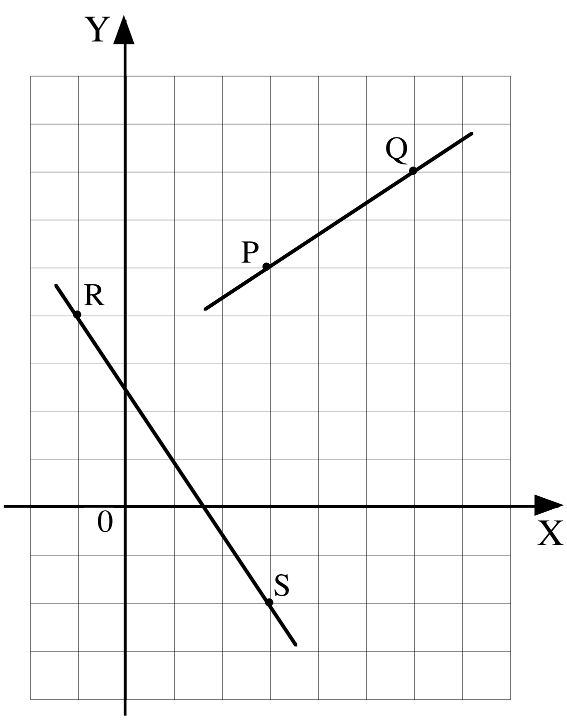
$m_{EF} = \dfrac{6}{3} = 2$

$m_{GH} = \dfrac{6}{-4} = -\dfrac{3}{2}$ negative m

m_{IJ} *is undefined (or infinite)* vertical

Using coordinates, **the gradient formula** is $m_{AB} = \dfrac{y_B - y_A}{x_B - x_A}$

For example,



$P\,(3,5)$, $Q\,(6,7)$

$$m_{PQ} = \frac{y_Q - y_P}{x_Q - x_P} = \frac{7-5}{6-3} = \frac{2}{3}$$

note: same result for

$$\frac{5-7}{3-6} = \frac{-2}{-3} = \frac{2}{3}$$

$R\,(-1,4)$, $S\,(3,-2)$

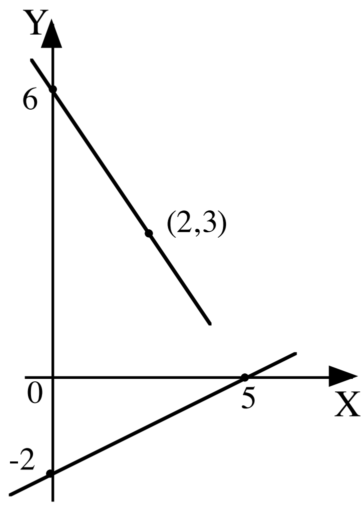
$$m_{RS} = \frac{y_S - y_R}{x_S - x_R} = \frac{-2-4}{3-(-1)} = \frac{-6}{4} = -\frac{3}{2}$$

EQUATION OF A STRAIGHT LINE

gradient m $\quad\quad$ $y = mx + C$

y-intercept C units ie. meets the y-axis at $(0,C)$

For example,



$(2,3)$ $\quad m = \frac{3-6}{2-0} = -\frac{3}{2}$ $\quad y = mx + C$

$(0,6)$ $\quad C = 6$ $\quad y = -\frac{3}{2}x + 6$

$(5,0)$ $\quad m = \frac{0-(-2)}{5-0} = \frac{2}{5}$ $\quad y = mx + C$

$(0,-2)$ $\quad C = -2$ $\quad y = \frac{2}{5}x - 2$

The form of the equation can be rearranged to $Ax + By + C = 0$

For example,

$$y = -\frac{3}{2}x + 6$$

$$2y = -3x + 12 \qquad \textit{multiplied each side by } 2$$

$$3x + 2y - 12 = 0 \qquad \textit{added } 3x \textit{ and subtracted } 12 \textit{ from each side}$$

Rearrange the equation to $y = mx + C$ for the gradient and y-intercept.

For example,

$$3x + 2y - 12 = 0$$

$$2y = -3x + 12 \qquad \textit{isolate } y\textit{- term}$$

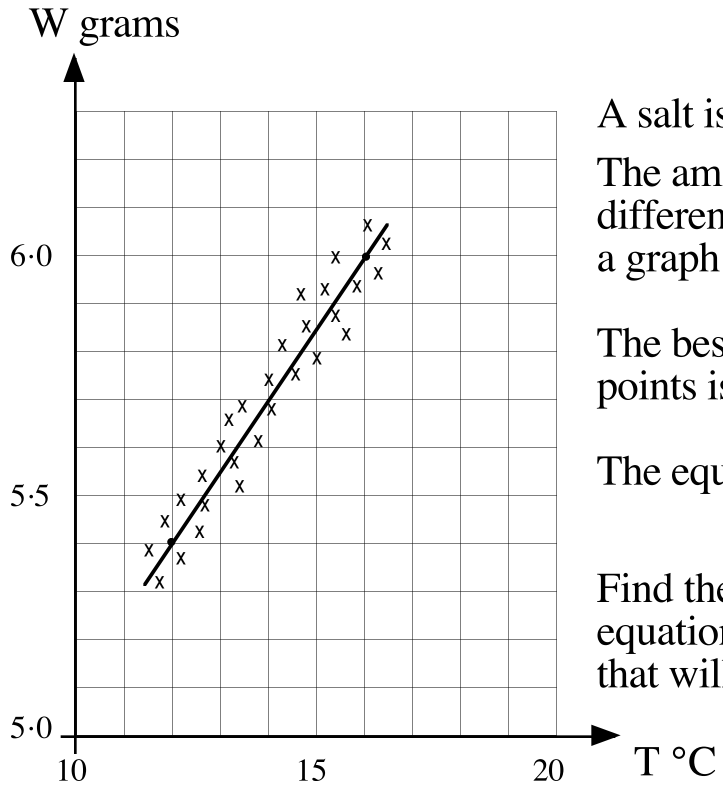
$$y = -\frac{3}{2}x + 6 \qquad \textit{obtain } 1y =$$

$$y = \ mx + C \qquad \textit{compare to the general equation}$$

$$m = -\frac{3}{2}\ ,\ C = 6 \quad ,\ \textit{line meets the } y\textit{- axis at } (0,6)$$

LINE OF BEST FIT

Example:



A salt is dissolved in a litre of solvent.

The amount of salt that dissolves at different temperatures is recorded and a graph plotted.

The best-fitting straight line through the points is drawn.

The equation of the graph is of the form

$$W = mT + C.$$

Find the equation of the line and use the equation to calculate the mass of salt that will dissolve at 30 °C.

using two well-separated points on the line

$(16\ ,\ 6\cdot 0)$

$(12\ ,\ 5\cdot 4)$

$$m = \frac{6\cdot 0 - 5\cdot 4}{16-12} = \frac{0\cdot 6}{4} = 0\cdot 15$$

$$y = mx + C$$
$$W = 0\cdot 15\,T + C$$

substituting for one point on the line $\overset{T}{(16}\ ,\ \overset{W}{6\cdot 0)}$

$$6\cdot 0 = 0\cdot 15 \times 16 + C$$
$$6\cdot 0 = 2\cdot 4 + C$$
$$C = 3\cdot 6$$
$$\underline{\underline{W = 0\cdot 15\,T + 3\cdot 6}}$$

$T = 30$

$$W = 0\cdot 15 \times 30 + 3\cdot 6$$
$$= 4\cdot 5 + 3\cdot 6$$
$$= 8\cdot 1$$
$$\underline{\underline{8\cdot 1 \ \ grams}}$$

CHAPTER 2: FUNCTIONS

A function pairs elements of one set of numbers with those of another set such that each element of the first set, the DOMAIN, has only one IMAGE under the function.

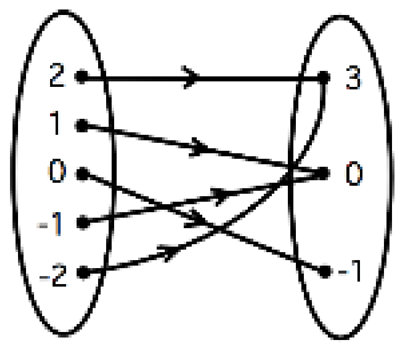
The set of images is the RANGE.

The pairings can be given by (i) an image rule and domain, (ii) listed pairings (iii) diagram. For example,

(i) function notation or formula: $f : x \to x^2 - 1$ *or* $f(x) = x^2 - 1$, $\{-2, -1, 0, 1, 2\}$

(ii) ordered pairs: $\{ (-2, 3), (-1, 0), (0, -1), (1, 0), (2, 3) \}$

(iii) arrow diagram:

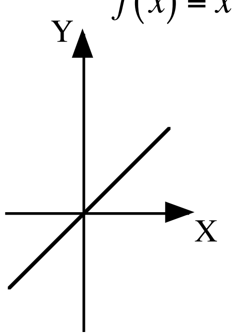


The domain is $\{-2, -1, 0, 1, 2\}$, the range is $\{-1, 0, 3\}$.

Since for example $f(-2) = (-2)^2 - 1 = 4 - 1 = 3$, the image of -2 is 3.

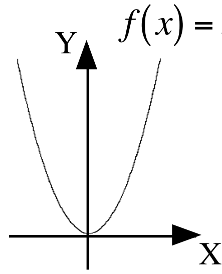
If the domain is any possible value of *x* then a graph is the only suitable diagram. For example,

$f(x) = x$



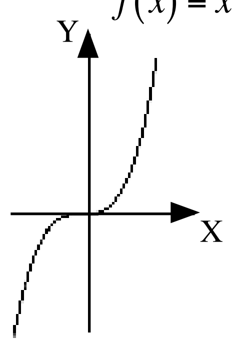
LINEAR

$f(x) = x^2$



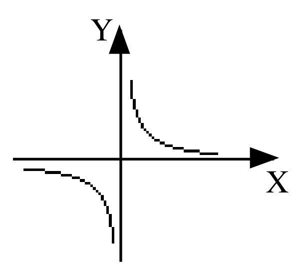
QUADRATIC

$f(x) = x^3$



CUBIC

$f(x) = \frac{1}{x}$, $x \neq 0$



RECIPROCAL

QUADRATIC FUNCTIONS

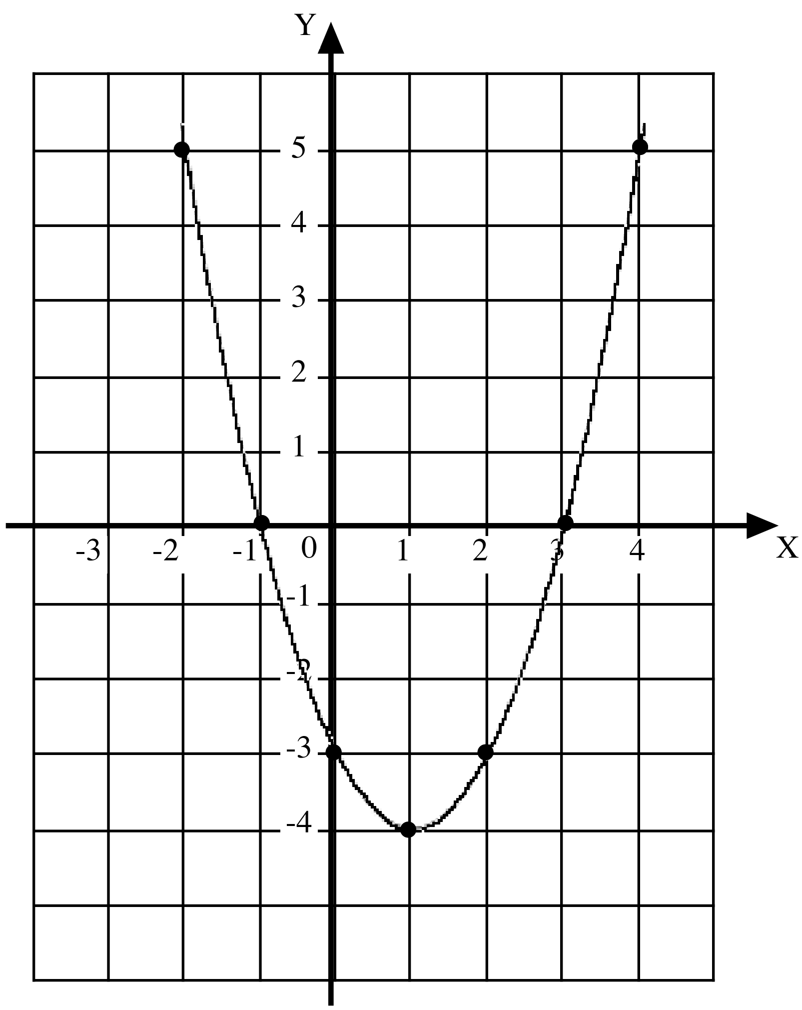
$f(x) = ax^2 + bx + c$, $a \neq 0$, where a, b and c are constants.

