

MATHEMATICS



Unit 3

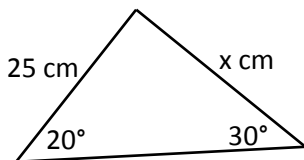
Applications

Trigonometry – Non Right Angled Triangles

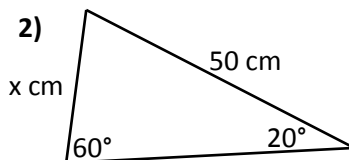
Exercise 1- Sine Rule (Side)

Calculate the side marked **x** in each triangle below

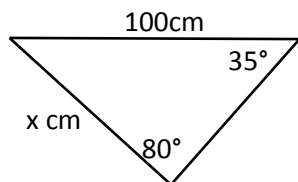
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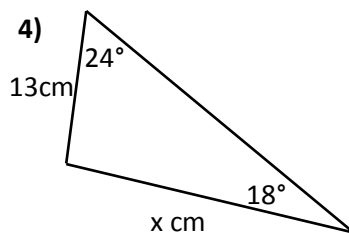
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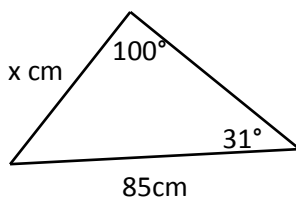
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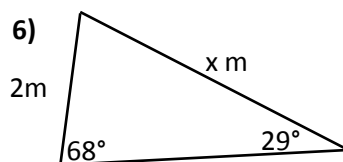
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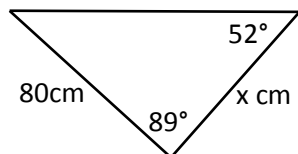
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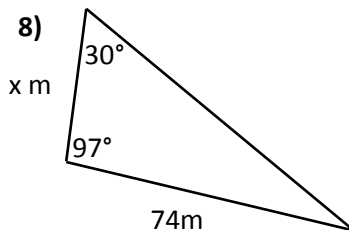
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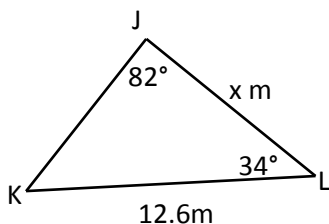
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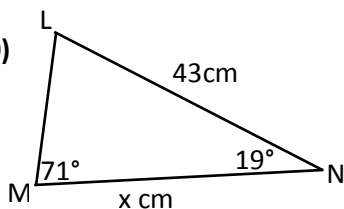
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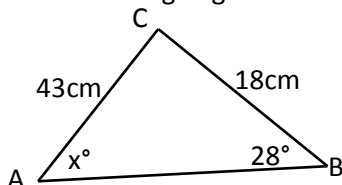
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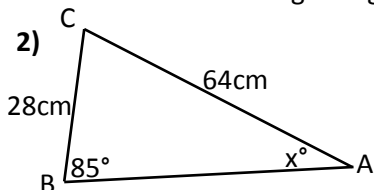
Exercise 2– Sine Rule (Angle)

Calculate the missing angle marked x in each of the following triangles:

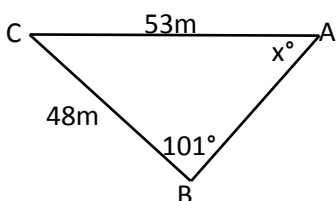
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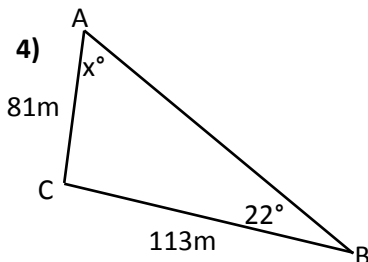
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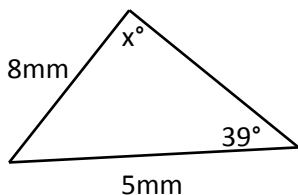
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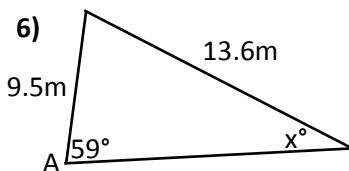
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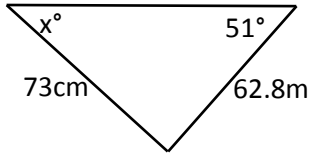


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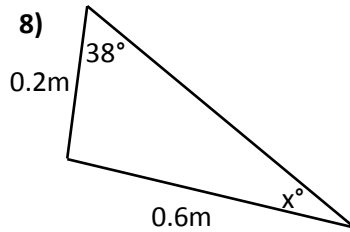


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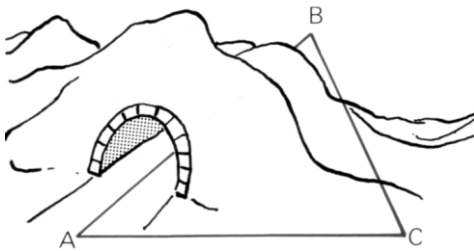


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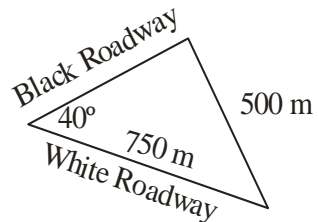
Exercise 3 – Mixed Sine Rule

- 1) A tunnel is to be made along the line AB. To help its planning, a point C is chosen, from where A and B can both be seen.



- a) Sketch triangle ABC, marking angle $A = 46^\circ$, angle $C = 68^\circ$, $AC = 400$ m.
 b) Calculate, to the nearest metre: i) BC ii) AB

- 2) Underground in the Redhill Coal Mine, the Black and White Roadways run at 40° to each other. A new 500 metre roadway link is being planned. Calculate the acute angle between the new link and the White Roadway.



3) Tariq measures his new bicycle:

$AB = 40 \text{ cm}$, $AD = 45 \text{ cm}$,

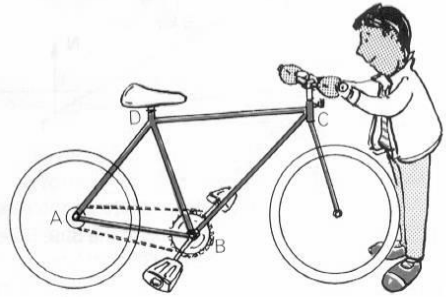
$DC = 64 \text{ cm}$,

angle $ABD = 62^\circ$,

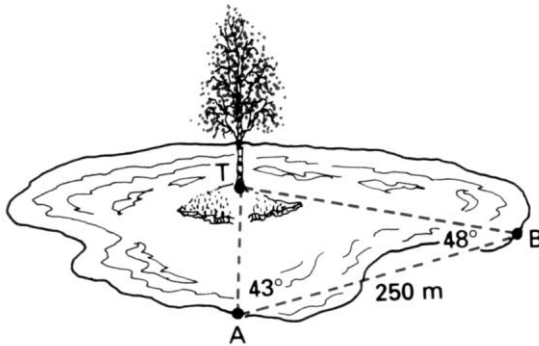
angle $DBC = 66^\circ$

a) Make a sketch of the bicycle frame,

b) Calculate: i) Angle ADB ii) DB iii) Angle DCB



4) A tree, T, stands on an island in the pond. Calculate the distance AT.



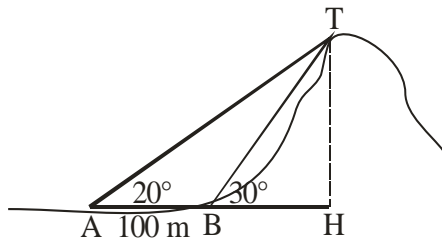
5) a) Sketch $\triangle ABC$, with $BC = 10 \text{ cm}$, $AC = 12 \text{ cm}$ and angle $ABC = 50^\circ$.

b) Find the size of the largest angle in the triangle.

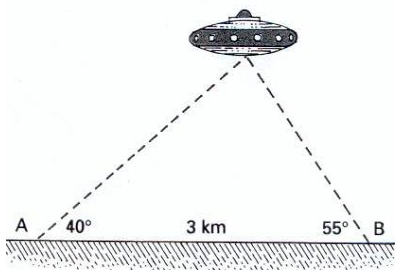
6) Observations of a hilltop, T, are made from points A and B, 100 m apart. The angles of elevation are 20° and 30° . Calculate:

a) TB

b) the height of the hill (TH).



- 7) There are reports of a UFO sighting. Adam says its angle of elevation is 40° . Barbara, who lives 3 km away from Adam, measures the angle of elevation as 55° . Calculate:

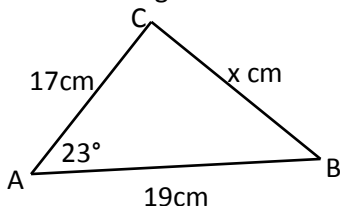


- the distance from Adam to the UFO
- the height of the UFO above the ground.

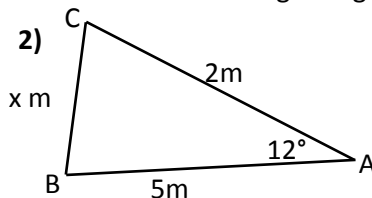
Exercise 4 – Cosine Rule (Side)

Calculate the missing side marked x in each of the following triangles:

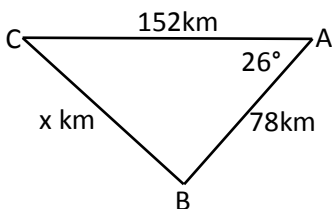
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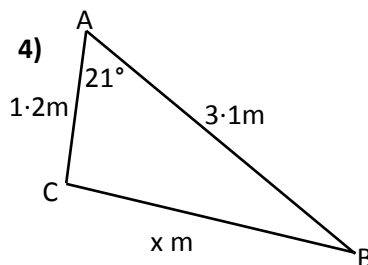
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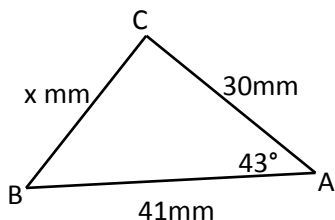
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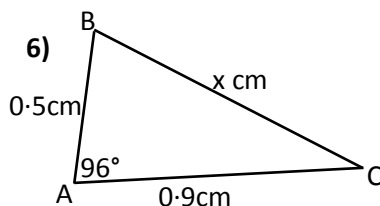
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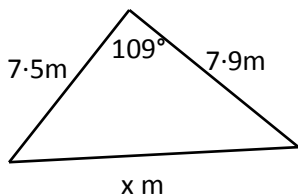
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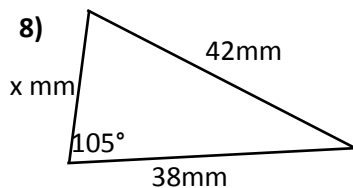
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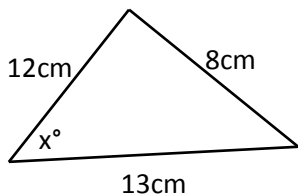
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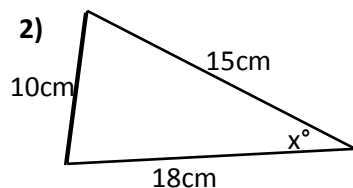
Exercise 5 – Cosine Rule (Angle)

