Proving Trigonometric Identities - Lesson 2

Proving Trigonometric Identities (Difficult Types)

LI

- Know what a Trigonometric Identity is.
- Prove difficult trigonometric identities.

SC

- Pythagorean Identity.
- Link between sin x, cos x and tan x.

Example 1

Show that $\sin^4 x - \cos^4 x = 1 - 2\cos^2 x$.

$$
\begin{aligned}
\text{LHS} &= \sin^4 x - \cos^4 x \\
&= (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) \\
&= (\sin^2 x - \cos^2 x)(1) \\
&= \sin^2 x - \cos^2 x \\
&= (1 - \cos^2 x) - \cos^2 x \\
&= 1 - 2\cos^2 x \\
&= \text{RHS}
\end{aligned}
$$

$$\boxed{\text{LHS} = \text{RHS; hence, } \sin^4 x - \cos^4 x = 1 - 2\cos^2 x}$$

Example 2

Show that $\dfrac{\sin^4 x - \cos^4 x}{\sin^2 x \cos^2 x} = \dfrac{1}{\cos^2 x} - \dfrac{1}{\sin^2 x}$.

$$\text{LHS} = \frac{\sin^4 x - \cos^4 x}{\sin^2 x \cos^2 x}$$

$$= \frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)}{\sin^2 x \cos^2 x}$$

$$= \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x}$$

$$= \frac{\sin^2 x}{\sin^2 x \cos^2 x} - \frac{\cos^2 x}{\sin^2 x \cos^2 x}$$

$$= \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x}$$

$$= \text{RHS}$$

$$\boxed{\text{LHS} = \text{RHS}; \text{ hence, } \frac{\sin^4 x - \cos^4 x}{\sin^2 x \cos^2 x} = \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x}}$$

Example 3

Show that $\dfrac{1}{\cos^4 x} - \tan^4 x = \dfrac{1 + \sin^2 x}{\cos^2 x}$.

$$\text{LHS} = \frac{1}{\cos^4 x} - \tan^4 x$$

$$= \frac{1}{\cos^4 x} - \left(\frac{\sin x}{\cos x}\right)^4$$

$$= \frac{1}{\cos^4 x} - \frac{\sin^4 x}{\cos^4 x}$$

$$= \frac{1 - \sin^4 x}{\cos^4 x}$$

$$= \frac{(1 - \sin^2 x)(1 + \sin^2 x)}{\cos^4 x}$$

$$= \frac{\cos^2 x(1 + \sin^2 x)}{\cos^4 x}$$

$$= \frac{1 + \sin^2 x}{\cos^2 x}$$

$$= \text{RHS}$$

$$\boxed{\text{LHS} = \text{RHS; hence, } \frac{1}{\cos^4 x} - \tan^4 x = \frac{1 + \sin^2 x}{\cos^2 x}}$$

Example 4

Show that $$\frac{\sin^2 x}{(1 - \cos x)^2} = \frac{1 + \cos x}{1 - \cos x}.$$

$$\text{RHS} = \frac{1 + \cos x}{1 - \cos x}$$

$$= \frac{1 + \cos x}{1 - \cos x} \times \frac{1 - \cos x}{1 - \cos x}$$

$$= \frac{(1 + \cos x)(1 - \cos x)}{(1 - \cos x)^2}$$

$$= \frac{1 - \cos^2 x}{(1 - \cos x)^2}$$

$$= \frac{\sin^2 x}{(1 - \cos x)^2}$$

$$= \text{LHS}$$

$$\boxed{\text{RHS} = \text{LHS}; \text{ hence, } \frac{\sin^2 x}{(1 - \cos x)^2} = \frac{1 + \cos x}{1 - \cos x}}$$

Questions

Prove these trigonometric identities :

1) $$\frac{\cos^4 x - \sin^4 x}{\cos^2 x} = 1 - \tan^2 x$$

2) $$\frac{1 - \sin^4 x}{1 + \sin^2 x} = \cos^2 x$$

3) $$\frac{1 - \cos^4 x}{1 + \cos^2 x} = \sin^2 x$$

4) $$\frac{\cos x}{1 + \sin x} - \frac{1 - \sin x}{\cos x} = 0$$

5) $$\frac{\sin x}{1 + \cos x} - \frac{1 - \cos x}{\sin x} = 0$$

6) $$\frac{(1 - \sin x)^2}{\cos^2 x} = \frac{1 - \sin x}{1 + \sin x}$$

7) $$\frac{\sin^3 x - 8}{\sin x - 2} = \sin^2 x + 2\sin x + 4$$

8) $$\frac{\cos^4 x - \sin^4 x}{\sin^2 x} = \frac{1}{\tan^2 x} - 1$$

9) $$\frac{1}{\sin^4 x} - \frac{1}{\tan^4 x} = \frac{1 + \cos^2 x}{\sin^2 x}$$

10) $$\frac{1}{\sin x} - \sin x = \frac{\cos x}{\tan x}$$

11) $$\frac{1}{\cos x} - \cos x = \sin x \tan x$$

12) $$\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} = \frac{2}{\cos x}$$

13) $$\frac{1 - \cos^3 x}{1 - \cos x} = \cos^2 x + \cos x + 1$$

14) $$\frac{1 - \sin x}{\cos^3 x} = \frac{1}{\cos x (\sin x + 1)}$$

15) $$\sin x (1 - \cos x) = \frac{\sin^3 x}{1 + \cos x}$$

16) $$\frac{\cos x}{1 - \sin x} - \frac{\cos x}{1 + \sin x} = 2 \tan x$$

17) $$\frac{\cos^4 x - \sin^4 x}{\cos^2 x - \sin^2 x} = 1$$

18) $$\frac{\tan^2 x}{\tan^2 x + 1} = \sin^2 x$$

19) $1 - 2\cos^2 x = \dfrac{\tan^2 x - 1}{\tan^2 x + 1}$

20) $\dfrac{1}{1 - \sin x} - \dfrac{1}{1 + \sin x} = \dfrac{2\tan x}{\cos x}$

21) $\dfrac{1 + \sin x}{1 - \sin x} - \dfrac{1 - \sin x}{1 + \sin x} = \dfrac{4\tan x}{\cos x}$

22) $\dfrac{1 + \tan^2 x}{1 - \tan^2 x} = \dfrac{1}{\cos^2 x - \sin^2 x}$

23) $\dfrac{\cos x}{1 - \sin x} - \tan x = \dfrac{1}{\cos x}$

24) $\tan x + \dfrac{\cos x}{1 + \sin x} = \dfrac{1}{\cos x}$

25) $\tan^2 x + 1 + \dfrac{\tan x}{\cos x} = \dfrac{1 + \sin x}{\cos^2 x}$

26) $\dfrac{1 - \tan^3 x}{1 - \tan x} = \dfrac{1}{\cos^2 x} + \tan x$

27) $\dfrac{\tan^3 x + 1}{\tan^3 x + \tan^2 x} = \dfrac{1}{\sin^2 x} - \dfrac{1}{\tan x}$