MATHEMATICS

N5

Unit 2

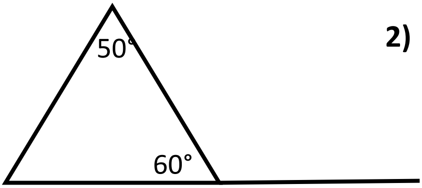
Part 2 of 2

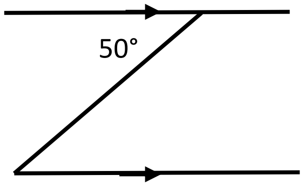
Relationships

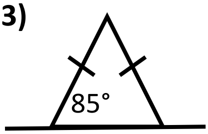
Angles

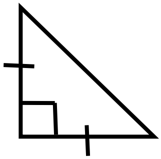
Exercise 1

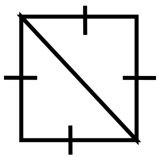
Copy the following diagrams into your jotter and fill in the sizes of all the angles:-

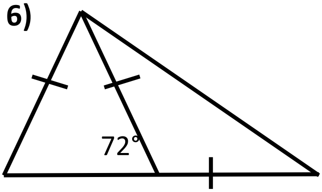
1) 

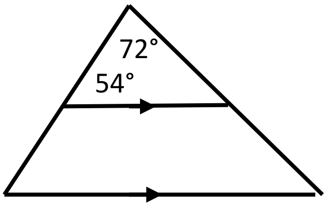
2) 

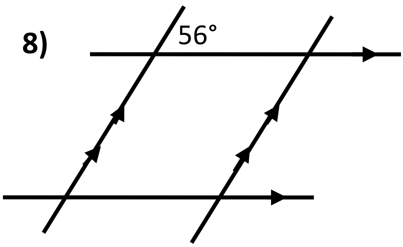
3) 

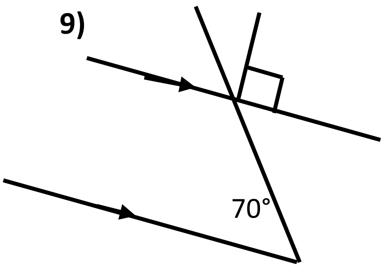
4) 

5) 

6) 

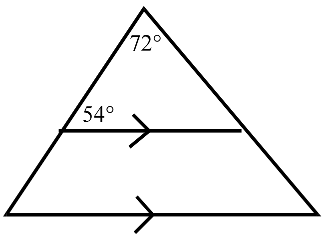
7) 

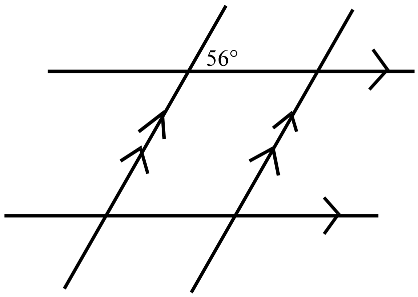
8) 

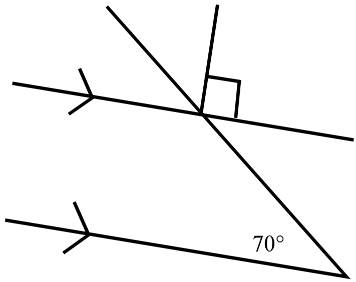
9) 

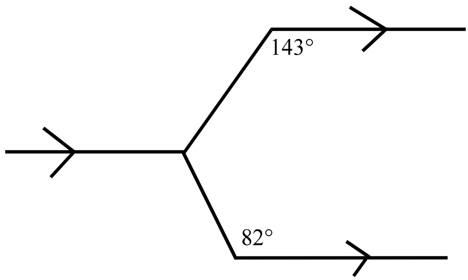
Exercise 2

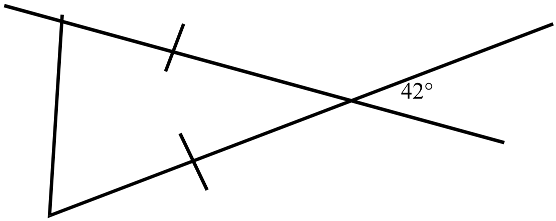
Make a copy of each diagram and fill in the sizes of all the angles