Calculate the missing angle marked x in each of the following triangles:

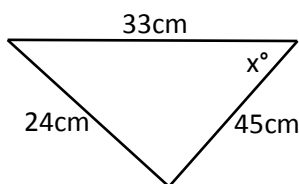
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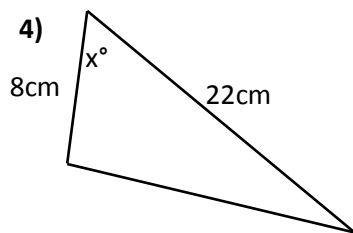
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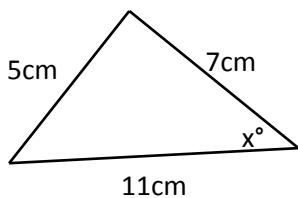
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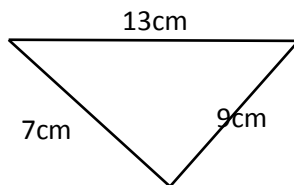
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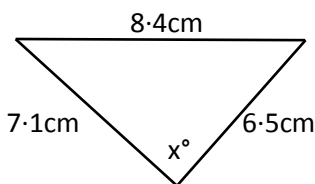
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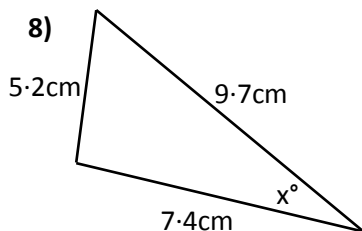
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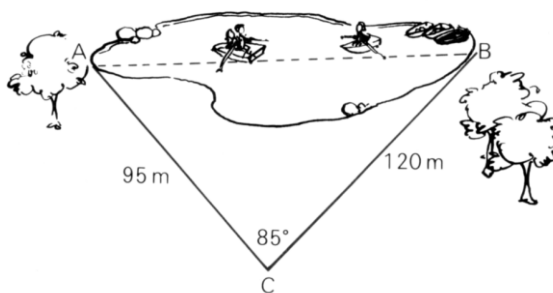


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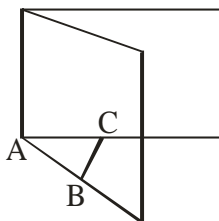


Exercise 6 – Mixed Cosine

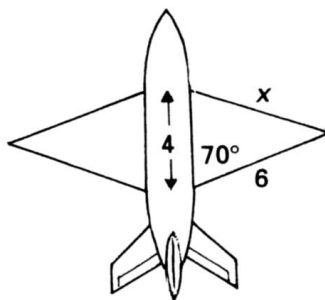
- 1) Carla, an apprentice surveyor, sketches the boating pond, and marks in these measurements. Calculate the width of the pond (AB).



- 2) This window is held open by the strut BC. Calculate the length of BC when the window is open at an angle of 40° ($AB = 45$ cm and $AC = 50$ cm).



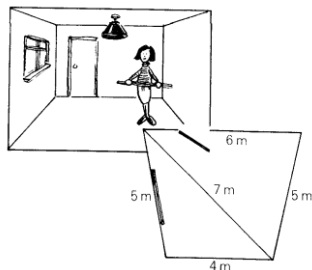
- 3) Calculate x for the wing position of this aircraft. Lengths are in metres.



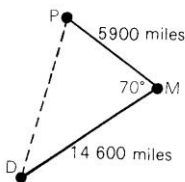
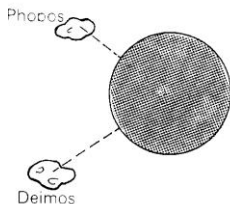
- 4) Steven's ladder has legs 150 cm and 146 cm long. When the ladder is fully open, the feet are 86 cm apart.



- Calculate the angles of the triangle formed when the ladder is fully open.
 - In a narrow space, Steven can only get the feet 80 cm apart. Calculate the angle between the legs of the ladder in this position.
- 5) Jillian is measuring the floor of her lounge for a fitted carpet. Calculate:
- the sizes of the angles at the corners of the room
 - the length of the other diagonal.

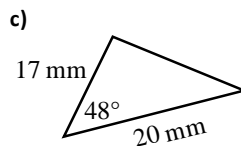
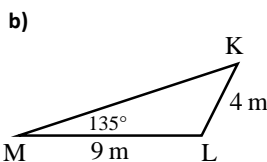
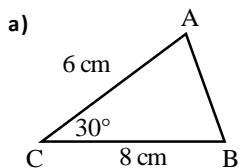


- 6) The planet Mars (M) has two moons. Phobos (P) is 5900 miles and Deimos (D) is 14600 miles from the planet's centre, to the nearest 100 miles. How far apart are the moons, to the nearest 100 miles, when:
- Angle $\text{PMD} = 70^\circ$
 - Angle $\text{PMD} = 140^\circ$



Exercise 7 – Area of a triangle

1) Calculate the area of each triangle below.



2) Calculate the area of $\triangle ABC$, with:

a) angle $BAC = 100^\circ$, $AB = 2.7$ cm, and $AC = 4$ cm.

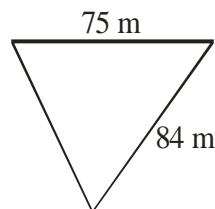
b) $BC = 7.1$ m, $AC = 3.5$ m and angle $ACB = 21.5^\circ$.

3) The area of $\triangle ABC$ is 12 cm^2 . $AC = 5$ cm and $BC = 6$ cm.

a) Calculate two possible sizes of angle ACB .

b) Sketch the two possible triangles.

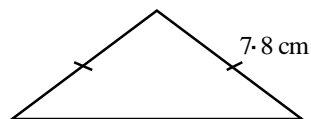
4) This field is in the shape of an acute angled triangle. Its area is 2580 m^2 . Calculate its perimeter, correct to 3 significant figures.



5) The area of $\triangle ABC$ is 20.72 cm^2 . $AB = 6.42$ cm and $AC = 8.54$ cm. Find the two possible sizes of angle BAC .

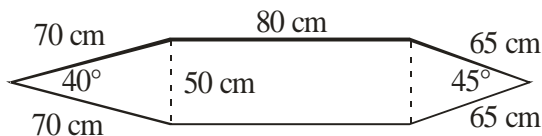
6) The equal sides of an isosceles triangle are each 4.2 cm long, and the area of the triangle is 6 cm^2 . Calculate the two possible lengths of the third side.

- 7) The area of the triangle shown is 29.52 cm^2 .
Calculate the length of the longer third side.

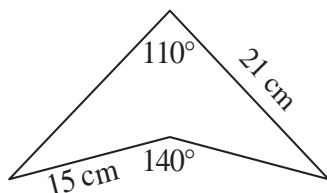


- 8) A regular hexagon ABCDEF is inscribed in a circle, centre O and radius 12 cm long. Calculate the area of
- a) $\triangle OAB$ b) the hexagon
- 9) Calculate the area of the following regular polygons inscribed in circles:
- a) a pentagon in a circle of radius 10 cm
- b) an octagon in a circle of radius 1 m

- 10) Find the area of the shape to the nearest square centimetre.

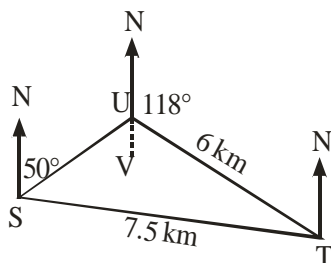


- 11) Find the area of the v-kite below, giving your answer correct to 3 significant figures.

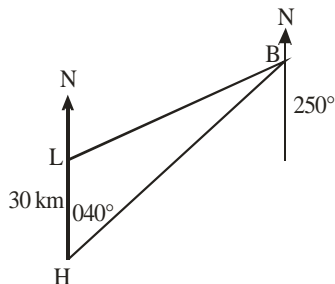


Exercise 8 – Bearings

- 1) This diagram shows the positions of three ships at S, T and U.

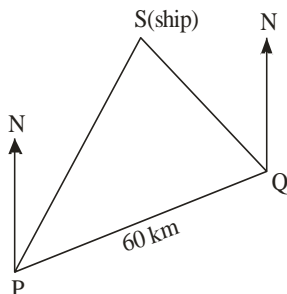


- Write down the size of angles: i) $\angle SUV$ ii) $\angle VUT$ iii) $\angle SUT$
 - Calculate the size of angle $\angle TSU$.
 - What is the bearing of T from S?
- 2) A ship leaves H and sails to B on a course bearing 040° . From B, the lighthouse L bears 250° .



- Using the parallel lines, explain why $\angle HLB = 110^\circ$.
- Calculate the distance HB.

- 3) Two oil platforms in the North Sea are 60 km apart.



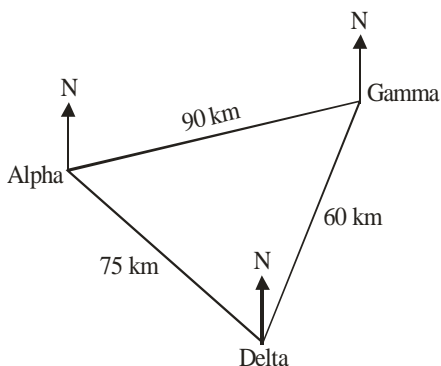
Platform P is on a bearing of 225° from platform Q.

A ship is on a bearing of 020° from Platform P and 330° from Platform Q.

How far is the ship from Platform Q?

- 4) Three oil platforms, Alpha, Gamma and Delta are situated in the North Sea as shown in the diagram below.

The distances between the oil platforms are shown in the diagram.

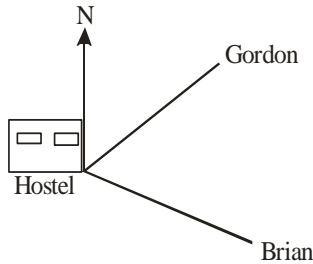


If the bearing of Delta from Alpha is 125° , what is the bearing of Gamma from Alpha?

- 5) Gordon and Brian leave a hostel at the same time.

Gordon walks on a bearing of 045° at a speed of 4.4 km/hr .

Brian walks on a bearing of 100° at a speed of 4.8 km/hr .



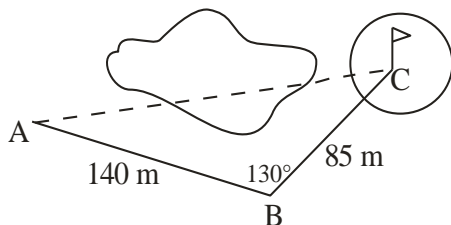
If they both walk at steady speeds, how far apart will they be after 2 hours?

