## **Balfron High School**

# Numeracy Information Booklet

update 2012













### **Numeracy Skills**

Literacy and numeracy are regarded as foundation stones for other learning. Numerical and graphical skills are essential for the expression and interpretation of quantitative information.

It is important that all teachers whose work involves the use of numerical and graphical information are aware of the skills it is reasonable to expect pupils to have at various stages in their school careers.

In this booklet you will find outlined the numerical and information handling skills to be expected in S1 and 2 for pupils working within the CfE levels 1-4 and information about the teaching methods used by teachers in this school and in our associated primary schools.

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3 – 6	Expectations at Level 1 to 4
7 – 23	Teaching methods for basic skills
24 - 26	Mental calculation strategies

### **Mental Agility Levels**

**Level Early** add single digits and 10 to single digits e.g. 5 + 8, 10 + 9subtract single digits from single digits and from 10 e.g. 10-3, 7-4Level 1 add numbers to 20 e.g. 15 + 9, 17 + 16subtract numbers up to 20 e.g. 20-5, 14-6add and subtract single digit numbers to/from numbers bigger than 20 e.g. 46 + 5, 57 - 6use 2, 3, 4, 5 and 10 times tables to multiply e.g. 4 x 2, 5 x 7 use 2, 3, 4, 5 and 10 times tables to divide with no remainders e.g.  $24 \div 4$ ,  $90 \div 10$ find halves in the context of the 2 times table e.g. half of 12, half of 18 find quarters in the context of the 4 times table e.g. quarter of 32, quarter of 40 add single digits to 2 or 3 digits e.g. 76 + 7, 234 + 5subtract single digits from 2 or 3 digits e.g. 678 - 9, 456 - 8add multiples of 10 to 3 digits e.g. 235 + 40, 789 + 50subtract multiples of 10 from 3 digits e.g. 536 - 40, 125 - 60find thirds, fifths, tenths using the 3, 5 and 10 times table e.g. a third of 18, a fifth of 35 find halves and quarters beyond 2 and 4 times tables e.g. half of 70, quarter of 200 Level 2 use all times tables to 10 x 10 to multiply e.g. 6 x 7, 4 x 9 use all times tables to 10 x 10 to divide e.g.  $72 \div 8, 45 \div 9$ divide to get remainders e.g.  $14 \div 3$ ,  $26 \div 4$ multiply 2 and 3 digit numbers by 10 e.g. 34 x 10, 450 x 10 add 2 digits to 2 digits e.g. 23 + 89, 45 + 86subtract 2 digits from 2 digits from 2 digits e.g. 87 - 45, 86 - 23add multiples of 10 and 100 to 3 digits e.g. 123 + 80, 457 + 600 subtract multiples of 10 and 100 from 3 digits e.g. 256 - 50, 874 - 500multiply 2 or 3 digits by a single digit e.g. 25 x 6, 345 x 4 divide 2 or 3 digits by a single digit e.g.  $64 \div 4$ ,  $600 \div 5$ multiply decimals by 10 and 100 e.g. 4.5 x 10, 6.7 x 100 divide decimals by 10 and 100 e.g.  $4.8 \div 10$ ,  $36.98 \div 100$ add decimals e.g. 3.2 + 6.5, 10.3 + 5.4subtract decimals e.g. 5-2.1, 4.56-1.2multiply decimals by 10, 100 and 1000 e.g. 3.5 x 1000, 6.24 x 1000 divide decimals by 10, 100 and 1000 e.g.  $4.3 \div 1000$ ,  $2.59 \div 1000$ multiply 2 and 3 digits by multiples of 10 and 100 e.g. 23 x 400, 64 x 30 divide 2 and 3 digits by multiples of 10 and 100 e.g.  $340 \div 20$ ,  $8000 \div 200$ add decimals e.g. 1.23 + 0.67, 9.87 + 20.3subtract decimals e.g. 8.12 - 6.24, 6.25 - 0.7multiply decimals by a single digit e.g. 2.3 x 4, 5.67 x 8 divide decimals by a single digit e.g.  $0.6 \div 3$ ,  $5.6 \div 7$ Level 3 add negative numbers e.g. 6 + -3, -5 + -8subtract negative numbers e.g. 5 - -7, -8 - 6multiply negative numbers e.g.  $5 \times -6$ ,  $-7 \times -8$ divide negative numbers e.g.  $-36 \div -6$ ,  $-42 \div -7$ 