The graph is a curve called a PARABOLA.

For example,

$f(x) = x^2 - 2x - 3$

x	-2	-1	0	1	2	3	4
x^2	4	1	0	1	4	9	16
$-2x$	4	2	0	-2	-4	-6	-8
-3	-3	-3	-3	-3	-3	-3	-3
$f(x)$	**5**	**0**	**-3**	**-4**	**-3**	**0**	**5**
points	(-2,5)	(-1,0)	(0,-3)	(1,-4)	(2,-3)	(3,0)	(4,5)



FORMULAE

Examples:

(1) If $f(x) = \dfrac{2x}{x+2}$, $x \neq -2$, evaluate $f(-3)$.

$$f(-3) = \frac{2\times(-3)}{-3+2} = \frac{-6}{-1} = 6$$

Note: -2 is not allowed for x as it has no image under the function (cannot divide by 0).

(2) If $g(x) = 5 - 2x$ and $g(a) = 11$, find a.

$$\begin{aligned} g(a) &= 5 - 2a \\ 11 &= 5 - 2a \\ 2a &= 5 - 11 \\ 2a &= -6 \\ a &= -3 \end{aligned}$$

(3) If $h(x) = x^2 - 2x$, fully simplify $h(-a) - h(a)$

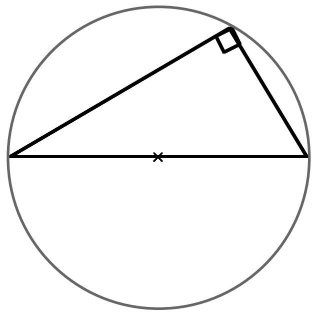
$$h(a) = a^2 - 2a$$

$$h(-a) = (-a)^2 - 2(-a) = a^2 + 2a$$

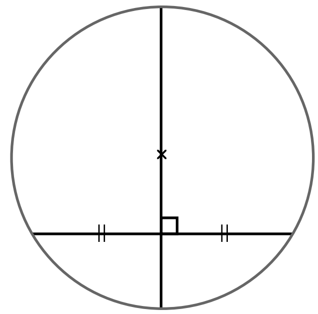
$$\begin{aligned} h(-a) - h(a) &= a^2 + 2a - \left(a^2 - 2a\right) \\ &= a^2 + 2a - a^2 + 2a \\ &= 4a \end{aligned}$$

CHAPTER 3: SYMMETRY IN THE CIRCLE

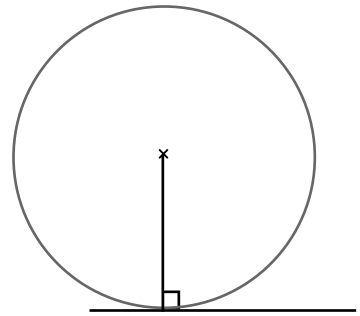
angle in a semicircle is a right-angle.



the perpendicular bisector of a chord is a diameter.



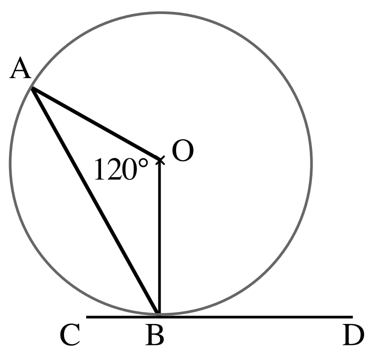
a tangent and the radius drawn to the point of contact form a right-angle.



ANGLES

Examples:

(1)



Calculate the size of angle ABC.

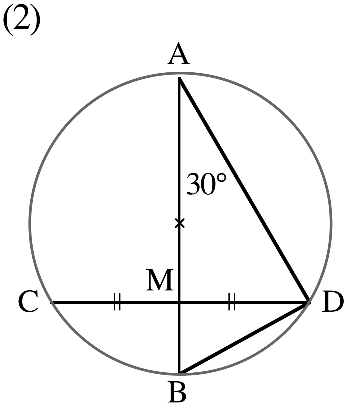
radius OA = OB so ΔAOB is isosceles and Δ angle sum 180° :

$\angle OBA = (180° - 120°) \div 2 = 30°$

tangent CD and radius OB : $\angle OBC = 90°$

$\angle ABC = 90° - 30° = 60°$

(2)



Calculate the size of angle BDC.

diameter AB bisects chord CD : $\angle AMD = 90°$

ΔAMD *angle sum* 180° :

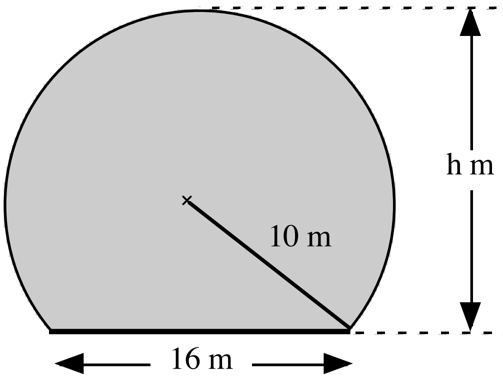
$\angle ADM = 180° - 90° - 30° = 60°$

angle in a semicircle : $\angle ADB = 90°$

$\angle BDC = 90° - 60° = 30°$

PYTHAGORAS' THEOREM

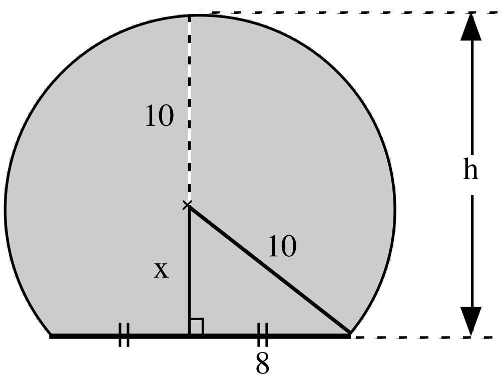
Example:



A circular road tunnel, radius 10 metres, is cut through a hill.

The road has a width 16 metres.

Find the height of the tunnel.

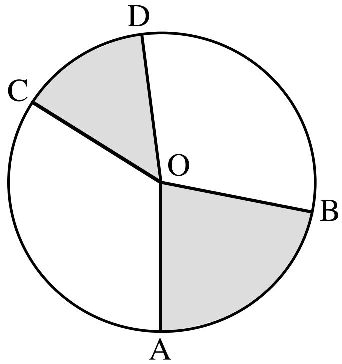


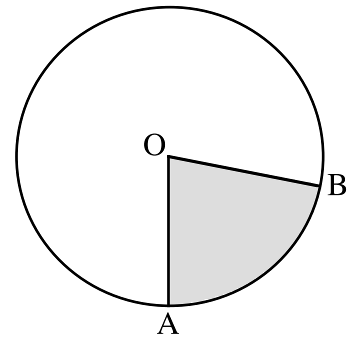
the diameter drawn is the perpendicular bisector of the chord:
Δ is right-angled so can apply Pyth. Thm.

$$\begin{aligned} x^2 &= 10^2 - 8^2 \\ &= 100 - 64 \\ &= 36 \\ x &= \sqrt{36} \\ x &= 6 \end{aligned} \qquad \begin{aligned} h &= x + 10 \\ &= 6 + 10 \\ h &= 16 \end{aligned}$$

$\underline{\underline{\textit{height } 16 \textit{ metres}}}$

SECTORS



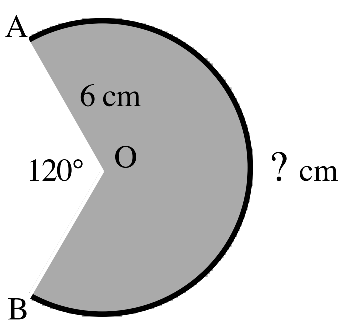


$$\frac{\angle AOB}{\angle COD}=\frac{arc\ AB}{arc\ \text{CD}}=\frac{area\ of\ sector\ AOB}{area\ of\ sector\ COD} \qquad \frac{\angle AOB}{360°}=\frac{arc\ AB}{\pi d}=\frac{area\ of\ sector\ AOB}{\pi r^2}$$

Choose the appropriate pair of ratios based on:
(i) the ratio which includes the quantity to be found
(ii) the ratio for which both quantities are known (or can be found).

Examples:

(1) Find the **exact** length of **major** arc AB.



$$\frac{\angle AOB}{360°}=\frac{arc\ AB}{\pi\, d}$$

$$\frac{240°}{360°}=\frac{arc\ AB}{\pi\times 12}$$

$$arc\ AB=\frac{240°}{360°}\times\pi\times 12$$

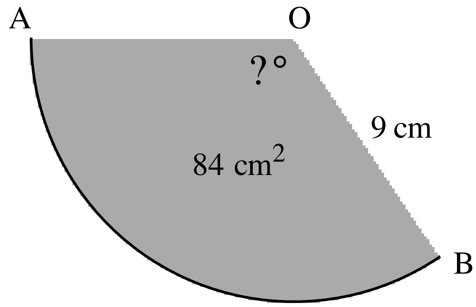
$$=8\pi\ cm \qquad (25\cdot 132...)$$

$$diameter\ d=2\times 6\ cm=12\ cm$$

$$reflex\ \angle AOB=360°-120°=240°$$

$$\frac{\overset{\checkmark}{\angle AOB}}{\underset{\checkmark}{360°}}=\frac{\overset{?}{arc\ AB}}{\underset{\checkmark}{\pi d}}=\frac{area\ of\ sector\ AOB\ \times}{\pi r^2\ \checkmark}$$

(2) Find the size of angle AOB.



$$\frac{\angle AOB}{360°} = \frac{area\ of\ sector\ AOB}{\pi r^2}$$

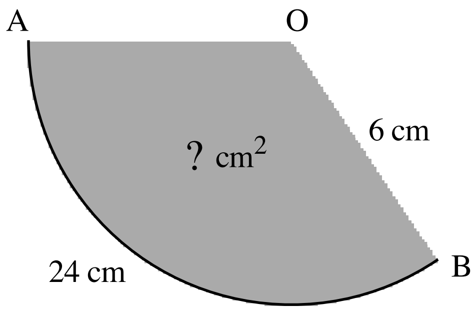
$$\frac{\angle AOB}{360°} = \frac{84}{\pi \times 9 \times 9}$$

$$\angle AOB = \frac{84}{\pi \times 9 \times 9} \times 360°$$

$$= 118 \cdot 835...$$

$$\angle AOB \approx 119°$$

(3) Find the **exact** area of sector AOB.