- 6) Two ships leave port together. One sails on a course of 045° at 9 km/hr , and the other on a course of 090° at 12 km/hr . How far apart will they be after $2\frac{1}{2} \text{ hrs}$?
- 7) Two ships P and Q are 8 km apart. From P, Q is on a bearing of 100° and another ship R is on a bearing of 160° . From Q, R is on a bearing of 200° . Calculate the distances of R from P and from Q.

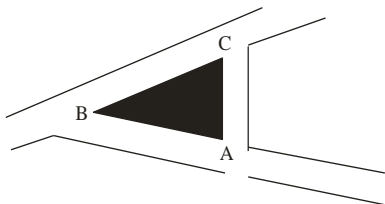
Exercise 9 – Mixed Exam Type Questions

- 1) Two golfers, Ann and Julie, are playing the most difficult hole on their golf course. Ann decides to avoid playing over the pond and hits a shot of 140 m from A to B followed by a second shot of 85 m from B to C. The angle between Ann's two shots is 130° . Julie, however, is more confident and plays a single shot from A over the pond.

How far would Julie have to hit this shot to reach C?

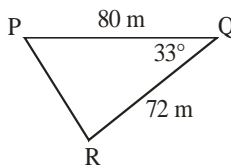
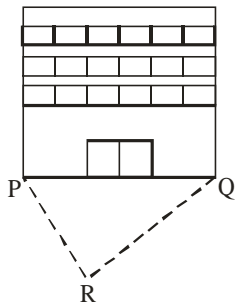


- 2) A traffic island, ABC, is shown below.



Find the area of the traffic island if $AB = 12.6$ m, $AC = 10$ m, angle $BAC = 72^\circ$.

3)

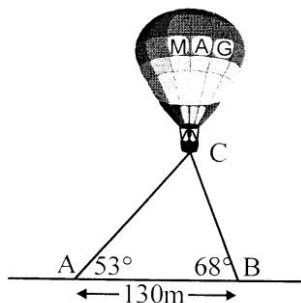


During a survey of the school, its length PQ was measured as 80 m.

The surveyor walked at an angle of 33° to the school for 72 m to the point R.

How far is R from the point P?

4) A newspaper group advertises a new magazine on a helium balloon.

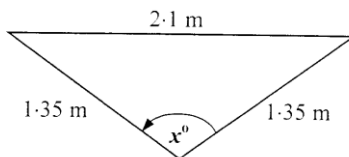
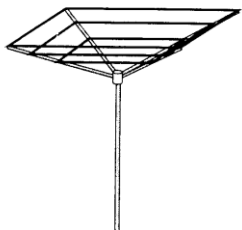


From the base of the balloon, C, two holding wires are attached to the ground at A and B. The distance from A to B is 130 m.

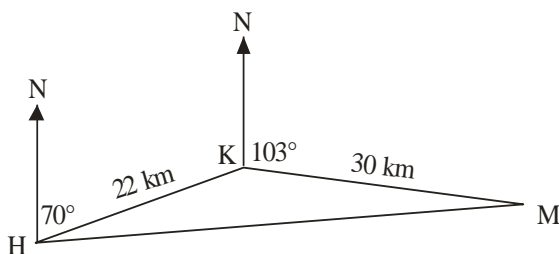
From A, the angle of elevation of C is 53° and from B it is 68° .

Calculate the length of AC and hence find the height of the point C above the ground.

- 5) The arms of a rotary clothes drier are 1.35 m long and the clothes line between them is 2.1 m long. Calculate the angle, x° , between the arms.



- 6) In the diagram below three towns, Holton, Kilter and Malbrigg are represented by the points H, K and M respectively.



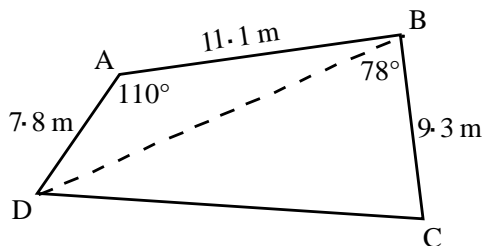
A helicopter flies from Holton for 22 km on a bearing of 070° to Kilter. It then flies from Kilter for 30 km on a bearing of 103° to Malbrigg. The helicopter then returns directly to Holton.

- a) i) Calculate the size of Angle HKM.
 ii) Calculate the total distance travelled by helicopter.

Do not use a scale drawing.

- b) A climber is reported missing somewhere in the triangle represented by HKM in the diagram. Calculate the area of this triangle.

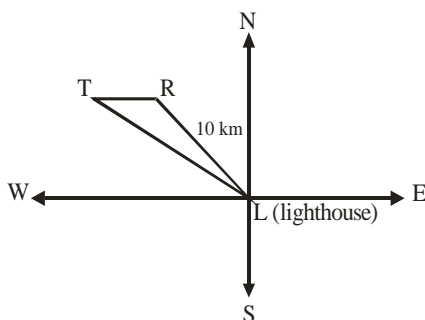
- 7) A garden, in the shape of a quadrilateral, is represented in the diagram below.



Calculate:

- the length of the diagonal BD
- the area of the garden

8)



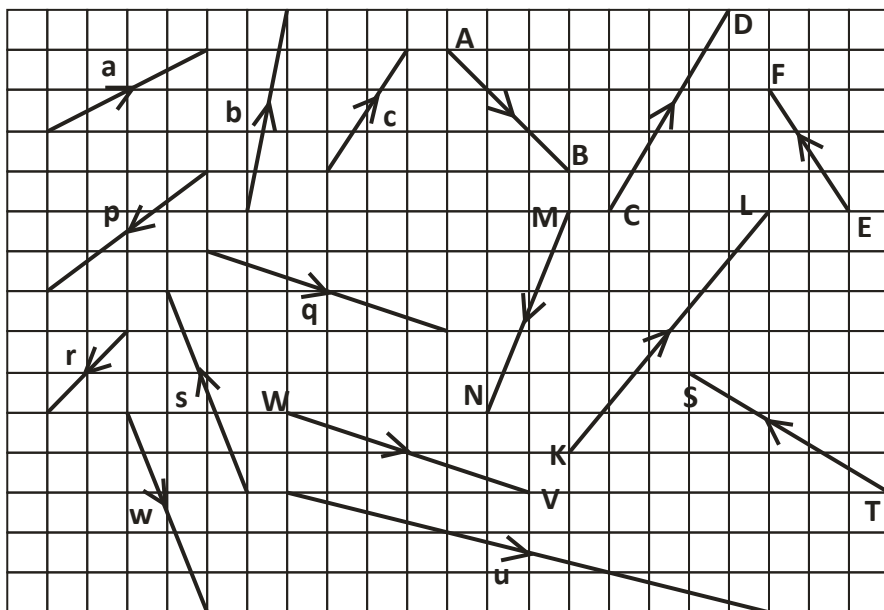
A ship is first spotted at a position R, which is on a bearing of 315° from a lighthouse L. The distance between R and L is 10 km. After the ship has travelled due West to position T, its bearing from the lighthouse is 300° .

How far has the ship travelled from R to T?

Vectors

Exercise 1

1) Express these vectors as **column vectors** and calculate their **magnitude**.



2) Using the vectors in question 1)

- Draw a vector which represents the following vectors
- Check your answer by calculating its column vector
- Calculate the magnitude of each new vector

a) $2\mathbf{a}$

b) $2\mathbf{b}$

c) $3\mathbf{c}$

d) $2\overline{AB}$

e) $3\overline{CD}$

f) $-\mathbf{p}$

g) $-\mathbf{q}$

h) $-2\overline{MN}$

i) $3\overline{FE}$

j) $\frac{1}{2}\mathbf{r}$

k) $-2\mathbf{s}$

l) $-\frac{1}{2}\overline{WV}$

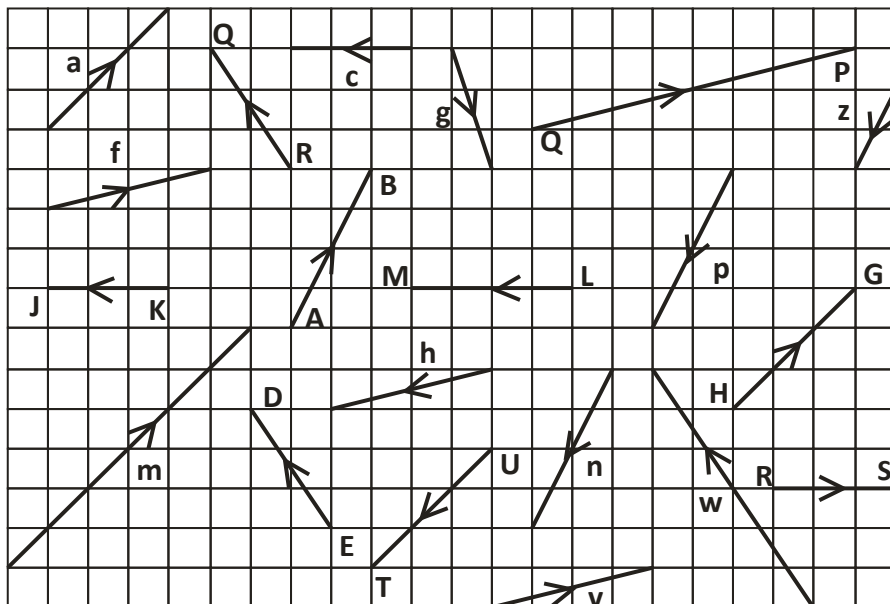
m) $-\overline{KL}$

n) $-2\overline{ST}$

o) $-\mathbf{w}$

p) $-\frac{1}{3}\mathbf{u}$

3) Using these vectors, copy and complete the table below.