### Numberwork

work with numbers		Level
	up to 20	1
	up to 1000	1
	up to 10 000	2
	up to 1 000 000	2 3
	negative numbers	3
	integers, powers, roots, standard form	4
calculate with	out a calculator	
	add / subtract 2 digit numbers	1
	add / subtract 2 digit and 3 digit numbers	1
	add / subtract 4 digit numbers including decimals	2
	multiply and divide 2 digits by 2, 3, 4, 5, 10	1
	multiply and divide 2 digit by single digit	
	multiply and divide 4 digit by single digit	$\frac{-}{2}$
	multiply and divide any number of digits up to 3 dp	3
	multiply and divide any number of digits	2 2 3 3
mounding numl	2040	
rounding num	2 digit to nearest 10	1
	<del>-</del>	1
	3 digit to nearest 10	
	any number to nearest 10, 100	2 3 3
	any number to 1 decimal place	3
	as required including significant figures	3
work with frac	tions and percentages	
	halves, quarters of quantities	1
	thirds, fifths, tenths of quantities	1
	simple fraction of a quantity	1
	widely used fractions of whole numbers	2
	percentage of whole number	2
	identify a simple ratio	2
	$+$ - x fraction ( $\square$ by a fraction is beyond Level 4)	4
	equivalence of widely used fractions and percentages	1
	equivalence of fractions, ratios, percentages	2
work with time	e	
	days, seasons, tell time in hours	early
	read digital and half and quarter on analogue clock	1
	12 hour clock, time intervals less than 1 hour, calendar	1
	24 hour clock, time interval in hours and minutes	2
	tenths, hundredths of seconds from stopwatches	3
	speed, distance, time calculations	3
	1 , , , , , , , , , , , , , , , , , , ,	-

### Measurement

length	handspans, non-standard units metre and centimetre millimetre, kilometre, common imperial	early 1 2
weight	non-standard units kilogram, gram, weight conserved common imperial	early 1 2
capacity	non-standard units litres millilitres volume conserved	early 1 2 1
area	non-standard units area conserved square cm/m/km, hectare find area using squared paper area triangle using squared paper area of square and rectangle using formula kite, parallelogram etc	early 1 2 1 1 3 3
volume	rules for cube and cuboid	3
temperature	above zero below zero	2 2
measuring	to nearest labelled graduation to nearest graduation by estimating between graduations	1 1 2

### **Handling Information**

collection by	survey	Level
_	1 direct question	1
	yes / no questionnaire	1
	questionnaire with several responses	2
	simple sampling strategy	3
	structured questionnaire multi-response	4
	sampling avoiding bias	4
organising inf	Formation	
	tally without grouping	1
	tally in groups	2
	use tables to record	2
	design and use tables	3
	grouping discrete / continuous data	4
displaying inf	Formation	
	bar graph with unit scale	1
	bar graph with scale in multiples	1
	bar graph, line graph, pie chart (simple fractions)	2
	line and curved graphs	2 2 2 3
	pie charts in percentages	2
	pie charts raw data	3
	scatter graphs / stem and leaf	3
interpreting in	nformation	
	answer direct question	1
	identify most and least	1
	retrieve information subject to 1 condition	1
	retrieve information subject to more than one condition	2
	describe features and trends	2
	retrieve information from range of displays	2 3 3 3 3
	retrieve information from extended range of displays	3
	use means for comparisons	3
	describe correlation	3
	mean, median, mode, range	3
probability		
	simple probability, certain events, impossible events	2

#### **Estimating and Rounding**

#### **Estimating**

```
As a guide pupils should be able to:
```

```
at Level 1 estimate height and length in centimetres and metres e.g. length of pencil = 10 \, \text{cm}, width of classroom = 7 \, \text{m} at Level 2 estimate small weights, small areas, small volumes e.g. bag of sugar = 1 \, \text{kg}, area of text book page = 200 \, \text{cm}^2, volume of a mug = 300 \, \text{ml} at Level 3 estimate areas in square metres, lengths in mm e.g. area of the room = 70 \, \text{m}^2, diameter of 1 \, \text{p} = 15 \, \text{mm}
```