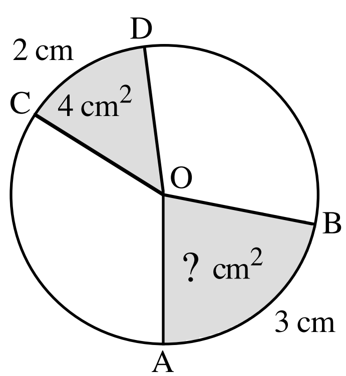
$$\frac{arc\ AB}{\pi d} = \frac{area\ of\ sector\ AOB}{\pi r^2}$$

$$\frac{24}{\pi \times 12} = \frac{area\ of\ sector\ AOB}{\pi \times 6 \times 6}$$

$$area\ of\ sector\ AOB = \frac{24}{\pi \times 12} \times \pi \times 6 \times 6$$

$$= 72\ cm^2$$

(4) Find the **exact** area of sector AOB.



$$\frac{arc\ AB}{arc\ CD} = \frac{area\ of\ sector\ AOB}{area\ of\ sector\ COD}$$

$$\frac{3}{2} = \frac{area\ of\ sector\ AOB}{4}$$

$$area\ of\ sector\ AOB = \frac{3}{2} \times 4$$

$$= 6\ cm^2$$

CHAPTER 4: INEQUALITIES

Simplify by following the rules for equations:

addition and subtraction

$$\begin{aligned} x + a &> b \\ x &> b - a \end{aligned} \qquad \begin{aligned} x - a &> b \\ x &> b + a \end{aligned}$$

multiplication and division

by a **positive** number

$$\frac{x}{a} > b \qquad ax > b$$

$$x > ab \qquad x > \frac{b}{a}$$

by a **negative** number
reverse the direction of the inequality sign

$$\frac{x}{a} > b \qquad ax > b$$

$$x < ab \qquad x < \frac{b}{a}$$

Examples:

(1)

$$8 + 3x > 2$$

$$+3x > -6$$

$$x > \frac{-6}{+3}$$ *divided each side by* $+3$
notice sign unchanged

$$x > -2$$

(2)

$$8 - 3x > 2$$

$$-3x > -6$$ *subtracted* 8 *from each side*

$$x < \frac{-6}{-3}$$ *divided each side by* -3
notice sign reversed

$$x < 2$$ *simplified*

(3)

$$\begin{aligned} 4x - 6 &\le x - 1 \\ 3x - 6 &\le -1 \\ 3x &\le 5 \\ x &\le \frac{5}{3} \end{aligned}$$

(4)

$$\begin{aligned} x - 6 &\le 4x - 1 \\ -3x - 6 &\le -1 \\ -3x &\le 5 \\ x &\ge \frac{5}{-3} \\ x &\ge -\frac{5}{3} \end{aligned}$$

or

$$\begin{aligned} x - 6 &\le 4x - 1 \\ -6 &\le 3x - 1 \\ -5 &\le 3x \\ \frac{-5}{3} &\le x \\ x &\ge -\frac{5}{3} \end{aligned}$$

RESTRICTIONS ON SOLUTIONS

Examples:

(1)

$$x \le \frac{5}{2} \quad \textit{where } x \textit{ is a whole number}$$

$$x = 0,1,2$$

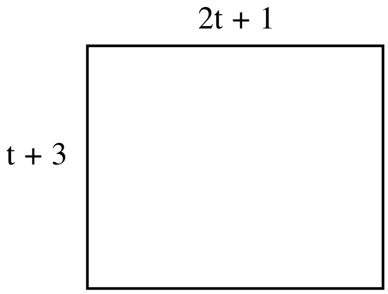
(2)

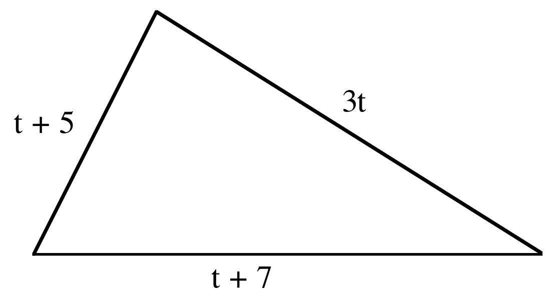
$$-2 \le x < 2 \quad \textit{where } x \textit{ is an integer}$$

$$x = -2,-1,0,1$$

MODELLING

Example:





The **perimeter** of the rectangle is less than that of the triangle.
Find the possible values of t where t is a **positive integer**.

$$2(2t+1)+2(t+3) < 3t+t+5+t+7$$

$$4t+2 + 2t+6 < 3t+t+5+t+7$$

$$6t+8 < 5t+12$$

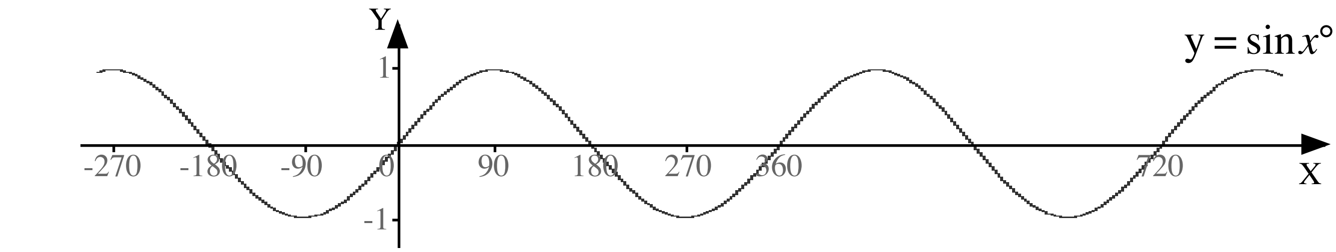
$$t+8 < 12$$

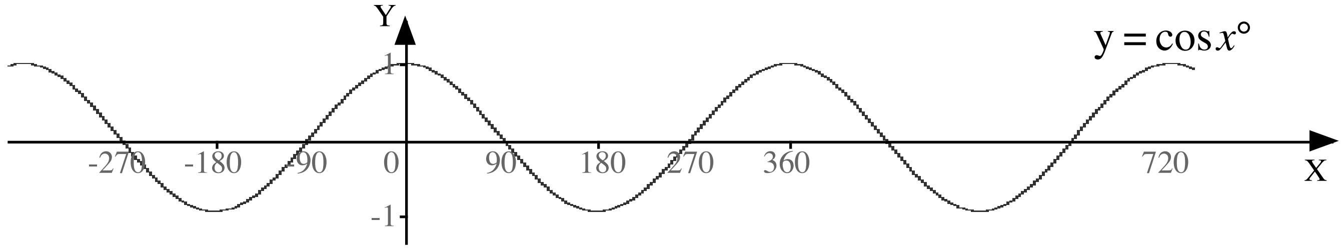
$$t < 4$$

$$t = 1,2,3$$

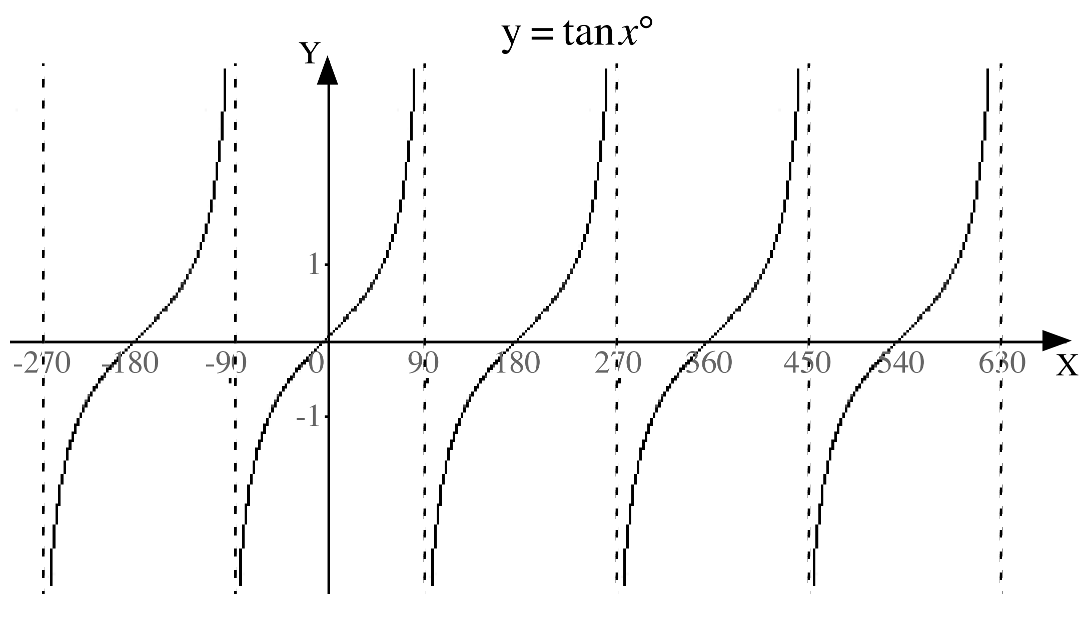
CHAPTER 5: TRIGONOMETRY: GRAPHS & EQUATIONS

GRAPHS





Each graph has a PERIOD of 360° (repeats every 360°).
The maximum value of each function is +1 , the minimum is -1.
The cosine graph is the sine graph shifted 90° to the left.



The tangent graph has a PERIOD of 180°.
The maximum value is positive infinity , the minimum is negative infinity.

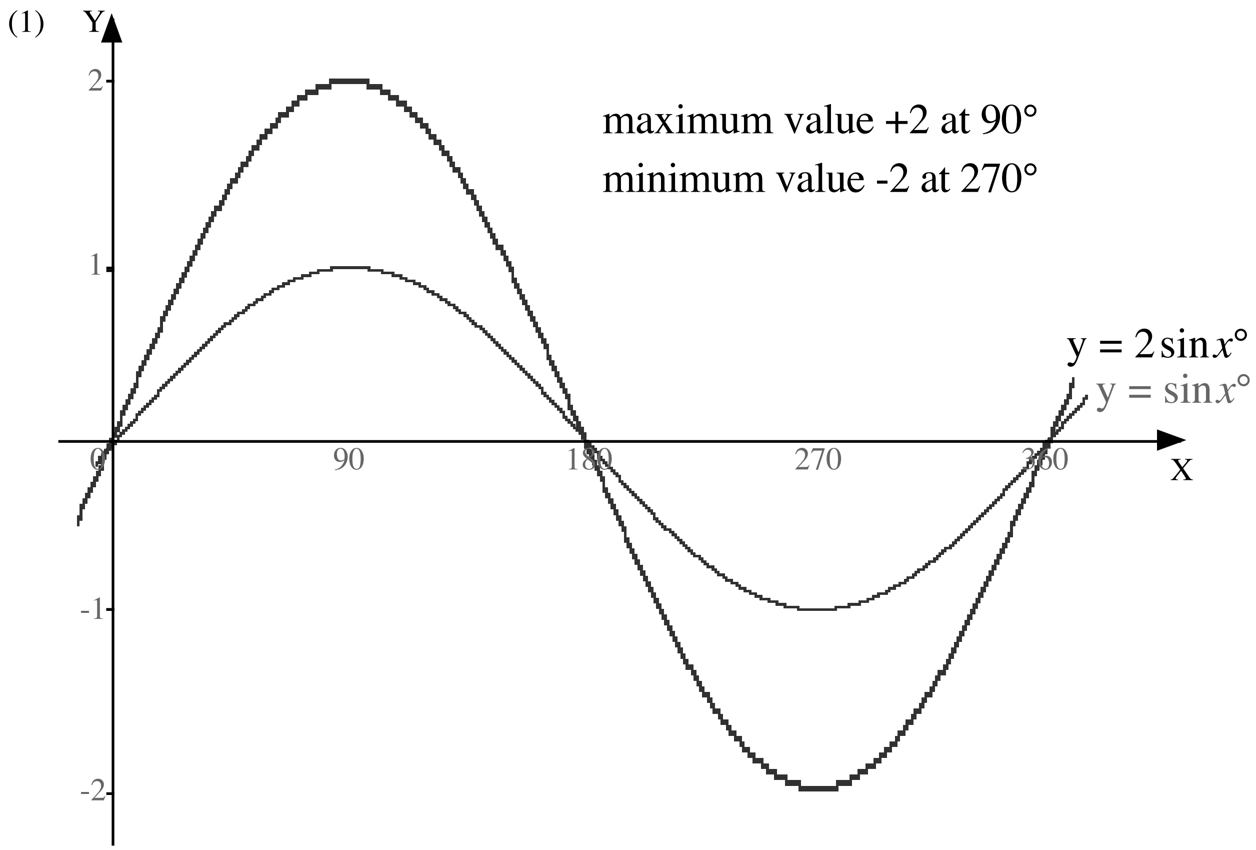
TRANSFORMATIONS Same rules for y = $\sin x°$ and y = $\cos x°$.