Vector	3 parallel vectors	equal vector	negative vector
\vec{a}			
\vec{AB}			
\vec{c}			
\vec{DE}			
\vec{f}			

4) On squared paper draw and label the following vectors.

a) $\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

b) $\vec{b} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

c) $\vec{CD} = \begin{pmatrix} -2 \\ -5 \end{pmatrix}$

d) $\vec{s} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

e) $\vec{TR} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

f) $\vec{MN} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$

g) $\vec{f} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$

h) $\vec{PQ} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

i) $\vec{g} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$

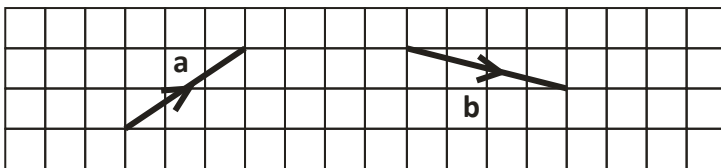
j) $\vec{y} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

k) $\vec{w} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

l) $\vec{UV} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$

Exercise 2

- 1) Here are the vectors **a** and **b**



Draw a diagram to represent the following.

a) $a + b$

b) $2a + b$

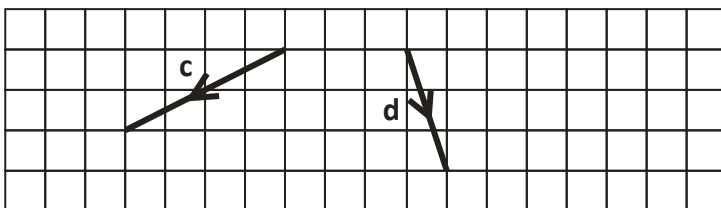
c) $a + 2b$

d) $3a + 2b$

e) $3a + b$

f) $2a + 2b$

- 2) Here are the vectors **c** and **d**



Draw a diagram to represent the following.

a) $c + d$

b) $d + c$

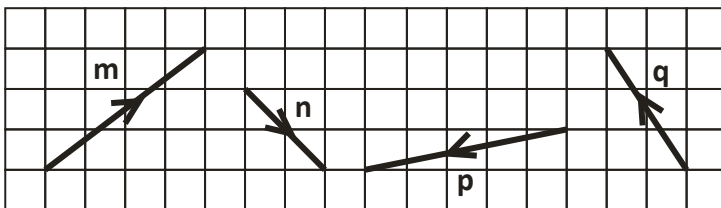
c) $3c + d$

d) $2c + 2d$

e) $3d + 2c$

f) $c + 3d$

- 3) Here are the vectors **m**, **n**, **p** and **q**



Draw a diagram to represent the following.

a) $m + n$

b) $n + p$

c) $m + q$

d) $2m + 3n$

e) $2p + 2q$

f) $3m + q$

g) $m + n + p$

h) $2m + n + 3q$

i) $3n + p + 2q$

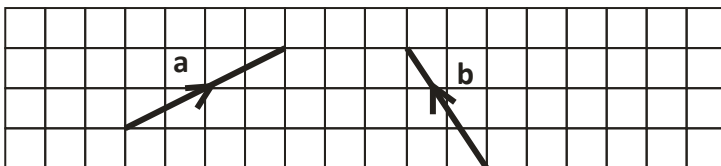
j) $p + 2q + n$

k) $3p + 2m + q$

l) $2m + 2n + 4p$

Exercise 3

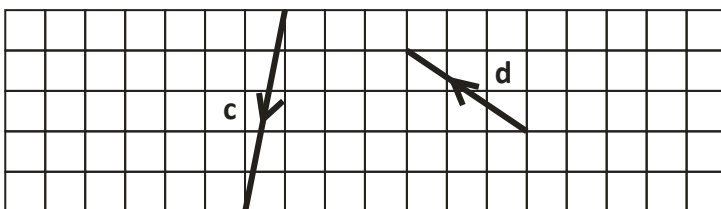
- 1) Here are the vectors **a** and **b**



Draw a diagram to represent the following.

- | | | |
|--------------|-------------|--------------|
| a) $a - b$ | b) $b - a$ | c) $2a - b$ |
| d) $3a - 2b$ | e) $2b - a$ | f) $4b - 2a$ |

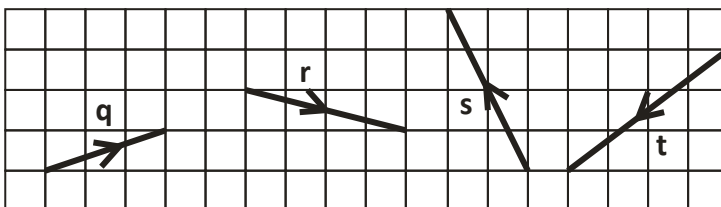
- 2) Here are the vectors **c** and **d**



Draw a diagram to represent the following.

- | | | |
|-------------|--------------|-------------|
| a) $c - d$ | b) $d - c$ | c) $2c - d$ |
| d) $3d - c$ | e) $2c - 2c$ | f) $c - 4d$ |

- 3) Here are the vectors **q**, **r**, **s** and **t**

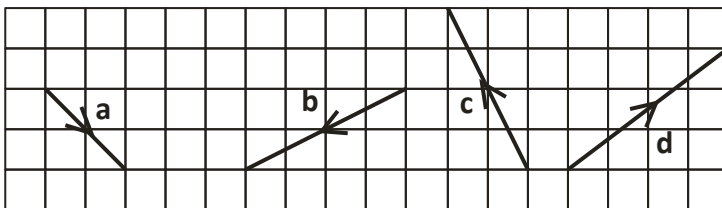


Draw a diagram to represent the following.

- | | | |
|------------------|------------------|------------------|
| a) $q - r$ | b) $r - s$ | c) $t - r$ |
| d) $s - q$ | e) $2r - t$ | f) $3q - 2s$ |
| g) $4r - 2q$ | h) $s - 3t$ | i) $2r - 2s$ |
| j) $2q - 2s - t$ | k) $3r - t - 2q$ | l) $2r - 3t - s$ |

Exercise 4

1) Here are the vectors **a**, **b**, **c** and **d**



Draw a diagram to represent the following.

a) $a + b - c$

b) $b - c + d$

c) $a + b - d$

d) $d + b - a$

e) $c - a + b$

f) $2a + b - 3c$

g) $a - 2c + 3b$

h) $2a + 3b - d$

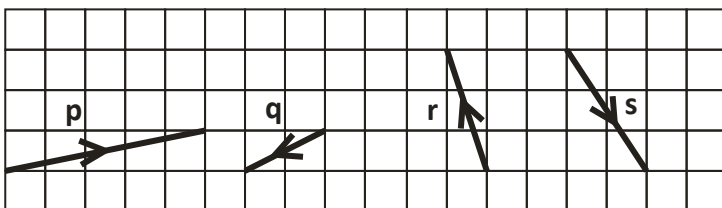
i) $2d - 3b + a$

j) $4c + 2a - 2d$

k) $a + b + c + d$

l) $a - b + c - d$

2) Here are the vectors **p**, **q**, **r** and **s**



Draw a diagram to represent the following.

a) $p - q + r$

b) $2q - r + s$

c) $s - 3p + r$

d) $2r - 3s + p$

e) $p - 2s + 3r$

f) $2s + q - r$

g) $p + q - 2r$

h) $3r - 2p + 2q$

i) $3p - 2s + 2q$

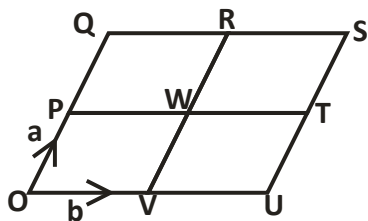
j) $p + q + r + s$

k) $2p + q - 3r$

l) $p - q - r + s$

Exercise 5

- 1) OQSU is made up from 4 congruent parallelograms



$$\overrightarrow{OP} = \mathbf{a} \quad \overrightarrow{OV} = \mathbf{b}$$

Express the following in terms of \mathbf{a} and \mathbf{b} .

a) \overrightarrow{OQ}

b) \overrightarrow{OU}

c) \overrightarrow{OW}

d) \overrightarrow{OT}

e) \overrightarrow{OR}

f) \overrightarrow{OS}

g) \overrightarrow{VS}

h) \overrightarrow{QU}

i) \overrightarrow{RT}

j) \overrightarrow{SP}

k) \overrightarrow{TQ}

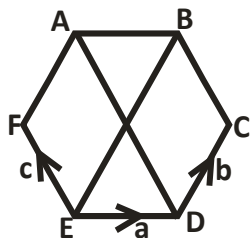
l) \overrightarrow{UR}

m) \overrightarrow{US}

n) \overrightarrow{PT}

o) \overrightarrow{VR}

- 2) ABCDEF is a regular hexagon.



$$\overrightarrow{ED} = \mathbf{a} \quad \overrightarrow{DC} = \mathbf{b} \quad \overrightarrow{EF} = \mathbf{c}$$

Express the following in terms of \mathbf{a} and \mathbf{b} .

a) \overrightarrow{EC}

b) \overrightarrow{EA}

c) \overrightarrow{EB}

d) \overrightarrow{DA}

e) \overrightarrow{DF}

f) \overrightarrow{BD}

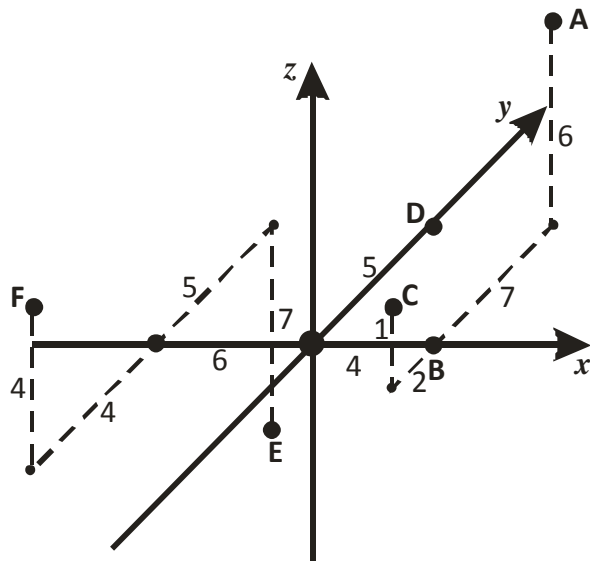
g) \overrightarrow{AE}

h) \overrightarrow{FC}

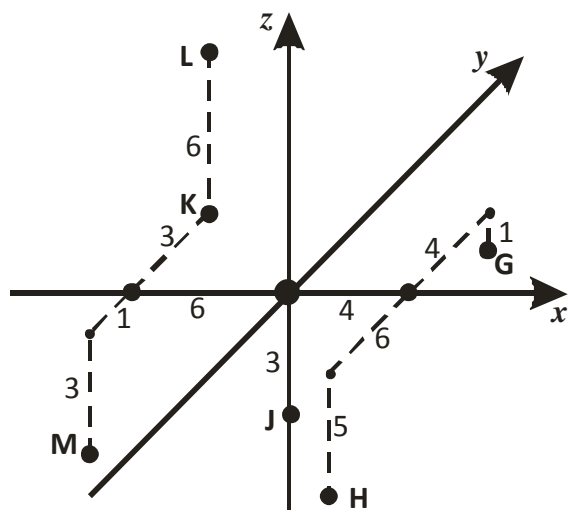
i) \overrightarrow{CF}

Exercise 6

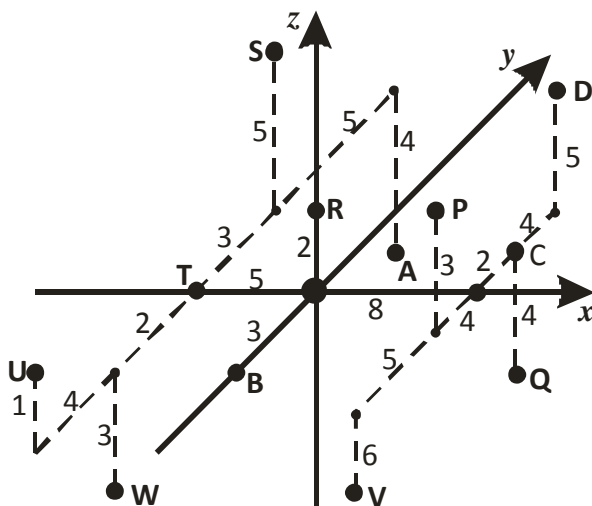
1) Write down the coordinates for each point.