### Rounding

As a guide pupils should be able to:

```
at Level 1 round 2 digit whole numbers to the nearest 10 eg 63 to the nearest 10 → 60

at Level 1 round 3 digit whole numbers to the nearest 10 eg 749 to the nearest 10 → 750

at Level 2 round any number to the nearest whole number, 10 or 100 eg 234.7 → 235 to nearest whole number, 234.7 → 230 to nearest 10, 234.7 → 200 to nearest 100

at Level 3 round any number to 1 decimal place eg 9.76 → 9.8 to 1 decimal place

at Level 3 round to any number of decimal places or significant figures eg 2.3456 → 2.35 to 2 decimal places, 2.3456 → 2.3 to 2 significant figures
```

We always round up for 5 or above eg  $1.253 \rightarrow 1.3$  to 1 decimal place,  $1.257 \rightarrow 1.3$  to 1 decimal place

We only consider the first digit after the required accuracy digit for rounding purposes eg  $3.141592654 \rightarrow 3.1$  to 1 dp,  $3.141592654 \rightarrow 3.142$  to 3 dp,  $3.141592654 \rightarrow 3.14159$  to 5 dp

#### **Subtraction**

As a written method, we do subtraction using decomposition.

Alternative methods are used for mental calculations eg counting on, breaking up the number being subtracted. Examples are given in the section on Short Methods.

Decomposition

we do not back" and pay back"

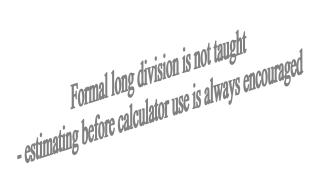
### **Division**

 $37 \div 5$ ,  $\frac{37}{5}$  and  $5)\overline{37}$  should all be recognised as division calculations

Each of the following answer styles may be appropriate dependant upon context

$$\frac{7 \text{ rem } 2}{5)37}$$
  $\frac{7^2}{5)37}$   $\frac{7.4}{5)37.0}$ 

Alternative methods are used for mental calculations eg breaking up the divisor. Examples are given in the section on Short Methods.



#### **Fractions**

At **Level 1** pupils should be able to find 1 part of a quantity eg 1 of 12 = 4 and to undertake a division calculation,  $12 \div 3 = 4$ 3

At Level 2 pupils should be able to find a simple fraction of a quantity eg  $\underline{4}$  of 56 = 32

and to undertake a division calculation followed by a multiplication,  $(56 \div 7 = 8, 8 \times 4 = 32)$ 

At Level 2 pupils should be able to find and use equivalent fractions, percentages and decimals of widely used fractions

$$\frac{7}{10} = 0.7 = 70\%$$

At **Level 4** pupils undertake formal +, -, x, of fractions

(÷ by a fraction is beyond Level 4 but is undertaken by most pupils with work on other fraction calculations)

#### **Addition and Subtraction**

$$\frac{2}{3} + \frac{4}{7}$$

$$=$$
  $\frac{14}{21}$  +  $\frac{12}{21}$  find and change to common denominator

$$=$$
  $\frac{26}{21}$  add numerators

$$= 1^{5}/_{21}$$
 convert improper fractions to mixed numbers

#### Multiplication

$$\frac{2}{3}$$
 x  $\frac{5}{6}$ 

$$=\frac{5}{9}$$
 simplify the answer

#### **Division**

$$\frac{2}{3} \div \frac{5}{6}$$

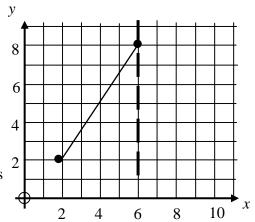
$$\frac{1}{3}$$
 5 then multiply

$$= 12$$
 multiply the numerators

#### **Coordinates**

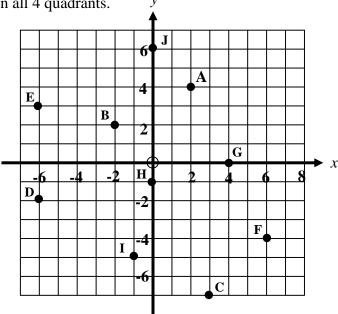
At **Level 2/3** pupils should be able to use coordinates in the  $1^{st}$  quadrant where both numbers are positive and should know and use the terms x-axis, y-axis, x coordinate, y coordinate and origin