Y-STRETCH y = $\mathbf{n}\sin x°$ maximum value $+n$, minimum value $-n$.

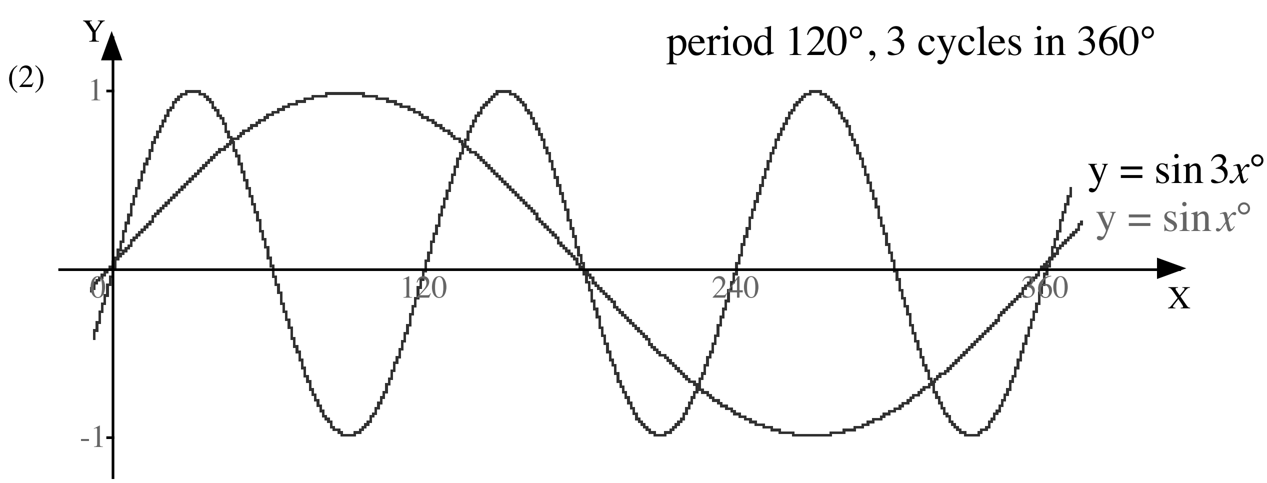
X-STRETCH y = $\sin \mathbf{n}x°$ has period $\dfrac{360°}{n}$. There are n cycles in 360°.

For example,

(1)



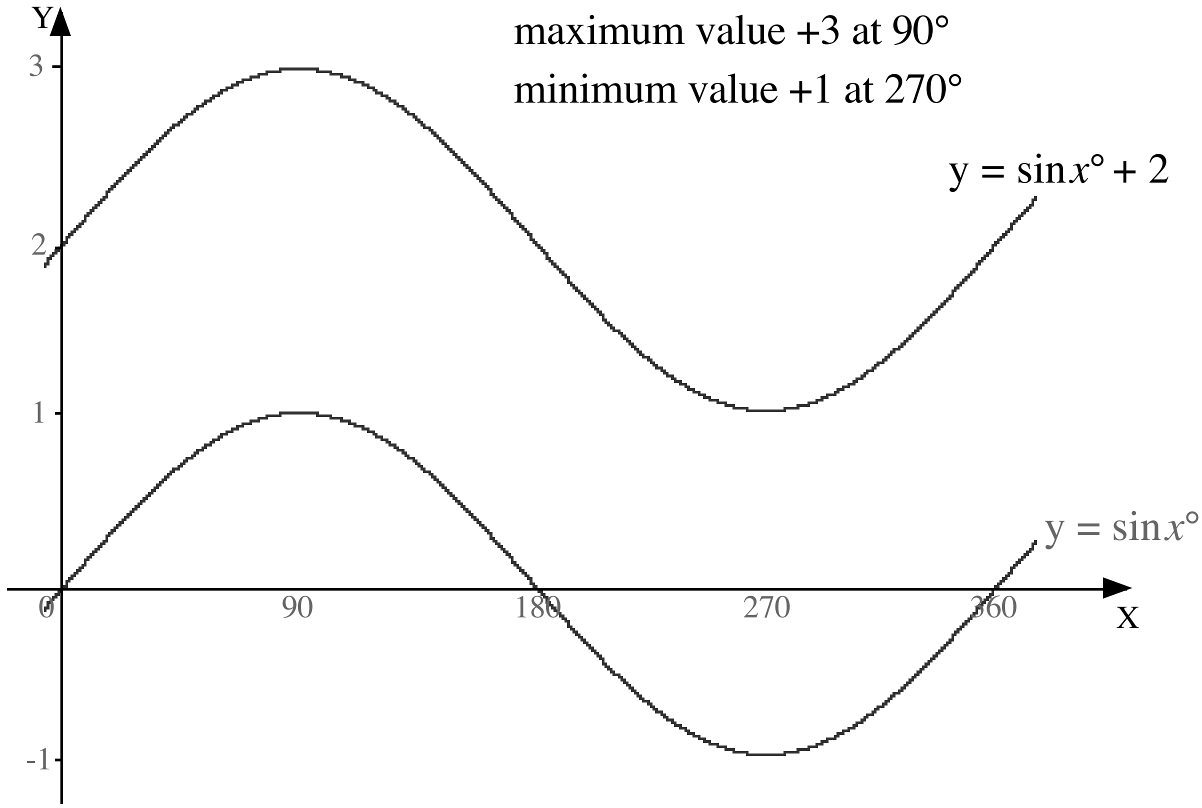
(2)



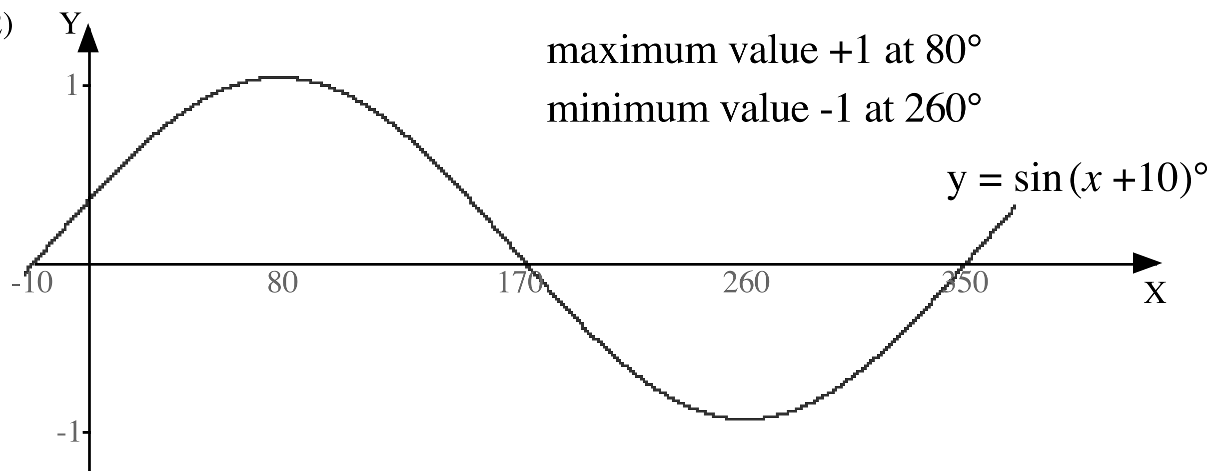
Y-SHIFT	$y = \sin x° + a$	graph shifted a units vertically.
X-SHIFT	$y = \sin(x + a)°$	graph shifted $-a°$ horizontally.

For example,

(1)



(2)