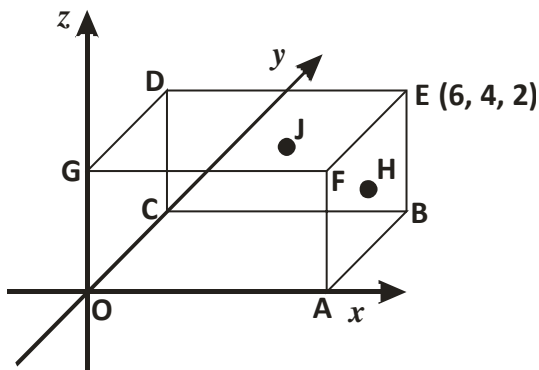
2) Write down the coordinates for each point.



3) Write down the coordinates for each point.



4) OABCDEFG is a cuboid



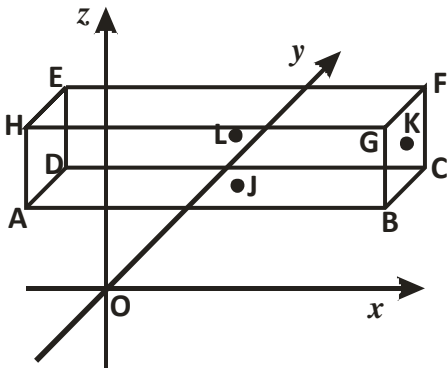
H is the centre of face ABEF.

J is the centre of face BCDE.

E is the point (6, 4, 2).

- Write down the coordinates of all the vertices.
- Write down the coordinates of points H and J.

5) ABCDEFGH is a cuboid



J is the centre of face ABCD.

K is the centre of face BCFG.

L is the centre of face CDEF.

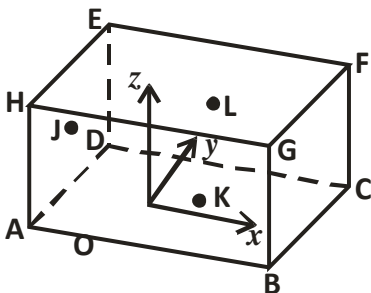
A is the point $(-2, 3, 0)$.

F is the point $(8, 7, 6)$

a) Write down the coordinates of all the vertices.

b) Write down the coordinates of points J, K and L.

6) ABCDEFGH is a cuboid



F is the point $(8, 3, 5)$.

EF has length 12 units and is parallel to the x -axis.

FG has length 8 units and is parallel to the y -axis.

BG has length 6 units and is parallel to the z -axis.

J is the centre of face ADEH.

K is the centre of face ABCD.

L is the centre of face CDEF.

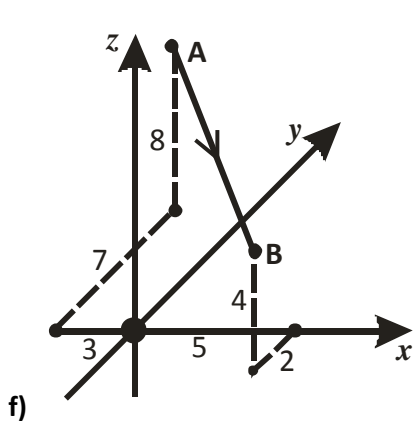
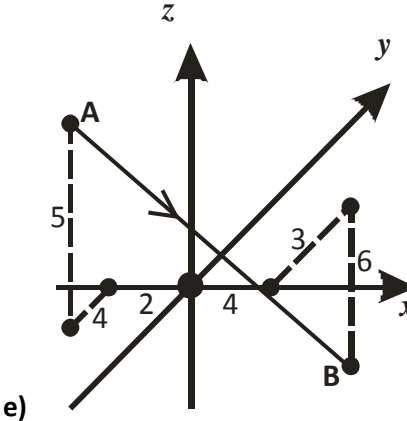
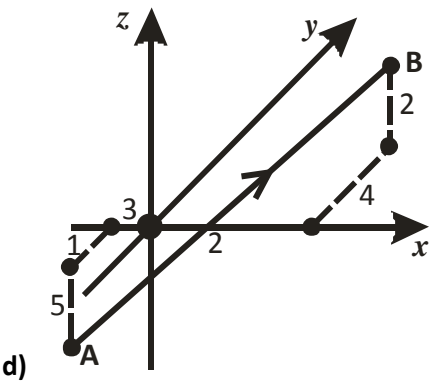
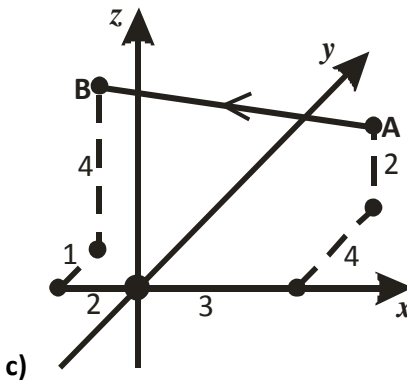
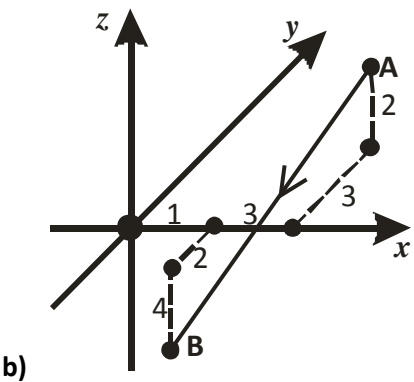
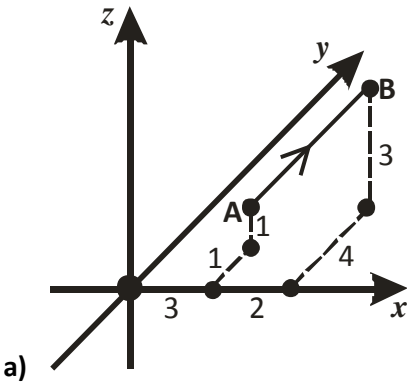
a) Write down the coordinates of all the vertices.

b) Write down the coordinates of points J, K and L.

Exercise 7

1) Write down the position vectors for points A and B.

Calculate the column vector \overrightarrow{AB} and its magnitude.



2) For each diagram write down the position vectors for points A, B, C and D.

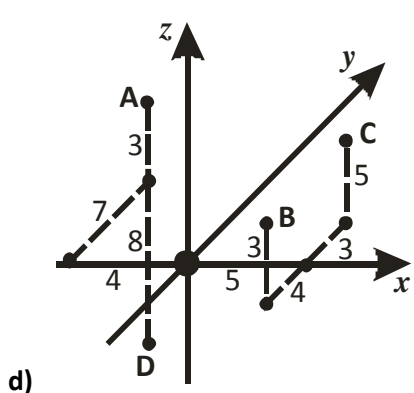
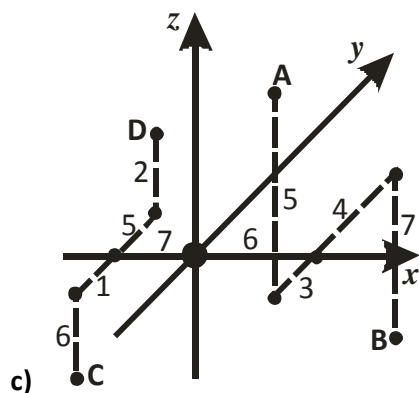
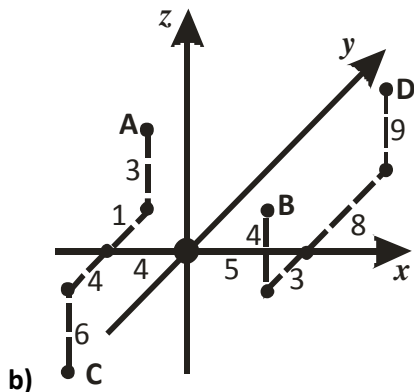
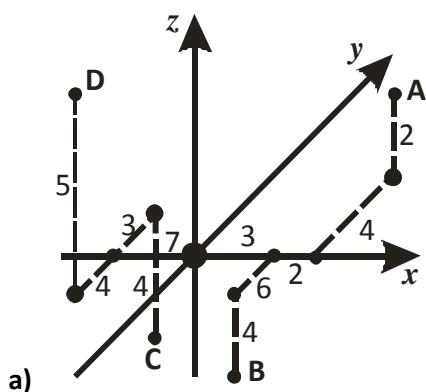
Calculate the following vectors and their magnitude:

\overrightarrow{AB} , \overrightarrow{AC} , \overrightarrow{AD}

\overrightarrow{BA} , \overrightarrow{BC} , \overrightarrow{BD}

\overrightarrow{CA} , \overrightarrow{CB} , \overrightarrow{CD}

\overrightarrow{DA} , \overrightarrow{DB} , \overrightarrow{DC}



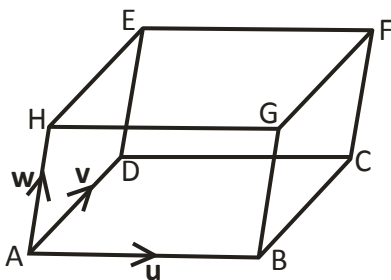
3) Given the vectors

$$\mathbf{a} = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix} \quad \mathbf{d} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \quad \mathbf{e} = \begin{pmatrix} 0 \\ 4 \\ -3 \end{pmatrix} \quad \mathbf{f} = \begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix}$$

- | | | |
|---|---|--|
| a) $2\mathbf{a} + 3\mathbf{b}$ | b) $\mathbf{a} + \mathbf{b} - \mathbf{c}$ | c) $3\mathbf{b} - 2\mathbf{c} + \mathbf{f}$ |
| d) $2\mathbf{e} - 4\mathbf{f}$ | e) $2\mathbf{b} + 2\mathbf{c} - 3\mathbf{d}$ | f) $4\mathbf{f} - 5\mathbf{c} + \mathbf{a}$ |
| g) $2\mathbf{b} + 5\mathbf{c}$ | h) $3\mathbf{d} - 4\mathbf{f}$ | i) $\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d}$ |
| j) $3\mathbf{a} - 2\mathbf{c} + \mathbf{e}$ | k) $5\mathbf{b} - \mathbf{d} + 4\mathbf{a}$ | l) $2\mathbf{a} - 2\mathbf{b} - 3\mathbf{c}$ |
| m) $\mathbf{a} + \mathbf{b} - \mathbf{c} - \mathbf{d}$ | n) $4\mathbf{a} - 2\mathbf{b} + 5\mathbf{e}$ | o) $2\mathbf{d} - 3\mathbf{e} + 4\mathbf{f}$ |
| p) $10\mathbf{a} - \mathbf{b} + 6\mathbf{c}$ | q) $2\mathbf{c} + 3\mathbf{d} - 2\mathbf{e} + 4\mathbf{f}$ | r) $5\mathbf{b} - 2\mathbf{a} + \mathbf{c}$ |
| s) $2\mathbf{f} - \mathbf{e} - 6\mathbf{d}$ | t) $4\mathbf{b} + 2\mathbf{a} - 3\mathbf{c}$ | u) $5\mathbf{a} - 2\mathbf{b} + 6\mathbf{e} - 4\mathbf{f}$ |
| v) $\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} + \mathbf{e}$ | w) $\mathbf{a} - \mathbf{b} - 2\mathbf{c} + 4\mathbf{d} - \mathbf{e}$ | x) $5\mathbf{f} + 5\mathbf{e} - 6\mathbf{d}$ |

Exercise 8

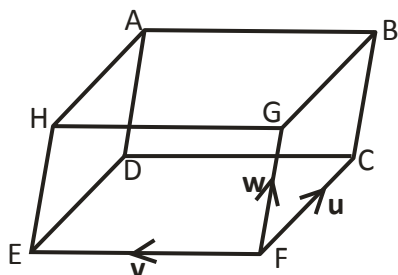
1) ABCDEFGH is a parallelepiped.



