Proving Trigonometric Identities - Lesson 1

Proving Trigonometric Identities
(Easy Types)

LI

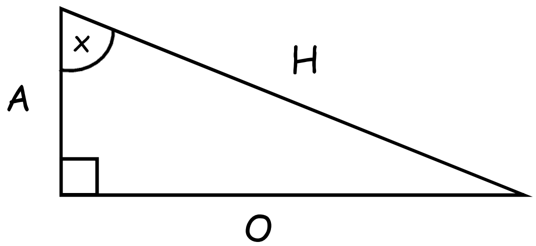
- Know what a Trigonometric Identity is.
- Prove simple trigonometric identities.

SC

- Pythagorean Identity.
- Link between sin x, cos x and tan x.

We will use the following shorthands :

- $\sin x$ for $\sin x^\circ$ etc. .
- $\sin^2 x$ for $(\sin x)^2$ etc. .



$$\sin x = \frac{O}{H}$$

$$\cos x = \frac{A}{H}$$

$$\tan x = \frac{O}{A}$$

$$H^2 = O^2 + A^2$$

$$\begin{aligned}
\sin^2 x + \cos^2 x &= \left(\frac{O}{H}\right)^2 + \left(\frac{A}{H}\right)^2 \\
&= \frac{O^2}{H^2} + \frac{A^2}{H^2} \\
&= \frac{O^2 + A^2}{H^2} \\
&= \frac{H^2}{H^2} \\
&= 1
\end{aligned}$$

We thus have the Pythagorean Identity :

$$\boxed{\sin^2 x + \cos^2 x = 1}$$

$$\begin{aligned}
\frac{\sin x}{\cos x} &= \frac{O}{H} \div \frac{A}{H} \\
&= \frac{O}{H} \times \frac{H}{A} \\
&= \frac{O\,H}{H\,A} \\
&= \frac{O}{A} \\
&= \tan x
\end{aligned}$$

Hence,

$$\boxed{\frac{\sin x}{\cos x} = \tan x}$$

Trigonometric Identity - equation involving trig. functions that is true for all allowable x - values.

To show that an identity is true, normally start with the more complicated side and try to show that it equals the other side

Sometimes, it is easier to work out both sides separately and show they equal the same expression

The two sides of an equation have a standard shorthand :

LHS : Left - Hand Side

RHS : Right - Hand Side

Example 1

Show that $\cos x \tan x = \sin x$.

$$\text{LHS} = \cos x \tan x$$

$$= \cos x \left(\frac{\sin x}{\cos x}\right)$$

$$= \frac{\cos x}{1} \times \frac{\sin x}{\cos x}$$

$$= \frac{\cos x \sin x}{\cos x}$$

$$= \sin x$$

$$= \text{RHS}$$

$\boxed{\text{LHS} = \text{RHS}\text{; hence, } \cos x \tan x = \sin x}$

Example 2

Show that $7\cos^2 x + 7\sin^2 x = 7$.

$$\begin{aligned} \text{LHS} &= 7\cos^2 x + 7\sin^2 x \\ &= 7(\cos^2 x + \sin^2 x) \\ &= 7(1) \\ &= 7 \\ &= \text{RHS} \end{aligned}$$

$$\boxed{\text{LHS} = \text{RHS; hence, } 7\cos^2 x + 7\sin^2 x = 7}$$

Example 3

Show that $\dfrac{1 - \cos^2 x}{\cos^2 x} = \tan^2 x$.

$$\text{LHS} = \frac{1 - \cos^2 x}{\cos^2 x}$$

$$\sin^2 x + \cos^2 x = 1$$
$$ - \cos^2 x \qquad - \cos^2 x$$
$$\sin^2 x = 1 - \cos^2 x$$

$$= \frac{\sin^2 x}{\cos^2 x}$$

$$= \left(\frac{\sin x}{\cos x}\right)^2$$

$$= (\tan x)^2$$

$$= \tan^2 x$$

$$= \text{RHS}$$

$$\boxed{\text{LHS} = \text{RHS}; \text{ hence, } \frac{1 - \cos^2 x}{\cos^2 x} = \tan^2 x}$$

Questions

Prove these trigonometric identities :