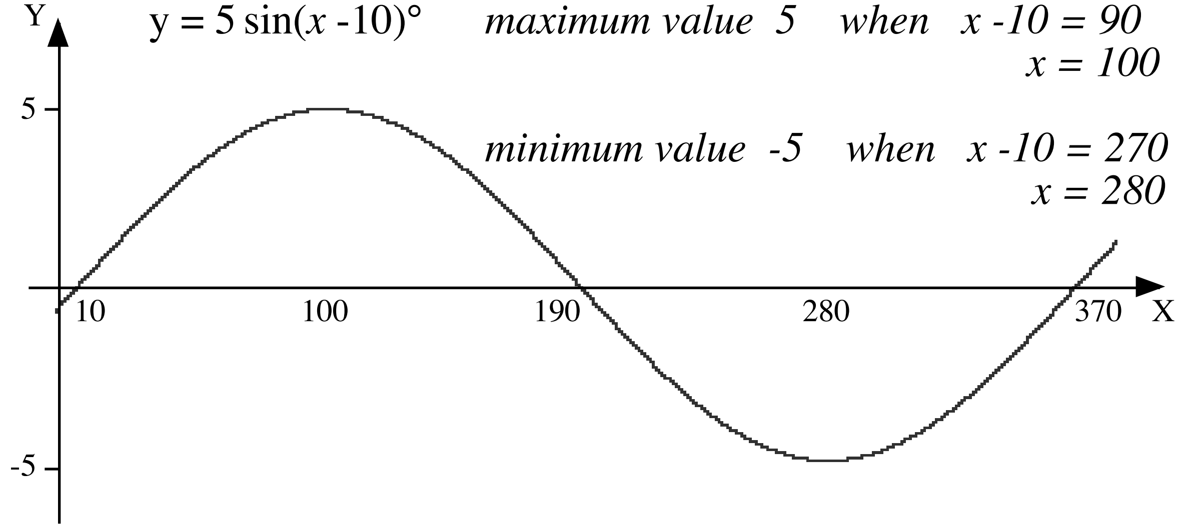


COMBINING TRANSFORMATIONS

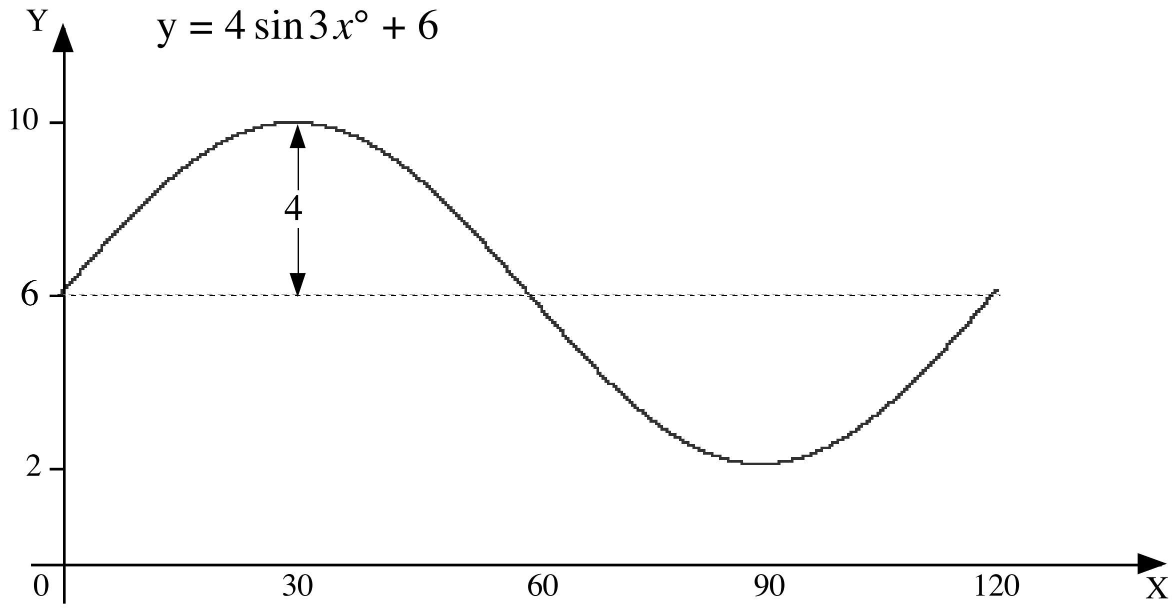
For example,

(1) $y = 5\sin(x - 10)°$ *maximum value* 5 *when* $x - 10 = 90$
$x = 100$

minimum value -5 *when* $x - 10 = 270$
$x = 280$



(2) $y = 4\sin 3x° + 6$



by $\frac{90}{3}$ $\frac{180}{3}$ $\frac{270}{3}$ $\frac{360}{3}$

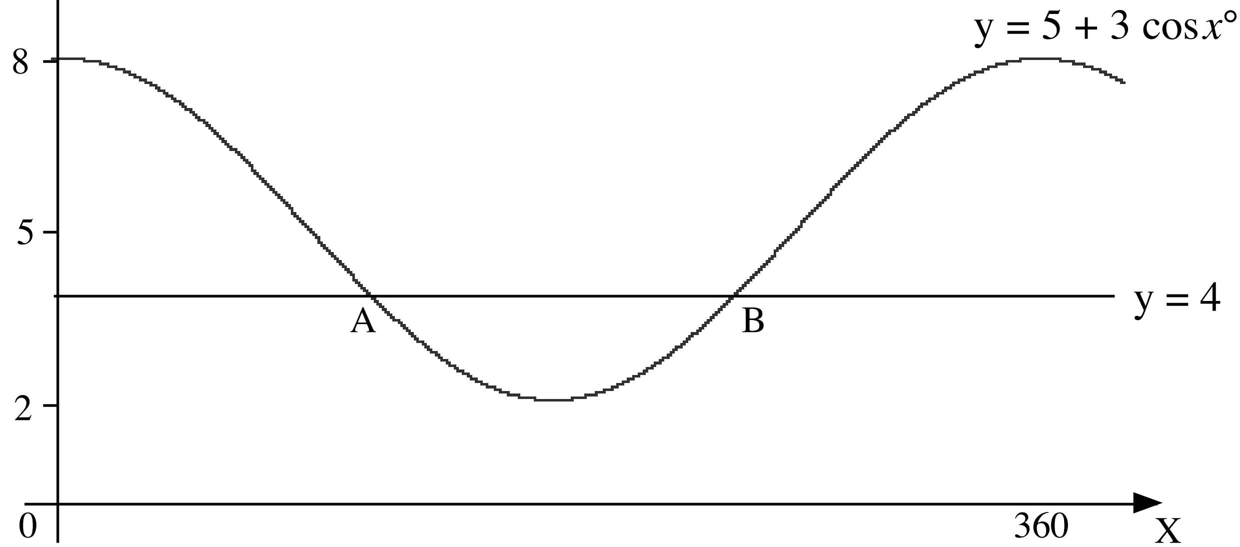
maximum value $4 \times 1 + 6 = 10$ *when* $3x = 90$ *maximum turning point* $(30, 10)$
$x = 30$

minimum value $4 \times (-1) + 6 = 2$ *when* $3x = 270$ *minimum turning point* $(90, 2)$
$x = 90$

EQUATIONS

Example:

The graphs with equations y = 5 + 3 cos x° and y = 4 are shown.
Find the x coordinates of the points of intersection A and B.



$$5 + 3\cos x° = 4$$
$$3\cos x° = -1$$
$$\cos x° = -\frac{1}{3}$$

$$\underline{\underline{x = 109 \cdot 5 \ \ or \ \ 250 \cdot 5}}$$

* **A, S, T, C** is where functions are **positive**:

✓S	A ×
cos -	cos +
180 - a = 109·5	a = cos $^{-1}$ $^{1}/_{3}$ = 70·528...
180 + a = 250·5	360 - a = 289·5
cos -	cos +
✓T	C ×

* A all functions are positive
S sine function only is positive
T cosine function only is positive
C tangent function only is positive

IDENTITIES

$$\sin^2 x° + \cos^2 x° = 1 \qquad\qquad \tan x° = \frac{\sin x°}{\cos x°}$$

Example:

If $\sin x° = \frac{1}{2}$, without finding x, find the **exact** values of $\cos x°$ and $\tan x°$.

$$\sin^2 x° + \cos^2 x° = 1$$

$$\left(\frac{1}{2}\right)^2 + \cos^2 x° = 1$$

$$\frac{1}{4} + \cos^2 x° = 1$$

$$\cos^2 x° = \frac{3}{4}$$

$$\cos x° = \frac{\sqrt{3}}{2}$$

$$\tan x° = \frac{\sin x°}{\cos x°}$$

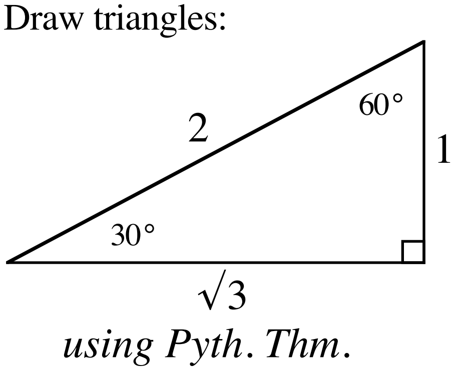
$$= \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

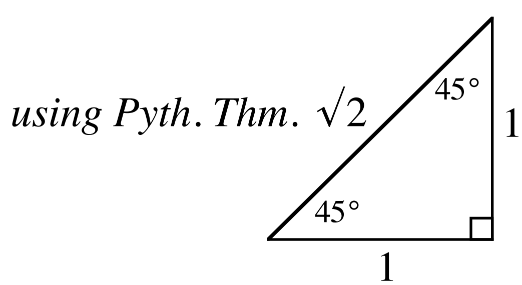
$$\tan x° = \frac{1}{\sqrt{3}}$$

EXACT VALUES

Remember: $\sin 30° = \frac{1}{2}$ $\qquad$ $\tan 45° = 1 \quad ie.\ \frac{1}{1}$

Draw triangles:





For example,

$$\sin 60° = \frac{\sqrt{3}}{2}\ ,\ \tan 30° = \frac{1}{\sqrt{3}} \qquad\qquad \cos 45° = \frac{1}{\sqrt{2}}\ ,\ \sin 45° = \frac{1}{\sqrt{2}}$$

CHAPTER 6: QUADRATIC EQUATIONS

An equation of the form $ax^2 + bx + c = 0,\ a \neq 0$, where a, b and c are constants.

The value(s) of x that satisfy the equation are the **roots** of the equation.

FACTORISATION

If $b^2 - 4ac =$ a square number ie. $0, 1, 4, 9, 16 \ldots..$
then the quadratic expression can be factorised to solve the equation.

Examples:

Solve:

(1) $4n - 2n^2 = 0$

$$2n(2-n) = 0$$

$$2n = 0 \quad \text{or} \quad 2 - n = 0$$

$$\underline{\underline{n = 0 \quad \text{or} \quad n = 2}}$$

(2) $2t^2 + t - 6 = 0$

$$(2t-3)(t+2) = 0$$

$$2t - 3 = 0 \quad \text{or} \quad t + 2 = 0$$

$$2t = 3$$

$$\underline{\underline{t = \frac{3}{2} \quad \text{or} \quad t = -2}}$$

The equation may need to be rearranged:

(3) $(w+1)^2 = 2(w+5)$

$$w^2 + 2w + 1 = 2w + 10$$

$$w^2 - 9 = 0$$

$$(w+3)(w-3) = 0$$

$$w + 3 = 0 \quad \text{or} \quad w - 3 = 0$$

$$\underline{\underline{w = -3 \quad \text{or} \quad w = 3}}$$

(4) $x + 2 = \frac{15}{x}\ ,\ x \neq 0$

$$x(x+2) = 15$$

$$x^2 + 2x = 15$$

$$x^2 + 2x - 15 = 0$$

$$(x+5)(x-3) = 0$$

$$x + 5 = 0 \quad \text{or} \quad x - 3 = 0$$

$$\underline{\underline{x = -5 \quad \text{or} \quad x = 3}}$$

QUADRATIC FORMULA

A quadratic equation $ax^2 + bx + c = 0$ can be solved using the **quadratic formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad , a \neq 0$$

Note: (1) **Use a calculator!**

(2) $b^2 - 4ac$ **will not be negative,** otherwise there is no solution.

Example:

Find the **roots** of the equation $3t^2 - 5t - 1 = 0$, correct to two decimal places.

$3t^2 - 5t - 1 = 0$

$at^2 + bt + c = 0$

$a = 3 \;,\; b = -5 \;,\; c = -1$

$b^2 - 4ac = (-5)^2 - 4 \times 3 \times (-1) = 37$

$-b = -(-5) = +5$

$2a = 2 \times 3 = 6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{5 \pm \sqrt{37}}{6}$$

$$= \frac{5 - \sqrt{37}}{6} \quad or \quad \frac{5 + \sqrt{37}}{6}$$

$$= \frac{-1 \cdot 0827....}{6} \quad or \quad \frac{11 \cdot 0827....}{6}$$

$$t = -0 \cdot 1804.... \quad or \quad 1 \cdot 8471....$$

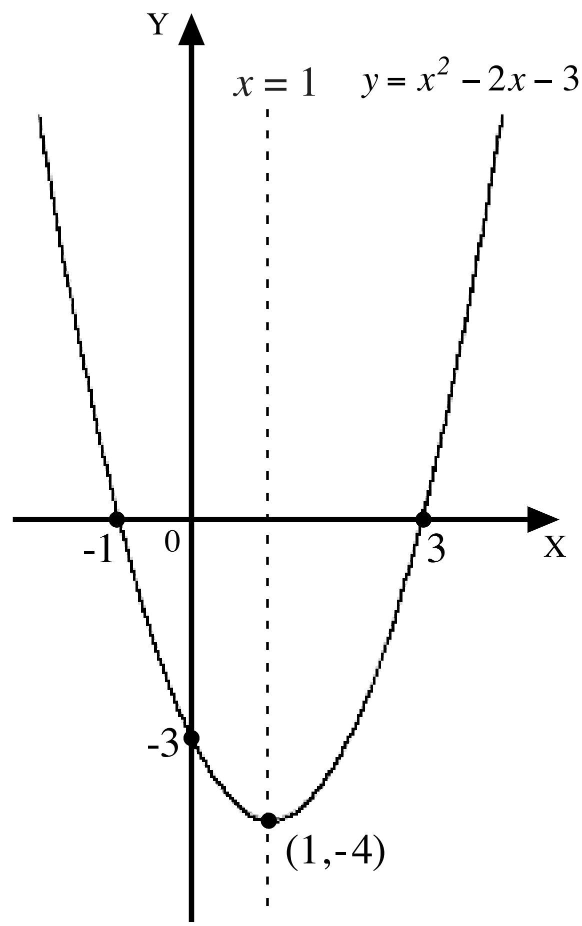
$\underline{\underline{\text{roots are } \; -0 \cdot 18 \text{ and } 1 \cdot 85}}$

GRAPHS

A sketch of the graph of a quadratic function should show where the parabola meets the axes and the maximum or minimum turning point.