Express the following in terms of \mathbf{u} , \mathbf{v} and \mathbf{w} .

- | | | |
|--------------------------|--------------------------|--------------------------|
| a) \overrightarrow{AC} | b) \overrightarrow{AF} | c) \overrightarrow{HC} |
| d) \overrightarrow{BE} | e) \overrightarrow{GD} | f) \overrightarrow{HB} |

2) ABCDEFGH is a parallelepiped.



Express the following in terms of \mathbf{u} , \mathbf{v} and \mathbf{w} .

a) \overrightarrow{FA}

b) \overrightarrow{BE}

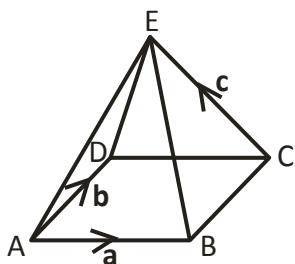
c) \overrightarrow{DF}

d) \overrightarrow{CH}

e) \overrightarrow{AC}

f) \overrightarrow{GD}

3) ABCDE is a square based pyramid.



Express the following in terms of \mathbf{a} , \mathbf{b} and \mathbf{c} .

a) \overrightarrow{AE}

b) \overrightarrow{DB}

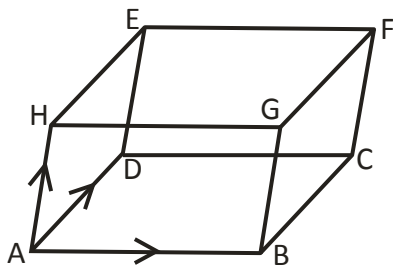
c) \overrightarrow{ED}

d) \overrightarrow{BE}

e) \overrightarrow{EA}

f) \overrightarrow{CA}

4) ABCDEFGH is a parallelepiped.



If $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$, $\overrightarrow{AD} = \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix}$ and $\overrightarrow{AH} = \begin{pmatrix} 5 \\ 0 \\ -3 \end{pmatrix}$,

find the following column vectors.

a) \overrightarrow{AF}

b) \overrightarrow{EC}

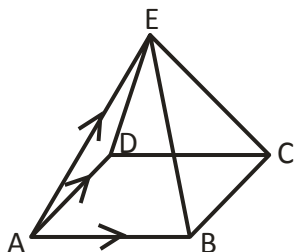
c) \overrightarrow{DG}

d) \overrightarrow{HF}

e) \overrightarrow{BE}

f) \overrightarrow{DB}

5) ABCDE is a square based pyramid.



If $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -4 \\ 8 \end{pmatrix}$, $\overrightarrow{AD} = \begin{pmatrix} 10 \\ 9 \\ 2 \end{pmatrix}$ and $\overrightarrow{AE} = \begin{pmatrix} 4 \\ -1 \\ 5 \end{pmatrix}$,

find the following column vectors.

a) \overrightarrow{CE}

b) \overrightarrow{BE}

c) \overrightarrow{DB}

d) \overrightarrow{AC}

e) \overrightarrow{EB}

f) \overrightarrow{EC}

Exercise 1

- 1) $\frac{1}{2} + \frac{5}{6} - \frac{3}{4}$ 2) $4\frac{1}{2} \times 1\frac{2}{9}$ 3) $\frac{8}{27} \div \frac{4}{7}$
- 4) $\frac{1}{2} + \frac{5}{6} - \frac{3}{4}$ 5) $4\frac{1}{2} \times 1\frac{2}{9}$ 6) $\frac{8}{27} \div \frac{4}{7}$
- 7) $\frac{3}{8} + \frac{5}{6} - \frac{1}{3}$ 8) $\frac{3}{4} \times \frac{8}{9}$ 9) $\frac{3}{16} \div \frac{2}{5}$
- 10) $1\frac{1}{5} + \frac{3}{4} - \frac{11}{12}$ 11) $\frac{7}{10} \times 5\frac{1}{3} \times 1\frac{1}{4}$ 12) $5\frac{1}{4} \div 4\frac{2}{3}$
- 13) $3\frac{1}{2} + 1\frac{4}{7} - 2\frac{3}{4}$ 14) $\frac{3}{8} \times 1\frac{5}{7} \times 4\frac{2}{3}$ 15) $6\frac{2}{3} \div 1\frac{1}{9}$
- 16) $3\frac{2}{3} - 2\frac{7}{8}$ 17) $\frac{2}{7} \times 2\frac{4}{5} \times \frac{1}{4}$ 18) $\frac{\frac{3}{8}}{\frac{1}{2} + 1\frac{1}{4}}$
- 19) $3\frac{1}{5} - 2\frac{3}{4}$ 20) $3\frac{3}{4} \times \frac{2}{9}$ 21) $\frac{1}{8} \div \left(\frac{1}{2} + \frac{1}{3}\right)$
- 22) $\left(\frac{5}{6} - \frac{1}{2}\right) \div 5$ 23) $\left(\frac{1}{2} + \frac{1}{3}\right)\left(\frac{1}{2} - \frac{1}{3}\right)$ 24) $2\frac{1}{5} \div 3\frac{1}{3}$
- 25) $1\frac{5}{9} \times 2\frac{1}{7} \times \frac{1}{5}$ 26) $\frac{1\frac{1}{8} + 2\frac{1}{4}}{4\frac{1}{2}}$ 27) $\frac{\frac{3}{7} \text{ of } 4}{\frac{1}{3} \text{ of } 8}$
- 28) $2\frac{1}{3} - \frac{3}{4}$ 29) $5 + \frac{5}{10} + \frac{5}{1000}$

Statistics

Exercise 1

In questions 1 to 19 calculate the mean (\bar{x}), mode, median (Q_2), quartiles (Q_1 and Q_3), range, semi-interquartile range and draw a box-plot.

- 1) 3, 4, 4, 4, 6, 7, 8, 8, 9, 16, 19.
- 2) 10, 10, 14, 15, 15, 18, 20, 21, 21, 22.
- 3) 1, 2, 4, 4, 5, 7, 8, 11, 12.
- 4) 3, 7, 7, 9, 11, 11, 16, 18, 20, 20, 20, 20.
- 5) 1, 3, 5, 7, 8, 9, 9.
- 6) 1, 2, 5, 8, 12, 13, 14, 14, 14, 19, 19.
- 7) 2, 4, 6, 7, 10, 13.
- 8) 1, 4, 7, 7, 8, 11, 11, 11, 12.
- 9) 1, 1, 2, 3, 5, 8, 13, 21, 34, 55.
- 10) 8, 9, 9, 9, 10, 11, 12, 12.
- 11) 5, 6, 7, 7, 9, 14.
- 12) 28, 28, 29, 31, 32, 32, 34, 35, 37, 37, 37, 39.
- 13) 101, 104, 106, 107, 109, 109, 111, 113, 113, 115, 116, 118, 118, 118, 120, 121, 125.
- 14) 15, 13, 7, 16, 11, 10, 13, 9, 16, 8, 10
- 15) 19, 21, 17, 14, 12, 4, 8, 17, 16, 20.
- 16) 10, 4, 3, 2, 1, 9, 7, 2, 8, 4, 9, 6, 5, 5, 8, 4, 3, 9, 8, 1, 2.
- 17) 41, 82, 29, 49, 78, 60, 56, 65, 69, 71, 36.

18) 7, 13, 4, 9, 3, 7, 8, 10, 11, 6, 12, 14.

19) 8, 18, 7, 16, 18, 8, 11, 18, 22, 9, 10, 14, 11, 15.

20) For each of the following distributions, calculate the mean, the median and the mode, and state which of the three would best represent the distribution as a whole.

a) Examination marks in English of a class of 15 girls:

58, 61, 48, 43, 62, 62, 51, 43, 54, 43, 70, 65, 42, 59, 48

b) Mathematics marks of the same class:

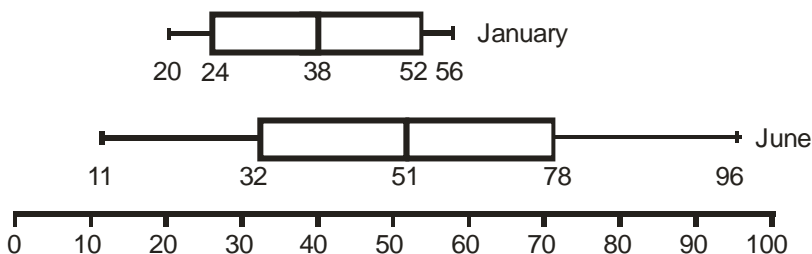
49, 53, 87, 56, 45, 38, 54, 39, 62, 70, 41, 43, 83, 49, 49

c) Daily noon temperatures in °C at a seaside resort over a 14 day period.

19, 20, 19, 17, 21, 18, 19, 24, 25, 25, 28, 25, 23, 18.

Exercise 2

1) These boxplots compare the results of two exams, one in January and one in June.

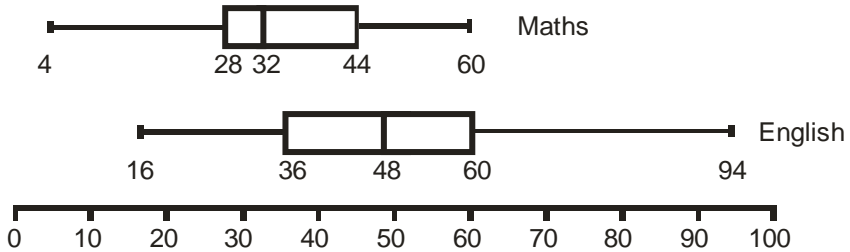


a) Write down 2 statements comparing the results in January and June.

b) What is the range of marks in January?