Grid <u>lines</u> are numbered Points should be denoted eg (2, 3) with a comma between the numbers and using round brackets



At **Level 4** pupils should be able to use coordinates in all 4 quadrants.

eg D(-6, -2)



#### **Percentages**

At **Level 2** pupils should be able to calculate common percentages, 25%, 50%, 10%, 1% without a calculator and combine these to find other amounts eg 35%,  $17^{1}/_{2}$ %

At Level 2 pupils should be able to find and use equivalent fractions, percentages and decimals of widely used fractions

$$\frac{7}{10} = 0.7 = 70\%$$

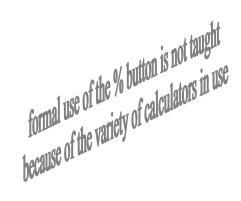
At **Level 3** pupils should be able to find percentages of quantities with a calculator eg 42% of £345 =  $0.42 \times 345 =$ £144.90

At **Level 3** pupils should be able to find and use all equivalent fractions, percentages and decimals.

At **Level 3** pupils should be able to find and use percentage increase and decrease eg A car is bought for £5000 and sold for £3500. Calculate the percentage loss. actual loss = £1500

% loss =  $\frac{1500}{5000}$  x 100%

% loss = 30%



### **Proportion**

At **Level 4** pupils should be able to identify direct and inverse proportion, use the unitary method for calculations and round only at the final answer stage.

#### eg Direct unitary method

If 5 pencils cost 90 pence, how much do 7 cost?

pencils cost  

$$5 \rightarrow 90$$
  
 $1 \rightarrow 90 \div 5 = 18$   
 $7 \rightarrow 18 \times 7 = £1.26$ 

eg Inverse unitary method

A journey takes 40 minutes at 60 mph, how long will the same journey take at 50 mph?

```
speed time
60 \rightarrow 40
1 \rightarrow 40 \times 60 = 2400
50 \rightarrow 2400 \div 50 = 48 \text{ minute}
```

### **Equations – change of subject of formulae**

Prior to level 3, equations are usually solved by "covering up" the variable and asking for solution eg 5 + x = 7 say 5 add what makes 7? eg 5x = 35 say 5 multiplied by what makes 35? x = 2 x = 7

At **Level 3** pupils should solve equations by use of inverse operations and balancing ie doing the same to both sides of the equation

eg 3x - 7 = 8 add 7 to both sides eg 7 + 5p = 34 subtract 9 from both sides 5p = 27 divide both sides by 5  $p = \frac{27}{5}$   $p = 5^{2}/5$ 

eg  $\underline{5} = 7$  now multiply both sides by x 5 = 7x now divide both sides by 7  $\underline{5} = x$   $\underline{7}$ 

At **Level 4** inverse operations will be applied to more complex equations

eg 5f + 6 = 7(f - 8) remove brackets by multiplying 5f + 6 = 7f - 56 subtract 5f from both sides ie "sort" variables 6 = 2f - 56 add 56 to both sides ie "sort" numbers 62 = 2f divide both sides by 2 31 = f

#### In general

"undo" the equation by applying the same inverse operation to both sides of the equation "undo" + with —, — with +,  $\times$  with  $\div$ ,  $\div$  with  $\vee$ 

the letter x should be written differently to the multiplication sign × only 1 equality sign per line equality signs should be beneath each other

We do not sign × only 1 equality signs should be beneath each other

#### **Time Calculations**

At **Level 2** pupils should be able to convert from  $12 \leftrightarrow 24$  hour times and calculate durations of time by counting up to the next hour and then adding on the required amount of time

At Level 2 pupils should be able to convert from hours ↔ minutes

At **Level 3** pupils should be able to convert decimal fractions of hours  $\leftrightarrow$  minutes eg 24 minutes =  $24 \div 60 = 0.4$  hours eg 0.45 hours =  $0.45 \times 60 = 27$  minutes

Time intervals are calculated as additions eg How long is it from 06.45 to 13.10?