Example:

Sketch the graph of the function $f(x) = x^2 - 2x - 3$.



(i) meets the Y-axis where x = 0

$f(0) = 0^2 - 2 \times 0 - 3 = -3$ *ie. y = -3*

point (0,-3)

(ii) meets the X-axis where y = 0

$$x^2 - 2x - 3 = 0$$
$$(x+1)(x-3) = 0$$
$$x + 1 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = -1 \quad \text{or} \quad x = 3$$

points (-1,0) and (3,0)

Note: **zeros** of the graph are -1 and 3.

(iii) axis of symmetry

vertical line half-way between the zeros

$x = \frac{-1+3}{2} = \frac{2}{2} = 1$, ***equation*** $x = 1$.

(iv) turning point

lies on the axis of symmetry $x = 1$

$f(1) = 1^2 - 2 \times 1 - 3 = -4$ *ie. y = -4*

point (1,-4)

Note: (1) from the graph, (1,-4) is a **minimum turning point**.

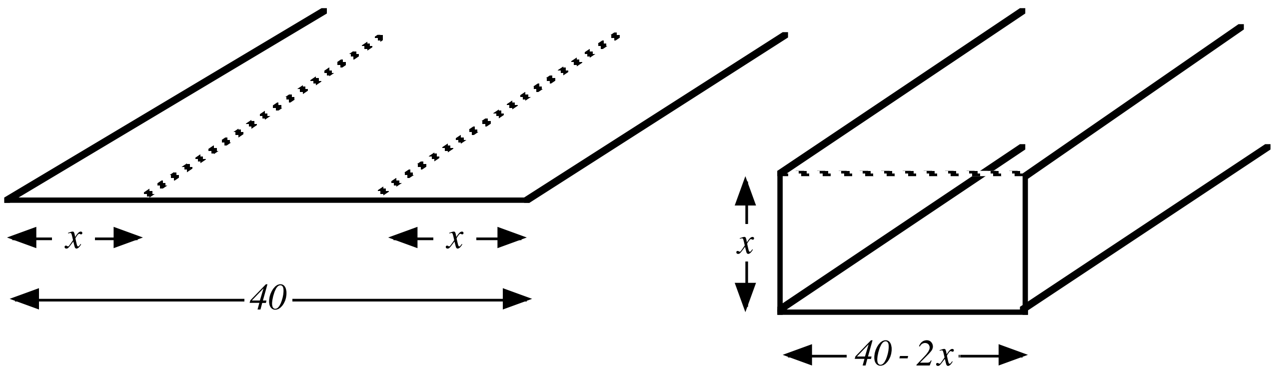
(2) the **minimum value** of the function is -4.

APPLICATIONS

Problems involving maxima or minima which can be modelled by a quadratic equation.

Example:

A sheet of metal 40 cm. wide is folded x cm from each end to form a gutter.
To maximise water flow the rectangular cross-section should be as large as possible.

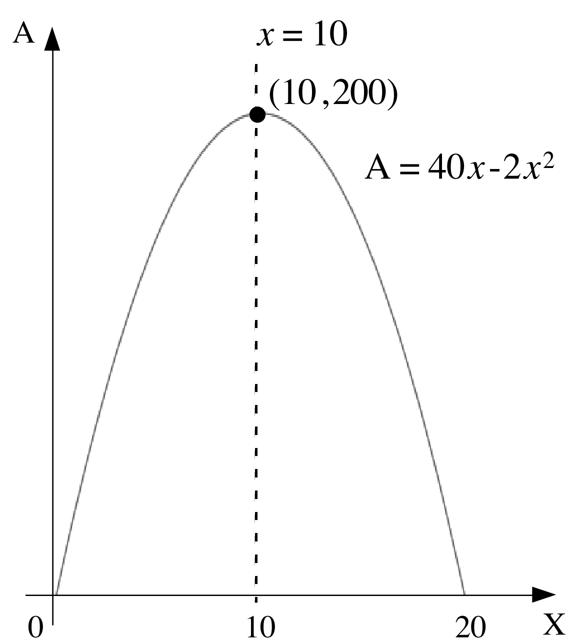


Find the maximum cross-sectional area possible.

$$\begin{aligned} A &= lb \\ &= x(40-2x) \qquad \textit{sketch the graph } A = 40x - 2x^2 \\ &= 40x - 2x^2 \end{aligned}$$

<u>*Zeros*</u>*:*

$40x - 2x^2 = 0$
$2x(20-x) = 0$
$2x = 0 \quad \text{or} \quad 20 - x = 0$
$x = 0 \quad \text{or} \quad x = 20$



<u>*Turning Point*</u>*:*

(0 + 20) ÷ 2 = 10
axis of symmetry x = 10

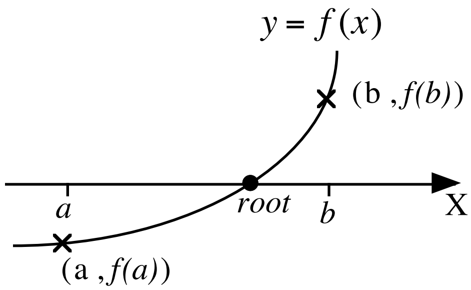
$y = 40x - 2x^2$
$y = 40 \times 10 - 2 \times 10^2 = 200$

maximum turning point(10,200)

Maximum area 200 square centimetres.

ITERATION

Repeating a process to improve the accuracy of an approximate root.



The root is where $f(x) = 0$
The value of $f(x)$, the y-coordinate, changes from negative to positive around the root.

To show a root lies between a and b, show $f(a)$ is negative and $f(b)$ is positive.

Example:

(a) Show that the cubic equation $x^3 - 2x - 3 = 0$ has a root between 1 and 2.

(b) Find the value of the root correct to **one decimal place**.

(a) $f(x) = x^3 - 2x - 3$

$x = 1$ $\quad f(1) = 1^3 - 2 \times 1 - 3 = -4$ *between 1 and 2 the function changed sign,*
$x = 2$ $\quad f(2) = 2^3 - 2 \times 2 - 3 = +1$ *so a root lies between 1 and 2*

(b)

$x = 1 \cdot 5$ $\quad 1 \cdot 5^3 - 2 \times 1 \cdot 5 - 3 = -2 \cdot 625$ *between 1·5 and 2 the sign changes*
$x = 1 \cdot 6$ $\quad 1 \cdot 6^3 - 2 \times 1 \cdot 6 - 3 = -2 \cdot 104$
$x = 1 \cdot 7$ $\quad 1 \cdot 7^3 - 2 \times 1 \cdot 7 - 3 = -1 \cdot 487$
$x = 1 \cdot 8$ $\quad 1 \cdot 8^3 - 2 \times 1 \cdot 8 - 3 = -0 \cdot 768$ *between 1·8 and 1·9 the sign changed,*
$x = 1 \cdot 9$ $\quad 1 \cdot 9^3 - 2 \times 1 \cdot 9 - 3 = +0 \cdot 059$ *so a root lies between 1·8 and 1·9*

$x = 1 \cdot 85$ $\quad 1 \cdot 85^3 - 2 \times 1 \cdot 85 - 3 = -0 \cdot 368375$

between 1·85 and 1·9 the sign changed, so a root lies between 1·85 and 1·9

root is 1·9

CHAPTER 7: PROPORTION

DIRECT PROPORTION or VARIATION

Two quantities A and B are in direct proportion if changing A by some factor changes B by the same factor.

The graph of A against B is a straight line through the origin.

The relation can be given as an equation $A = kB$, where k is the constant of variation.

"A varies directly as B"

INVERSE PROPORTION or VARIATION

Two quantities A and B are in inverse proportion if changing A by some factor changes B by the **reciprocal** (multiplcative inverse) of that factor.

The graph of A against $\frac{1}{B}$ is a straight line through the origin.

The relation can be given as an equation $A = k\frac{1}{B}$, where k is the constant of variation.

"A varies inversely as B"

JOINT VARIATION

Example:

P varies directly as N and as T and inversely as the square of r.

(a) If P = 15 when N = 8 , T = 10 and r = 4 , find a formula connecting P,N,T and r.

(b) Hence: (i) find P when N = 4 , T = 6 and r = 3.
(ii) find T when P = 15 , N = 5 and r = 2.

(a)

$$P = k\frac{NT}{r^2}$$

$$15 = k\frac{8 \times 10}{4^2}$$

$$15 = k \times 5$$

$$k = 3$$

$$P = \frac{3NT}{r^2}$$

(b) (i)

$$P = \frac{3NT}{r^2}$$

$$= \frac{3 \times 4 \times 6}{3^2}$$

$$P = 8$$

(ii)

$$P = \frac{3NT}{r^2}$$

$$15 = \frac{3 \times 5 \times T}{2^2}$$

$$60 = 15T$$

$$T = 4$$

CHAPTER 8: STATISTICS SUPPLEMENT

STANDARD DEVIATION

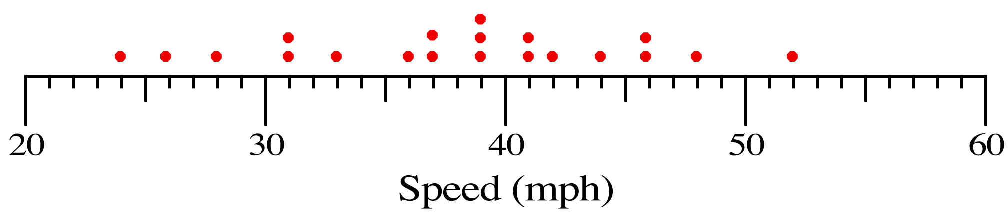
Is a measure of the spread (dispersion) of a set of data, giving a numerical value to how the data deviates from the mean.