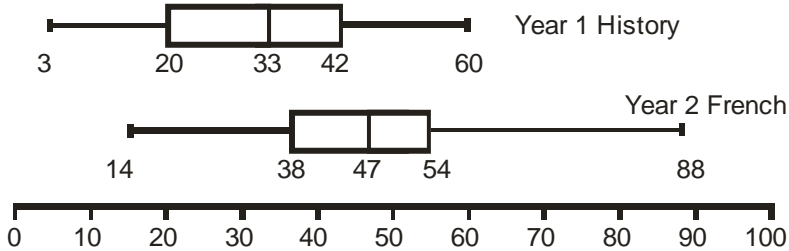
c) What is the semi-interquartile range for the June marks?

- 2) These boxplots compare the results of two exams, one in Maths and one in English.



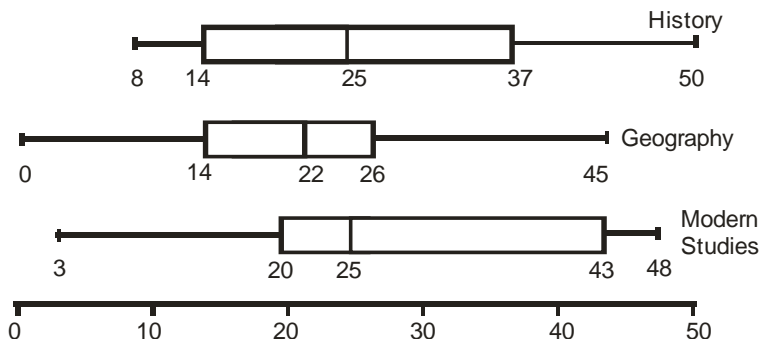
- a) Write down 2 statements comparing the marks in Maths and English.
b) Calculate semi-interquartile range for the Maths marks.
c) What is the range of marks in the English exam?

- 3) The results of two exams are illustrated in the box plots below



- a) Write down 2 statements comparing the marks for Year 1 History and Year 2 French.
b) Calculate semi-interquartile range for both sets of marks.

- 4) The boxplots below show the distribution of marks in 3 examinations sat by the same group of pupils. All 3 examinations were out of 50.



- Compare statistically the 3 distributions of marks in the above diagram.
 - What fraction of pupils in History scored marks between 37 and 50?
 - What fraction of pupils in Geography scored marks between 14 and 26?
 - Calculate semi-interquartile range for the Modern Studies marks?
- 5) A factory manager noted the number of absences, due to 'illness', both the men and women had during 2004.
- Men** 2, 2, 2, 3, 3, 3, 5, 5, 5, 6, 7, 7, 7, 7, 7, 8, 8, 8, 10, 11, 11, 12, 12, 13, 17
- Women** 0, 0, 0, 0, 1, 1, 1, 2, 2, 2, 2, 3, 4, 4, 4, 6, 6, 7, 8, 8, 8, 8, 8, 9, 9
- Find the median and quartiles for each set of data.
 - On the same diagram, draw the two boxplots to represent the two data sets.
 - Make two observations about your results.
- 6) A fertiliser is being tested. Twelve plants grown using the fertiliser and twelve without have their heights measured in centimetres at harvesting.
- With fertiliser:** 5, 6, 7, 10, 15, 16, 18, 22, 23, 28, 29, 30
- Without fertiliser:** 3, 4, 4, 7, 11, 12, 13, 17, 17, 21, 24, 25
- Draw boxplots of each data set on the same diagram.
 - Compare statistically the 2 sets of data.

- 7) Castle Estates build houses. At present they use two suppliers of roof tiles. They decide only to use one in future, They don't mind waiting for a reasonable length of time for deliveries. For planning purposes, however, deliveries must be consistent. They check the recent records for their two suppliers

Reliable Roofs' last few orders took these numbers of days to arrive:

50, 30, 38, 49, 52, 21, 20, 36, 24, 47, 21

Top Tiles' last few orders took these numbers of days to arrive:

48, 24, 43, 34, 37, 40, 35, 41, 37, 15, 30, 60, 39, 45, 36, 49

- a) Draw a boxplot for each company (on the same scale).
 - b) Which company should be chosen if consistency of deliveries is
 - c) What comparison would you make about the actual delivery times?
- 8) Two bus companies tender for the school bus contract.

Within limits the declared arrival time is not vital.

For planning ahead, however, it is better if arrival times are consistent.

The head teacher checks the records for the two companies during earlier contracts.

Arrival times are given as *minutes* before 9 am.

Top Form Transport's last few arrivals:

3, 7, 11, 5, 10, 13, 15, 14, 18, 13, 12, 12

Student Service's last few arrivals:

10, 9, 4, 11, 14, 15, 11, 3, 6, 11, 13, 3, 10, 15, 4, 8

- a) Draw a boxplot for each company (on the same scale).
- b) Which company should be chosen if consistency of arrival time is
- c) What comparison would you make about the actual arrival times?

Exercise 3

NOTE: There are 2 formulae for a sample population. If you know the data, use the formula

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

If you do not know the data, use the formula

$$s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n - 1}}$$

- 1) a) The weights of a group of men are

78 kg 74 kg 82 kg 81 kg 87 kg 78 kg 87 kg

Calculate the standard deviation of this group of men.

- b) The weights of a group of women have a mean of 73 kg and a standard deviation of 6.5 kg.

Compare statistically the weights of the men and the women.

- 2) a) Four friends compare the lengths of their Christmas holidays:

Elaine 7 days, Frank 7 days, Gill 12 days and Harry 14 days.

Calculate the mean length of holiday and also the standard deviation.

- 3) a) The shoe sizes of five girls are 3, 4, 4, 7 and 7.

Calculate the mean and standard deviation of the shoe sizes.

- b) The shoe sizes of five boys are 5, 7, 7, 7 and 9.

Calculate the mean and standard deviation of the shoe sizes.

- c) Make 2 comparisons between the shoe sizes of the girls and the shoe sizes of the boys.

- 4) a)** The pulse rates of eight athletes are measured after completing strenuous exercise.

The rates are 70, 72, 73, 74, 75, 76, 76 and 76 beats per minute.

Calculate the sample mean rate and the standard deviation (correct to 1 dp).

- b)** Eight office staff take the same exercise as the athletes.

Their pulse rates are 80, 81, 83, 90, 94, 96, 96 and 100.

i) Calculate the sample mean rate for the office staff.

ii) Calculate the sample standard deviation correct to 1 dp.

- c)** Compare statistically the 2 sets of pulse rates.

- 5)** At the start of a typing course a student recorded her mistakes per minute as follows: 0, 1, 2, 2, 3, 5, 7, 8.

- a) i)** Calculate the mean number of errors she made per minute.

ii) Calculate the sample standard deviation.

- b)** After several practice sessions she again recorded her mistakes per minute, with these results: 2, 3, 3, 3, 4, 4, 4, 5.

i) Calculate her new mean and standard deviation.

ii) Has her mean improved?

iii) Has her consistency improved?

- 6)** The number of frogs in various local ponds was counted for a wildlife survey. The results were: 7, 21, 35, 37, 57, 68, 100.

- a)** Calculate

i) the range

ii) the semi-interquartile range

iii) the sample standard deviation.

- b)** A year later a second survey was done.

The results this time were: 12, 25, 34, 37, 52, 62, 76.

Calculate

- i)** the range
 - ii)** the semi-interquartile range
 - iii)** the standard deviation.
- c)** Which measure of spread was most affected by the small changes in the data?

- 7)** For a set of 7 marks, it is found that $\sum x = 140$ and $\sum x^2 = 2982$.

Calculate the mean and standard deviation of the marks.

- 8)** For a set of 8 marks, it is found that $\sum x = 632$ and $\sum x^2 = 51864$.

Calculate the mean and standard deviation of the marks.

- 9)** The weights (in kilograms) of 5 pupils are taken.

It is discovered that $\sum x = 285$ and $\sum x^2 = 16541$.

Calculate the mean and standard deviation of the weights.

- 10)** The heights (in centimetres) of 10 pupils are taken.

It is discovered that $\sum x = 1450$ and $\sum x^2 = 212174$.

Calculate the mean and standard deviation of the heights.

- 11)** The speeds (in MPH) of 9 motorists on a motorway are measured.

The following data are then calculated: $\sum x = 549$ and $\sum x^2 = 39807$.

Calculate the mean and standard deviation of the speeds.

- 12)** The mean of a group of marks is 8 and the standard deviation is 5.
- a) What are the new mean and standard deviation when 7 is added to all of the marks?
 - b) What are the new mean and standard deviation when all of the marks are multiplied by 7?
- 13)** The mean of a group of numbers is 14 and the standard deviation is 12.
- a) What are the new mean and standard deviation when 8 is added to all of the numbers?
 - b) What are the new mean and standard deviation when all of the numbers are multiplied by 5?
 - c) What are the new mean and standard deviation when 9 is subtracted from all of the numbers?
 - d) What are the new mean and standard deviation when all of the numbers are divided by 2?
 - e) What are the new mean and standard deviation when 15 is added to all of the numbers?
 - f) What are the new mean and standard deviation when all of the numbers are multiplied by 3?
 - g) What are the new mean and standard deviation when 20 is subtracted from all of the numbers?
 - h) What are the new mean and standard deviation when all of the numbers are halved?

- 14)** The mean and standard deviation of the scores

30 40 50 60 70

are 50 and 15.8 respectively.

Use these 2 results to find the mean and standard deviation of

a) 20 30 40 50 60

b) 130 140 150 160 170

- c) -10 0 10 20 30
- d) 3 4 5 6 7
- e) 60 80 100 120 140
- f) 15 20 25 30 35
- g) 50 60 70 80 90
- h) 25 30 35 40 45

15) The mean and standard deviation of the numbers

12 18 18 30 42 60

are 30 and 18.2 respectively.

Use these 2 results to find the mean and standard deviation of

- a) 15 21 21 33 45 63
- b) 24 36 36 60 84 120
- c) 0 6 6 18 30 48
- d) 4 6 6 10 14 20
- e) 120 180 180 300 420 600
- f) -8 -2 -2 10 22 40
- g) 48 72 72 120 168 240
- h) 20 26 26 38 50 68

16) The mean and standard deviation of the numbers

3 4 6 7 8 8 10 10

are 7 and 2.56 respectively.

Use these 2 results to find the mean and standard deviation of

- a) 7 9 13 15 17 17 21 21
- b) 8 11 17 20 23 23 29 29
- c) 9 13 21 25 29 29 37 37

d)	19	24	34	39	44	44	54	54
e)	$2\frac{1}{2}$	3	4	$4\frac{1}{2}$	5	5	6	6
f)	0	4	12	16	20	20	28	28

Exercise 1

In all 8 questions in this exercise you will be asked in part a to draw a best-fitting straight line through points plotted on a scattergraph.

There are usually many possibilities for the position of the line, most very close to each other. You should first decide where you would position the line but don't actually draw it.

To ensure that you can check your answers (given at the back of the booklet) to parts b onwards you should draw the line which passes through the 2 points given below for each question in the exercise.