06.45 to 07.00 15mins 07.00 to 13.10 6h 10mins + 6 h 25mins

we do not use we do not use we do not use subtraction to calculate time intervals

### Using formulae

At Level 4 pupils should be able to use and construct simple formulae

eg Vol cuboid = lbh Calculate the volume of a cuboid with length 6 cm, breadth 4 cm and height 8 cm

$$V = lbh$$

$$V = 6 \times 4 \times 8$$

$$V = 192 \text{ cm}^3$$

eg Calculate the height of a cuboid with volume 24 cm<sup>3</sup>, length 2 cm and breadth 3 cm

method 1 substitute first then solve

$$V = lbh$$

$$24 = 2 \times 3 \times h$$

$$24 = 6h$$

$$h = 4 \text{ cm}$$

method 2 rearrange to give h as subject then substitute

$$V = lbh$$

$$\underline{V} = h$$

$$lb$$

$$\underline{24} = h$$

$$2 \times 3$$

$$h = 4 \text{ cm}$$

The method used will depend on complexity of the formula. In either case appropriate working will always be expected.

### **Scientific Notation or Standard Form**

At **Level 4** pupils should be able to write both large and small numbers in standard form. In Mathematics standard form consists of a number between 1 and 10 multiplied by some power of 10

eg 
$$27\ 800\ 000 = 2.78\ x\ 10^7$$
  
 $0.0000789 = 7.89\ x\ 10^{-5}$ 

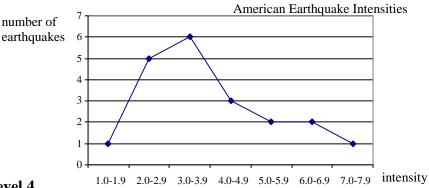
### **Line Graphs**

From **Level 2** pupils should be able to: choose an appropriate scale for the axes to fit the space available label the axes give the graph a title number the lines for frequencies plot the points neatly (using a cross or dot) fit a suitable line

eg Below is a list of the intensities of some earthquakes in America, measured on the Richter Scale: 3.4, 2.7, 5.6, 4.8, 7.6, 3.2, 6.2, 1.7, 3.2, 3.1, 4.0, 3.6, 5.4, 2.1, 2.9, 2.0, 4.4, 3.5, 2.7, 6.1 (a) Copy and complete the grouped Tally Table below.

Intensity	1.0 - 1.9	2.0 - 2.9	3.0 - 3.9	4.0 - 4.9	5.0 - 5.9	6.0 - 6.9	7.0 - 7.9
Tally							
Total							

- (b) Construct a line graph to display this information.
- (c) Which group do most of the earthquakes lie in?

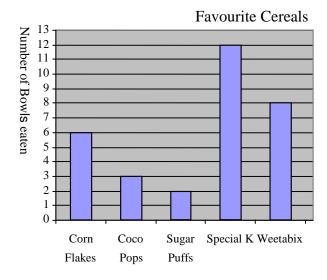


At Level 4

if necessary, make use of a jagged line to show that the lower part of a graph has been missed out.

### **Bar Graphs**

From Level 2 pupils should be able to choose an appropriate scale for the axes to fit the space available label the axes give the graph a title number the lines for frequencies label the bars plot the bars giving them equal widths leave spaces of equal width between the bars start the chart with a space



#### **Pie Charts**

At Level 2 pupils should be able to construct pie charts involving simple fractions or decimals

eg using quarters

eg What fraction of the spice mix is coriander?



At Level 3 pupils should be able to construct pie charts involving data expressed as percentages

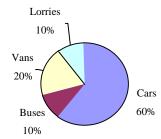
Karen did a survey of the traffic outside her house. The results were as follows:

Cars – 60% Buses – 10% Vans – 20% Lorries – 10%

Show this information on a pie chart

$$10\% \text{ of } 360^0 = 36^0$$

Cars 
$$60\% = 6 \times 36 = 216^{0}$$
  
Buses  $10\% = 1 \times 36 = 36^{0}$   
Vans  $20\% = 2 \times 36 = 72^{0}$   
Lorries  $10\% = 1 \times 36 = 36^{0}$ 



At Level 4 pupils should be able to construct pie charts from raw data

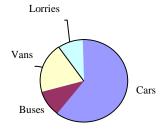
Karen did a survey of the traffic outside her house. The results were as follows

Cars – 48 Buses – 8 Vans – 16 Lorries – 8

Show this information on a pie chart

total vehicles = 80

Cars 
$$^{48}/_{80}$$
 x  $360 = 216^{0}$   
Buses  $^{8}/_{80}$  x  $360 = 36^{0}$   
Vans  $^{16}/_{80}$  x  $360 = 72^{0}$   
Lorries  $^{8}/_{80}$  x  $360 = 36^{0}$ 



### **Short Calculation Methods**

Calculations – useful methods most of which are familiar to pupils from primary school

Many of these methods illustrate the way most of us actually do mental calculations – ie we find the easiest way possible, which is not usually the same as our written approach to the same calculation.