Formulae:

mean $\bar{x} = \dfrac{\sum x}{n}$ standard deviation $s = \sqrt{\dfrac{\sum \left(x - \bar{x}\right)^2}{n-1}}$ or $s = \sqrt{\dfrac{\sum x^2 - \dfrac{\left(\sum x\right)^2}{n}}{n-1}}$

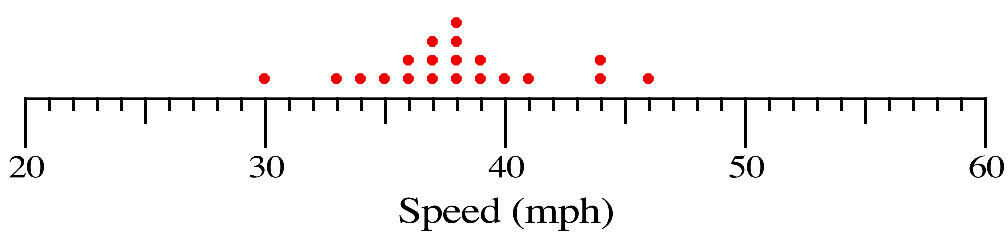
Examples,

(1) High Standard Deviation: results spread out



mean = 38 , standard deviation = 7·5

(2) Low Standard Deviation: results clustered around the mean



mean = 38 , standard deviation = 3·8

Calculations for Example (2):

	x	$x-\bar{x}$	$(x-\bar{x})^2$
	30	-8	64
	33	-5	25
	34	-4	16
	35	-3	9
	36	-2	4
	36	-2	4
	37	-1	1
	37	-1	1
	37	-1	1
	38	0	0
	38	0	0
	38	0	0
	38	0	0
	39	+1	1
	39	+1	1
	40	+2	4
	41	+3	9
	44	+6	36
	44	+6	36
	46	+8	64
totals	760	0	276

or

	x	x^2
	30	900
	33	1089
	34	1156
	35	1225
	36	1296
	36	1296
	37	1369
	37	1369
	37	1369
	38	1444
	38	1444
	38	1444
	38	1444
	39	1521
	39	1521
	40	1600
	41	1681
	44	1936
	44	1936
	46	2116
totals	760	29156

$$\bar{x} = \frac{\sum x}{n} = \frac{760}{20} = 38$$

$$s = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}} = \sqrt{\frac{276}{19}} = \sqrt{14\cdot 526....} = 3\cdot 811.... \approx 3\cdot 8$$

or

$$s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} = \sqrt{\frac{29156 - \frac{760^2}{20}}{19}} = \sqrt{\frac{276}{19}} = 3\cdot 811.... \approx 3\cdot 8$$

CHAPTER 9: INDICES AND SURDS

INDICES

$$\text{base} \longrightarrow a^n \longleftarrow \text{index or exponent}$$

INDICES RULES: require the same base. **Examples:**

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$\frac{w^2 \times w^5}{w^3} = \frac{w^7}{w^3} = w^4$$

$$\left(a^m\right)^n = a^{mn}$$

$$\left(3^5\right)^2 = 3^{10}$$

$$(ab)^n = a^n b^n$$

$$\left(2a^3b\right)^2 = 2^2a^6b^2 = 4a^6b^2$$

$$\frac{1}{a^p} = a^{-p}$$

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

$$a^0 = 1$$

$$\left(2\mathrm{b}^3\right)^0 = 1$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

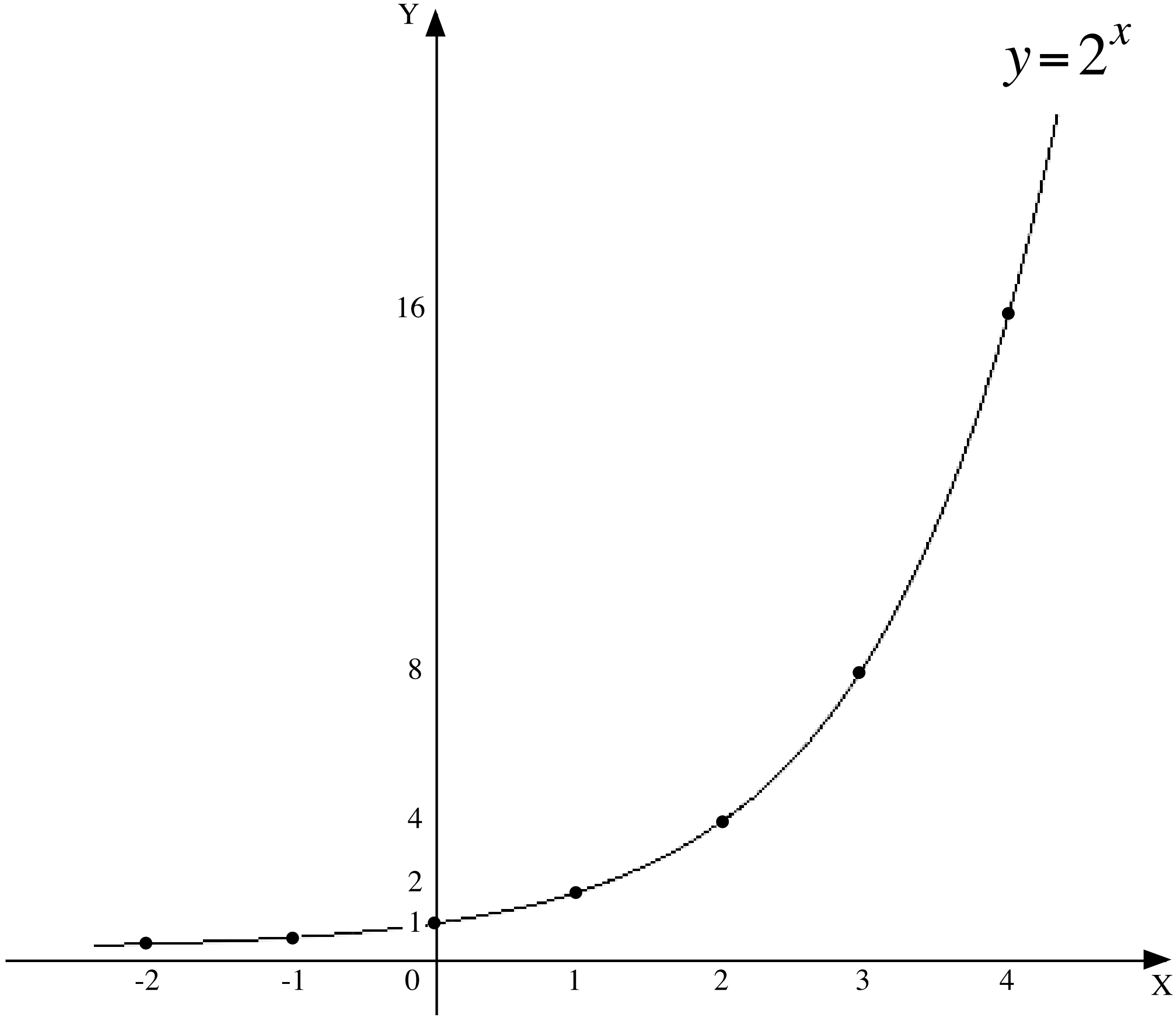
$$8^{\frac{4}{3}} = \left(\sqrt[3]{8}\right)^4 = 2^4 = 16$$

$$8^{-\frac{4}{3}} = \frac{1}{8^{\frac{4}{3}}} = \frac{1}{16}$$

EXPONENTIAL FUNCTION

A function where the variable x is the exponent $f(x) = a^x$

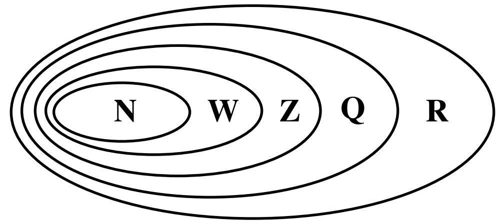
For example, $f(x) = 2^x$



SURDS

NUMBER SETS:

Natural numbers N = {1, 2, 3...}
Whole numbers W = {0, 1, 2, 3...}
Integers Z = {... -3, -2, -1, 0, 1, 2, 3...}



Rational numbers, Q , can be written as a division of two integers.
Irrational numbers **cannot** be written as a division of two integers.

Real numbers, R , are all rational and irrational numbers.

SURDS ARE IRRATIONAL ROOTS.

For example, $\sqrt{2}$, $\sqrt{\frac{5}{9}}$, $\sqrt[3]{16}$ are surds.

whereas $\sqrt{25}$, $\sqrt{\frac{4}{9}}$, $\sqrt[3]{-8}$ are **not** surds as they are 5 , $\frac{2}{3}$ and -2 respectively.

SIMPLIFYING SURDS:

RULES: $\sqrt{mn} = \sqrt{m} \times \sqrt{n}$ $\qquad$ $\sqrt{\frac{m}{n}} = \frac{\sqrt{m}}{\sqrt{n}}$

Examples:

(1) Simplify $\sqrt{24} \times \sqrt{3}$

$\sqrt{24} \times \sqrt{3}$

$= \sqrt{72}$

$= \sqrt{36} \times \sqrt{2}$ $\qquad$ 36 *is the largest square number which is a factor of* 72

$= 6 \times \sqrt{2}$

$= 6\sqrt{2}$

(2) Simplify $\sqrt{72} + \sqrt{48} - \sqrt{50}$

$\sqrt{72} + \sqrt{48} - \sqrt{50}$

$= \sqrt{36} \times \sqrt{2} + \sqrt{16} \times \sqrt{3} - \sqrt{25} \times \sqrt{2}$

$= 6\sqrt{2} + 4\sqrt{3} - 5\sqrt{2}$

$= 6\sqrt{2} - 5\sqrt{2} + 4\sqrt{3}$

$= \sqrt{2} + 4\sqrt{3}$

(3) Remove the brackets and fully simplify:

(a) $\left(\sqrt{3}-\sqrt{2}\right)^2$

$= \left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)$

$= \sqrt{3}\left(\sqrt{3}-\sqrt{2}\right) - \sqrt{2}\left(\sqrt{3}-\sqrt{2}\right)$

$= \sqrt{9}-\sqrt{6} - \sqrt{6}+\sqrt{4}$

$= 3-\sqrt{6} - \sqrt{6}+2$

$= 5-2\sqrt{6}$

(b) $\left(3\sqrt{2}+2\right)\left(3\sqrt{2}-2\right)$

$= \left(3\sqrt{2}+2\right)\left(3\sqrt{2}-2\right)$

$= 3\sqrt{2}\left(3\sqrt{2}-2\right) + 2\left(3\sqrt{2}-2\right)$

$= 9\sqrt{4}-6\sqrt{2} + 6\sqrt{2} - 4$

$= 18-6\sqrt{2} + 6\sqrt{2} - 4$

$= 14$

RATIONALISING DENOMINATORS:

Removing surds from the denominator.

Examples:

Express with a rational denominator:

(1) $\dfrac{4}{\sqrt{6}}$

$\dfrac{4}{\sqrt{6}}$

$= \dfrac{4 \times \sqrt{6}}{\sqrt{6} \times \sqrt{6}}$ *multiply the 'top' and 'bottom' by the surd on the denominator*

$= \dfrac{4\sqrt{6}}{6}$

$= \dfrac{2\sqrt{6}}{3}$

(2) $\dfrac{\sqrt{3}}{3\sqrt{2}}$

$\dfrac{\sqrt{3}}{3\sqrt{2}}$

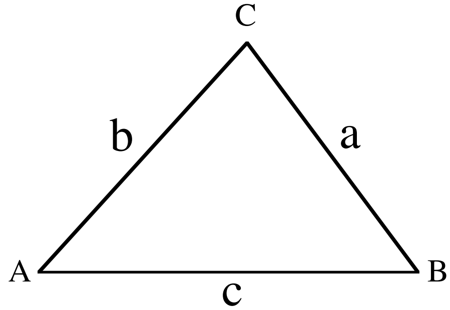
$= \dfrac{\sqrt{3} \times \sqrt{2}}{3\sqrt{2} \times \sqrt{2}}$

$= \dfrac{\sqrt{6}}{3 \times \sqrt{4}}$

$= \dfrac{\sqrt{6}}{6}$

CHAPTER 10: TRIGONOMETRY: TRIANGLE CALCULATIONS

SINE RULE

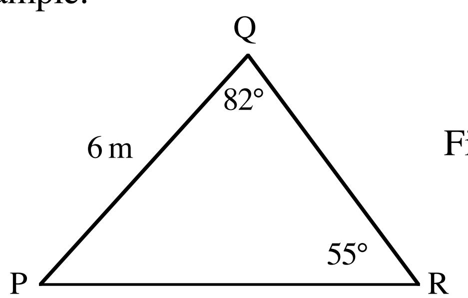


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

NOTE: requires at least one side and its opposite angle to be known.