1) $9 \sin^2 x = 9 - 9 \cos^2 x$

2) $(\sin x - \cos x)(\sin x - \cos x) = 1 - 2 \sin x \cos x$

3) $(\cos x + \sin x)^2 = 1 + 2 \sin x \cos x$

4) $(\cos x + \sin x)(\cos x - \sin x) = 2 \cos^2 x - 1$

5) $(\cos x - \sin x)(\cos x + \sin x) = 1 - 2 \sin^2 x$

6) $\sin x - \sin x \cos^2 x = \sin^3 x$

7) $(\sin x - \cos x)^2 + (\sin x + \cos x)^2 = 2$

8) $(\sin x + \cos x)^2 - (\sin x - \cos x)^2 = 4 \sin x \cos x$

9) $\sin x \cos^2 x + \sin^3 x = \sin x$

10) $\cos x \sin^2 x + \cos^3 x = \cos x$

11) $\cos^2 x (1 + \tan^2 x) = 1$

12) $(1 - \sin x)(1 + \sin x) = \cos^2 x$

13) $(1 - \cos x)(1 + \cos x) = \sin^2 x$

14) $2\cos^2 x - 1 = 1 - 2\sin^2 x$

15) $3\cos^2 x - 2 = 1 - 3\sin^2 x$

16) $4\sin^2 x - 3 = 1 - 4\cos^2 x$

17) $4\cos^2 x - 2\sin^2 x = 6\cos^2 x - 2$

18) $5\sin^2 x + 3\cos^2 x = 2\sin^2 x + 3$

19) $(3\cos x + 4\sin x)^2 + (4\cos x - 3\sin x)^2 = 25$

20) $(2\cos x + 5\sin x)^2 + (5\cos x - 2\sin x)^2 = 29$

21) $(1 - \sin x)(\sin^2 x + 4\sin x + 3) = (3 + \sin x)\cos^2 x$

22) $\sin^2 x(2 + \cos x) = (1 - \cos x)(\cos^2 x + 3\cos x + 2)$

23) $\sin^3 x + \cos^3 x = (1 - \sin x \cos x)(\sin x + \cos x)$

24) $\sin x \cos x \tan x = 1 - \cos^2 x$

25) $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$

26) $1 + \cos x = \dfrac{\sin^2 x}{1 - \cos x}$

27) $1 + \sin x = \dfrac{\cos^2 x}{1 - \sin x}$

28) $\dfrac{\sin^2 x}{1 - \sin^2 x} = \tan^2 x$

29) $\dfrac{1}{\cos^2 x} - \tan^2 x = 1$

30) $\tan x - \dfrac{1}{\tan x} = \dfrac{\sin^2 x - \cos^2 x}{\sin x \cos x}$

31) $\dfrac{1}{\cos x} - \dfrac{1}{\sin x} = \dfrac{\sin x - \cos x}{\sin x \cos x}$

32) $\dfrac{\cos x}{\tan x} + \sin x = \dfrac{1}{\sin x}$

33) $\tan x \sin x + \cos x = \dfrac{1}{\cos x}$

34) $$\frac{1}{1 + \cos x} + \frac{1}{1 - \cos x} = \frac{2}{\sin^2 x}$$

35) $$\frac{1}{1 - \sin x} + \frac{1}{1 + \sin x} = \frac{2}{\cos^2 x}$$

36) $$\frac{1}{\tan x} + \tan x = \frac{1}{\sin x \cos x}$$

37) $$\frac{1 + \tan x}{1 - \tan x} = \frac{\cos x + \sin x}{\cos x - \sin x}$$

38) $$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{1}{\sin x \cos x}$$

39) $$\frac{1 - \cos^2 x}{\cos^2 x} \times \frac{1 - \sin^2 x}{\sin^2 x} = 1$$

40) $$\frac{\sin^2 x}{\cos x} - \tan x = \tan x\,(\sin x - 1)$$

41) $$\frac{\cos^2 x}{\sin^2 x} = \frac{1}{\tan^2 x}$$

42) $$\frac{\tan^7 x}{\sin^7 x} = \frac{1}{\cos^7 x}$$

43) $$\tan x - \frac{\sqrt{1 - \cos^2 x}}{\sqrt{1 - \sin^2 x}} = 0$$

44) $$(\sin x \cos x \tan x)^6 = \sin^{12} x$$