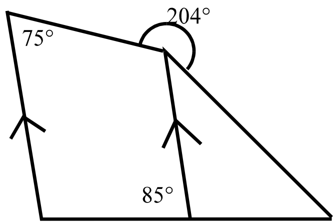


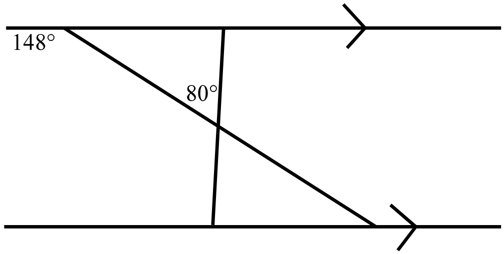


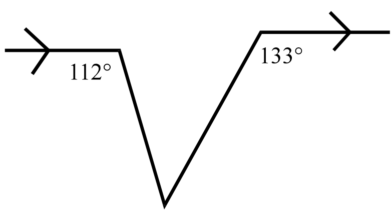








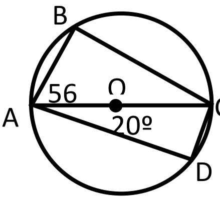




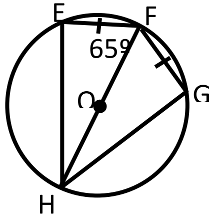
Exercise 3

1) Copy each of the following diagrams into your jotter, and mark the sizes of every angle in your diagram. (O is the centre of every circle)

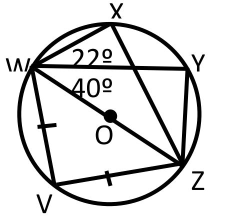
a



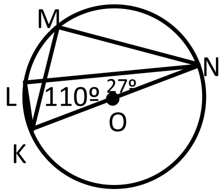
b



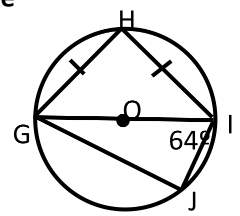
c



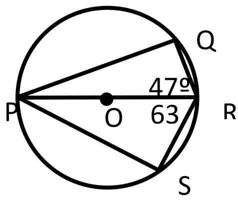
d



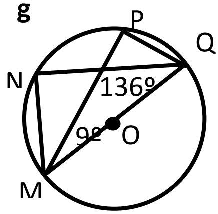
e



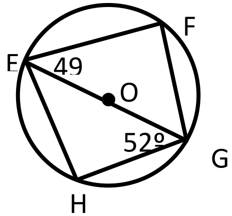
f



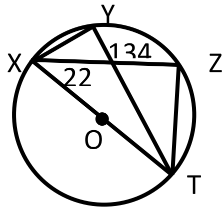
g



h



i

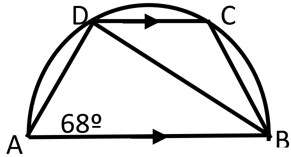


2) In the diagram ADCB is a semi-circle.

AB is parallel to DC and angle DAB = 68º.

a Calculate the size of angle BDC.

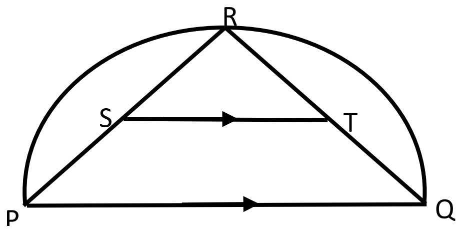
b Explain why angle ADB is a right angle.



3) In the diagram PRQ is an isosceles triangle drawn inside a semi-circle.

ST is parallel to PQ.

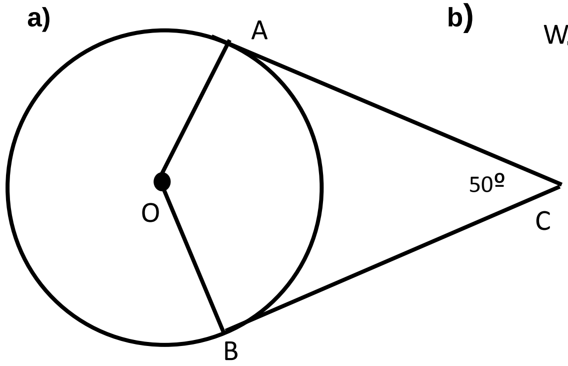
Find the size of angle PST giving a reason for your answer.