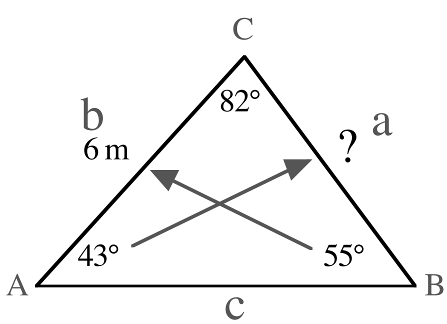
FINDING AN UNKNOWN SIDE

Example:



Find the length of side QR.

relabel triangle with a *as uknown side*
known angle/side pair labelled B and b



$$\frac{a\ ?}{\sin A \checkmark} = \frac{b \checkmark}{\sin B \checkmark} = \frac{c\ \times}{\sin C \checkmark}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 43°} = \frac{6}{\sin 55°}$$

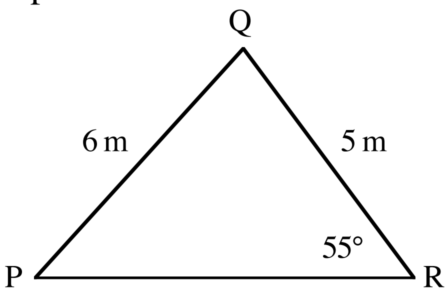
$$a = \frac{6}{\sin 55°} \times \sin 43°$$

$$= 4 \cdot 995.....$$

$$QR \approx 5 \cdot 0\ m$$

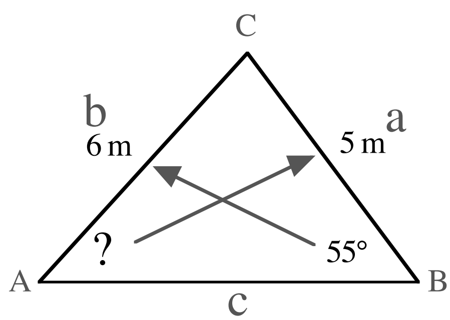
FINDING AN UNKNOWN ANGLE

Example:



Find the size of angle PQR.

cannot find angle PQR *directly but can find angle* QPR *first*
relabel triangle with A *as uknown angle* QPR
known angle/side pair labelled B and b
use the Sine Rule with the angles on the 'top'



$$\frac{\sin A}{a} \; ? = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{5} = \frac{\sin 55^\circ}{6}$$

$$\sin A = \frac{\sin 55^\circ}{6} \times 5$$

$$= 0 \cdot 682.....$$

$$A = \sin^{-1} 0 \cdot 682.....$$

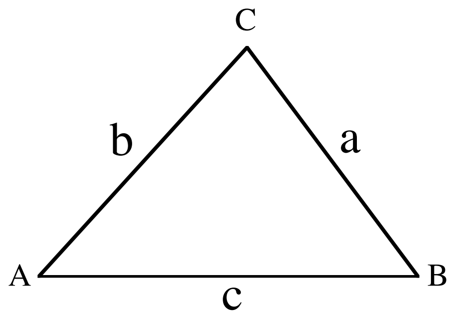
$$\angle QPR = 43 \cdot 049.....$$

$$\angle PQR = 180 - 55 - 43 \cdot 049.....$$

$$= 81 \cdot 950.....$$

$$\angle PQR \approx 82 \cdot 0^\circ$$

COSINE RULE



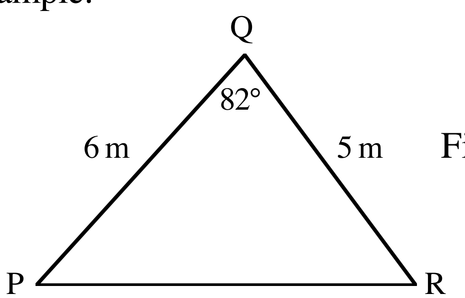
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

FINDING AN UNKNOWN SIDE $\quad a^2 = b^2 + c^2 - 2bc\cos A$

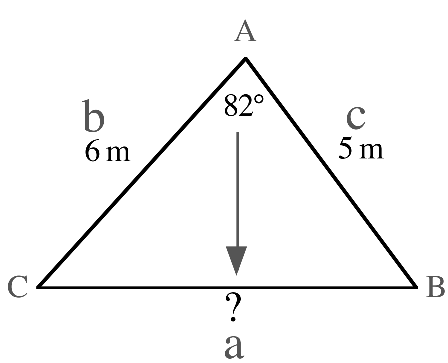
NOTE: requires knowing 2 sides and the angle between them.

Example:



Find the length of side PR.

relabel triangle with a *as uknown side*
known sides labelled b *and* c, *it doesn't matter which one is b or c*



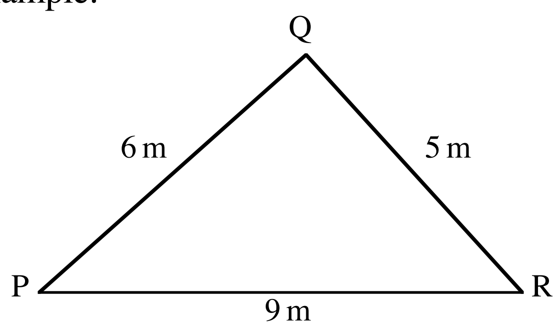
$$\begin{aligned}
a^2 &= b^2 + c^2 - 2bc\ \cos A \\
&= 6^2 + 5^2 - 2 \times 6 \times 5 \times \cos 82° \\
a^2 &= 52 \cdot 649..... \\
a &= \sqrt{52 \cdot 649.....} \\
&= 7 \cdot 256..... \\
PR &= 7 \cdot 3\ m
\end{aligned}$$

FINDING AN UNKNOWN ANGLE

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

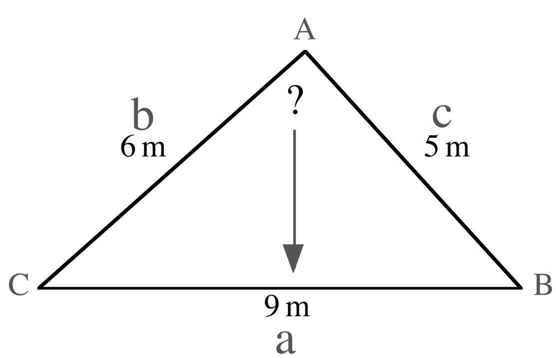
NOTE: requires knowing all 3 sides.

Example:



Find the size of angle PQR.

relabel the triangle with A *as the uknown angle and* a *as its opposite side*
other sides labelled b *and* c, *it doesn't matter which one is b or c*



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{6^2 + 5^2 - 9^2}{2 \times 6 \times 5}$$

$$= \frac{-20}{60}$$

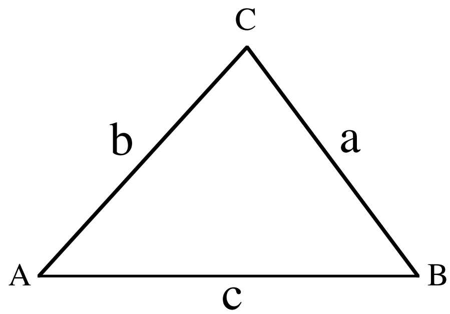
$$\cos A = -0 \cdot 333.....$$

$$A = \cos^{-1}(-0 \cdot 333.....)$$

$$= 109 \cdot 471.....$$

$$\angle PQR = 109 \cdot 5°$$

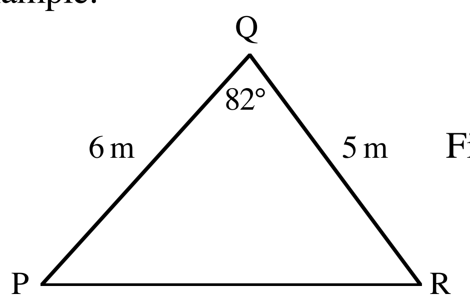
AREA FORMULA



$$Area\ \Delta ABC = \frac{1}{2}bc\sin A$$

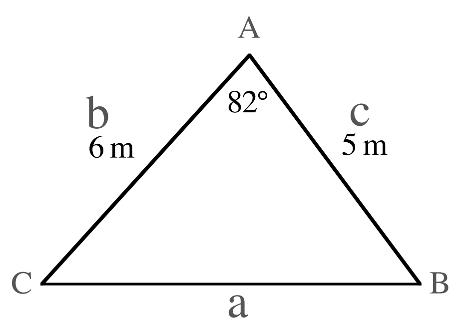
NOTE: requires knowing 2 sides and the angle between them.

Example:



Find the area of triangle PQR.

relabel triangle with A *as the known angle between 2 known sides*
the 2 known sides labelled b *and* c, *it doesn't matter which one is b or c*



$$\begin{aligned} Area\ \Delta ABC &= \frac{1}{2}bc\ \sin A \\ &= \frac{1}{2}\times 6\times 5\times \sin 82^\circ \\ &= 14\cdot 854..... \\ Area &= 14\cdot 8\ m^2 \end{aligned}$$

CHAPTER 11: FRACTIONS AND EQUATIONS

ALGEBRAIC FRACTIONS

SIMPLIFYING: (i) fully factorise 'top' and 'bottom'
(ii) 'cancel' common factors between 'top' and 'bottom'

Examples:

(1) $\dfrac{x^2-9}{x^2+2x-3}$

$= \dfrac{(x-3)(x+3)}{(x-1)(x+3)}$

$= \dfrac{x-3}{x-1}$

(2) $\dfrac{x-3}{2x^2-6x}$

$= \dfrac{1(x-3)}{2x(x-3)}$

$= \dfrac{1}{2x}$

(3) $\dfrac{3a^2b}{3a^2+3ab}$

$= \dfrac{3a \times ab}{3a(a+b)}$

$= \dfrac{ab}{a+b}$

ADD/SUBTRACT: a common denominator is required.

Examples:

(1) $\dfrac{1}{x} + \dfrac{3}{x(x-3)}$

$= \dfrac{1(x-3)}{x(x-3)} + \dfrac{3}{x(x-3)}$

$= \dfrac{x-3+3}{x(x-3)}$

$= \dfrac{x}{x(x-3)}$

$= \dfrac{1}{x-3}$

(2) $\dfrac{3}{x-3} - \dfrac{2}{x+3}$

$= \dfrac{3(x+3)}{(x-3)(x+3)} - \dfrac{2(x-3)}{(x-3)(x+3)}$

$= \dfrac{3(x+3)-2(x-3)}{(x-3)(x+3)}$

$= \dfrac{3x+9-2x+6}{(x-3)(x+3)}$

$= \dfrac{x+15}{(x-3)(x+3)}$

EQUATIONS WITH FRACTIONS

(i) write the fractions with common denominators
(ii) multiply both sides of the equation to remove the denominators

Examples:

(1) Solve $\frac{1}{2}(x+3)+\frac{1}{3}x=1$

$$\frac{3}{6}(x+3)+\frac{2}{6}x=\frac{6}{6} \qquad LCM(2,3)=6$$

$$3(x+3)+2x=6 \qquad \textit{multiplied both sides by } 6$$

$$3x+9+2x=6$$

$$5x=-3$$

$$x=-\frac{3}{5}$$

(2) Solve $\frac{4}{3-x} - \frac{1}{x} = 1$

$$\frac{4x}{x(3-x)} - \frac{3-x}{x(3-x)} = \frac{x(3-x)}{x(3-x)} \qquad \textit{common denominator}$$

$$4x - (3-x) = x(3-x) \qquad \textit{multiplied both sides by } x(3-x)$$

$$4x - 3 + x = 3x - x^2$$

$$x^2+2x-3 = 0$$

$$(x+3)(x-1) = 0$$

$$x+3=0 \quad \textit{or} \quad x-1=0$$

$$x=-3 \quad \textit{or} \quad x=1$$