This approach often depends on recognition of significant number pairs

eg 
$$18 = 100 - 82$$
,  $6 + 4 = 10$  and so on

Adding from the left

$$127 + 34 = 127 + 30 + 4 = 157 + 4 = 161$$
  
 $1.15 + 1.8 = 2.15 + 0.8 = 2.95$ 

Break and bridge – also adding from the left

Subtracting from the left – often but not always convenient

$$7.45 - 3.34 = 4.45 - 0.34 = 4.11$$
 convenient  $506 - 105 = 406 - 5 = 401$  convenient  $423 - 186 = 323 - 86 =$ not really convenient  $423 - 186 = 223 + 14 = 237$  is easier ie  $+200$  then  $-14$ 

Subtracting from the left in steps – more convenient when the "bottom" digit is bigger than the "top" digit

$$135 - 69 = 135 - 60 - 9 = 75 - 9 = 66$$
  
 $21.6 - 8.7 = 21.6 - 8 - 0.7 = 13.6 - 0.7 = 12.9$ 

Subtracting by balancing – using the same rules as solving equations by balancing

$$52-17 = (52-2) - (17-2) = 50-15 = 35$$
 -2 from each number  $125-36 = 129-40 = 89$  by +4 to each number  $243-124 = 239-120 = (23-12=11) = 119$  (mental step)  $387-199 = 388-200 = 188$ 

Multiplying from the left

Halving and Doubling – again the idea of balance to provide an easier calculation

#### Breaking the dividend

$$165 \div 5 = 150 \div 5 + 15 \div 5 = 30 + 3 = 33$$

$$648 \div 6 = 600 \div 6 + 48 \div 6 = 100 + 6 = 106$$

$$128 \div 8 = 80 \div 8 + 48 \div 8 = 10 + 6 = 16$$

$$256 \div 4 = 240 \div 4 + 16 \div 4 = 60 + 4 = 64$$

#### Dividing by balancing

$$12 \div 0.25 = 24 \div 0.5 = 48 \div 1 = 48$$

$$3 \div 0.2 = 15 \div 1 = 15$$

(multiply both terms by 5)

$$18 \div 25 = 36 \div 50 = 72 \div 100 = 0.72$$

$$26 \div 6.5 = 52 \div 13 = 4$$

$$10 \div 0.25 = 40 \div 1 = 40$$

#### Multiplying and dividing by multiples of 10

$$1000 \times 40 = 40000$$

$$600 \div 1000 = 0.6$$

#### Combining natural pairs

$$2 \times 46 \times 5 = 46 \times 10 = 460$$

$$6 \times 50 \times 2 = 6 \times 100 = 600$$

#### Division

$$15 \div 3 = 5$$

$$17 \div 3 = 5 \text{ r}2$$

$$17 \div 3 = 5.67 \text{ (2dp)}$$

$$17 \div 3 = 5^2/_3$$

#### Finding a remainder from a calculator

$$127 \div 17 = 7.47058...$$
  $17 \times 7 = 119$   $127 - 119 = 8$ 

$$17 \times 7 = 119$$

$$127 - 119 = 8$$

 $127 \div 17 = 7^8/_{17}$ 

#### Factors and Multiples

correct language is very important for use in algebraic work from early S1 onwards

3 is a factor of 12

12 is a multiple of 3

4 is a common factor of 20 and 24

4 is the highest common factor of 12 and 16

24 is a common multiple of 3 and 4

12 is the lowest common multiple of 3 and 4

finding factors of a number in an organised fashion is essential for algebraic fluency

List the factors of 48 - list as ordered pairs so that none are missed

- 1 48
- 2 24
- 3 16
- 4 12
- 6 8

#### Table facts

are constantly reinforced through all work done in Mathematics

need to be familiar for common factor work at first and then for use in fractions, algebra ...

"answers" also need to be recognised as factors

see 16 and think 2 x 8, 4 x 4

see 24 and think 2 x 12, 3 x 8, 4 x 6