CHECK POINTS

1) (0, 20)	(32, 60)	5) (0, 9)	(16, 5)
2) (0, 2)	(12, 26)	6) (16, 412)	(36, 427)
3) (0, 15)	(15, 165)	7) (27, 24)	(54, 72)
4) (10, 80)	(70, 20)	8) (10, 34)	(25, 28)

- 1)** Some pupils sat a test in French (F) [out of 80] and another in German (G) [out of 40]. The marks obtained by the pupils are shown in scattergraph **T1** of your worksheets.
 - a)** Draw a line of "best-fit" through the points.
 - b)** Find the equation of this line.
 - c)** If a pupil scores a mark of 22 in the German test, calculate an estimate for the mark the pupil will score in the French test.
 - d)** If a pupil scores a mark of 4 in the German test, calculate an estimate for the mark the pupil will score in the French test.

- e) Which estimate in **c** or **d** is likely to be more accurate and why?
- f) If a pupil scores a mark of 70 in the French test, calculate an estimate for the mark the pupil will score in the German test.
- g) If a pupil scores full marks in the French test, calculate an estimate for the mark the pupil will score in the German test.
Make an observation about your answer.
- 2)** Some pupils sat a test in Science (S) [out of 30] and another in Technical (T) [out of 12]. The marks obtained by the pupils are shown in scattergraph **2** of your worksheets.
- a) Draw a “best-fitting line” through the points.
- b) Find the equation of this line.
- c) If a pupil scores a mark of 7 in the Technical test, calculate an estimate for the mark the pupil will score in the Science test.
- d) If a pupil scores a full marks the Technical test, calculate an estimate for the mark the pupil will score in the Science test.
Is your answer possible?
- e) Which estimate in **c** or **d** is likely to be more accurate and why?
- f) If a pupil scores a mark of 22 in the Science test, calculate an estimate for the mark the pupil will score in the Technical test.
- g) If a pupil scores a mark of 0 in the Science test, calculate an estimate for the mark the pupil will score in the Technical test.
Make an observation about your answer.
- 3)** A general knowledge test (out of 200) was given to some pupils of different ages. The results are shown in scattergraph **3** of your worksheets.
- a) Draw a regression line through the points.
- b) Find the equation of this line.
- c) Calculate an estimate for the score a pupil aged 10 would be expected to obtain.
- d) A pupil scores 160 in the test. Calculate an estimate of the person’s age.
- e) A person scores full marks. Calculate an estimate of the person’s age.

- f)** Which of your answers to **d** or **e** is likely not to be a reasonable answer? Explain why.
- g)** A person scores 30 in the test. Calculate an estimate for the person's age. Explain why this is not a reasonable answer.
- h** Explain mathematically the principal underlying the unreasonable answers in **f** and **g**.
- 4)** Some people were given an aptitude test marked out of 80. Scattergraph 4 on your worksheet shows the marks of these people and the times required to complete the test.
- a)** Draw a best fitting straight line through the points.
- b)** Find the equation of the line.
- c)** A person takes 47 minutes to complete the test. Calculate an estimate for her test mark.
- d)** Another person scores a mark of 18. Calculate an estimate for the time he takes to complete the test.
- e)** Which of your answers to **c** or **d** is likely to be accurate? Explain why.
- f)** Calculate an estimate for the time taken to complete the test by a person who scores full marks.
- g)** Is your answer to **f** likely to be a very good estimate or not? Explain.
- 5)** Some rats were injected with varying doses of an anaesthetic. The times taken by the rats to lose consciousness are shown in scattergraph 5 of your worksheet.
- a)** Draw a line of "best-fit" through the points.
- b)** Find the equation of the line.
- c)** Calculate an estimate for the time a rat will remain conscious if it is given a dose of 11 mg.
- d)** Calculate an estimate for the time a rat will remain conscious if it is given a dose of 20 mg.
- e)** Which of your answers to **c** or **d** is likely to be more accurate and why?

- f) A rat remains conscious for $7\frac{1}{2}$ minutes. Calculate an estimate for the dose given to the rat.
- g) Calculate an estimate for the dose required to be given to a rat if it becomes unconscious immediately.
- h) A rat is given a dose of 50 mg. Explain why a calculated estimate for the time the rat remains conscious is not possible.
- 6) The lengths in centimetres (L) and weights in grams (W) of a sample of species of fish formed part of a scientific research project. The results of this part of the survey are shown on scattergraph 6 of your worksheet.
- a) Draw a regression line through the points.
- b) Find the equation of the line.
- c) Calculate an estimate for the weight of a fish of length 40 cm. Is this length likely to be accurate?
- d) Calculate an estimate for the length of a fish weighing 421g. Is this estimate likely to be accurate?
- e) Calculate an estimate for the length of a fish weighing 403g. Comment on your answer.
- 7) A group of people sat test A (out of 60) and test B (out of 80). The results are shown on scattergraph 7 of your worksheet.
- a) Draw a best fitting line through the points.
- b) Find the equation of the line.
- c) If a person scores a mark of 32 in test A, calculate an estimate for the mark he will score in test B, rounding your answer correct to 1 decimal place.
- d) If a person scores full marks in test B, calculate an estimate for the mark she will score in test A, rounding your answer correct to 1 decimal place.
- e) Which of your answers to c or d is likely to be more reliable? Explain.

- f) If a person scores no marks in test B, calculate an estimate for the mark he will score in test A, rounding your answer correct to 1 decimal place.
- g) If a person scores no marks in test A, calculate an estimate for the mark she will score in test B, rounding your answer correct to 1 decimal place.

Comment on your answer.

- 8) A substance is heated in an experiment and then left to cool. It cools approximately at a steady rate. The temperature of the substance is taken several times during the cooling process. The results are shown on scattergraph 8 of your worksheet.

- a) Draw a regression line through the points.
- b) [CAREFUL!] Write down the coordinates of the point where the line crosses the C-axis.
- c) Hence find the equation of the line.
- d) Calculate an estimate for the temperature of the substance after 17 minutes, $30\frac{1}{2}$ minutes and 2 hours.

[NB: The temperature can fall below freezing for this substance]

- e) Calculate an estimate for the time it will take the substance to cool to 24°C , 19°C and freezing point.
- f) Calculate an estimate for the fall in temperature during any $\frac{1}{2}$ hour period.
- g) When using this equation, there is an occasion when the time in minutes gives the same number estimate for the temperature. Calculate this time (in minutes).

Appreciation/Depreciation

Exercise 1

- 1) For each of the following deposits into a bank account find the simple interest made and the total amount in the bank account after the given time:

	Deposit	Interest rate per annum	Duration
a)	£120	6%	2 years
b)	£2,520	3.2%	9 months
c)	£85	4.9%	7 months
d)	£675	6.3%	16 months
e)	£3214	8%	75 days
f)	£64,050	12.6%	312 days
g)	£1,052	3½%	5 months
h)	£968	7.3%	162 days
i)	£3,259	5.5%	3 years 5 months
j)	£25,612	6¼%	2 years 11 months

- 2) For each of the following deposits into a bank account find, using **compound interest**, the total amount in the bank account after the given time:

	Deposit	Interest rate per annum	Duration
a)	£360	7%	3 years
b)	£90	8%	4 years
c)	£75	6.3%	2 years
d)	£3,248	5%	5 years
e)	£5,621	9.2%	6 years
f)	£975	4.6%	2 years
g)	£1,441	4.9% (yr 1), 9.3% (yr 2), 11.4% (yr 3)	3 years
h)	£398	3.2% (yr 1), 3.6% (yr 2), 4.1% (yr 3)	3 years
i)	£20,502	12.6% (yr 1), 9.3% (yr 2), 6.1% (yr 3), 3.1 (yr 4)	4 years
j)	£65,312	10.6% (yr 1), 9.2% (yrs 2&3)	3 years

- 3) The village of Strathmahey currently has a population of 8400. Its population is expected to increase by 4% each year over the next three years. What will its population be in three years time?
- 4) Jackie buys a new car for £11,695. It will depreciate at a rate of 6% each year for the first four years. What will the cars value be after 4 years?
- 5) There are 6248 cells present at the start of a research experiment at 11am. The amount of cells present increases at a rate of 12% per hour. How many cells will be present at 2pm?
- 6) There are 238mg/Litre of sewage in a loch at the start of May. It is cleaned monthly, resulting in 5% of the sewage being removed every month. How much sewage will there be in the loch at the start of October?

- 7) £150 is deposited in a bank account. The interest rates are 4% in year one, 6% in year two and 7% in year three. How much money will be in the account after 3 years?
- 8) Pedro buys a car for £12,899. It depreciates in value by 25% in the first year, 18% in the second year, 14% in the third year and a further 11% in the fourth year. How much is the car worth after 4 years?
- 9) There are 2050 trees in a forest, which is to be expanded. The amount of trees in the forest will increase by 9% per year for the first three years and by 7% per year thereafter. How many trees will be in the forest after
a) 3years b) 7years?
- 10) There are 846 rabbits in a colony. The amount of rabbits in the colony is expected to decrease by 14% in the next year and a further 11% the following year. How many rabbits will be in the colony after two years?
- 11) The city of Pythagorasgo has a population of 2.9 million. Its population is expected to increase by 4% per year for the next two years and a further 7% per year for the following two years. How many people are expected to live in the village in 4 years time?
- 12) Caroline can swim 80 lengths of a swimming pool. She starts training and the distance she can swim is expected to increase by 25% after 1 month, a further 20% the second month and a further 15% the 3rd month. How many lengths should she be able to do after 3 months of training?

Reverse Percentages

- 1) A special bottle of cola contains 10% more than the normal bottle.
The special bottle contains 660ml.
How much does the normal bottle contain?

- 2) Ross saves 25% of his monthly salary.
Each month he saves £100.
What is his monthly salary?

- 3) Mia gets a 5% wage rise.
Her new wage is £1186.50 a month.
What was her wage before her pay rise?

- 4) A house is valued at £350000.
This is a 12% increase on the value of the house a year ago.
What was the house valued at a year ago?

- 5) In a long jump event Anna jumps 2.7m.
This is 10% lower than her best jump.
What is the best length she can jump?

- 6) 30g of breakfast cereal provides 16.2mg of vitamin C.
This is 24% of the recommended daily intake.
What daily intake of vitamin C is recommended?

- 7) Billy gets a 2% increase in his salary.
His new salary is £2269.50 a month.
What was Billy's salary before his wage rise?

- 8) A one year old car is worth £7330.
This is a decrease of 18% of its value from new.
What was the price of the new car?
- 9) By selling a vase for £980 an antiques dealer makes a 40% profit on the price he paid for it.
How much did he pay for it?
- 10) In a sale a manager reduced the price of a pair of trainers by 10% and sold them for £36.
What was the price of the trainers before the sale?
- 11) The population of a village increased by 15% between 2003 and 2013.
The population in 2013 was 897.
What was the population in 2003?
- 12) Claire sells her laptop to Daniel and makes a 15% profit.
Daniel then sells the laptop to Ellie for £391.
Daniel makes a 15% loss.
How much did Claire pay for the laptop?
- 13) Frank sells his bike to Gemma.
Gemma sells it to Harry for £121.50.
Both Frank and Gemma make a 10% loss.
How much did Frank pay for the bike?