Exercise 3

1) Copy each of the following diagrams into your jotter. In each diagram mark the size of every angle. O. is the centre of each circle.

a)



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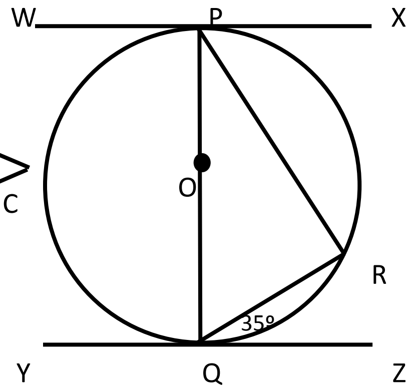

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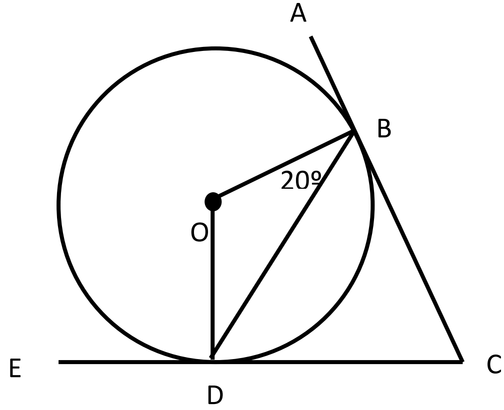

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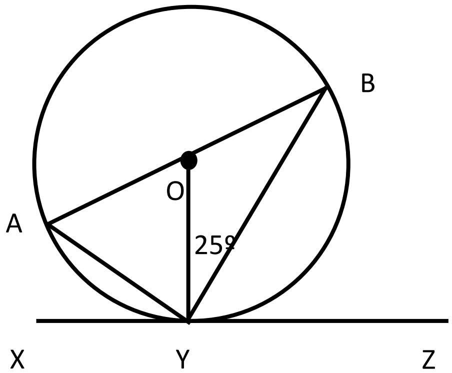
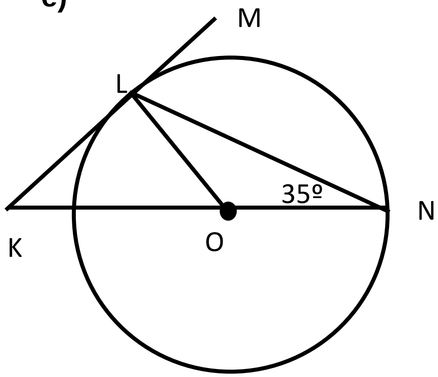


e)



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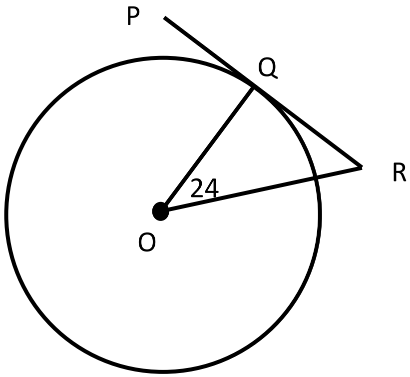

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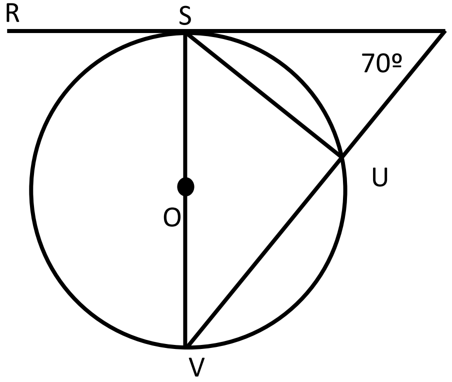

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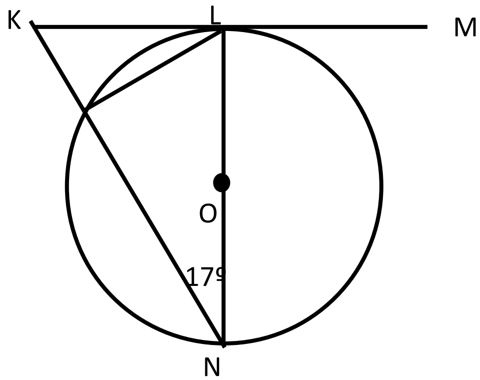
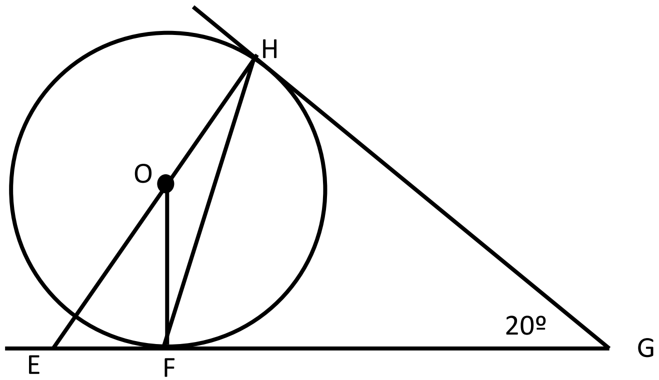
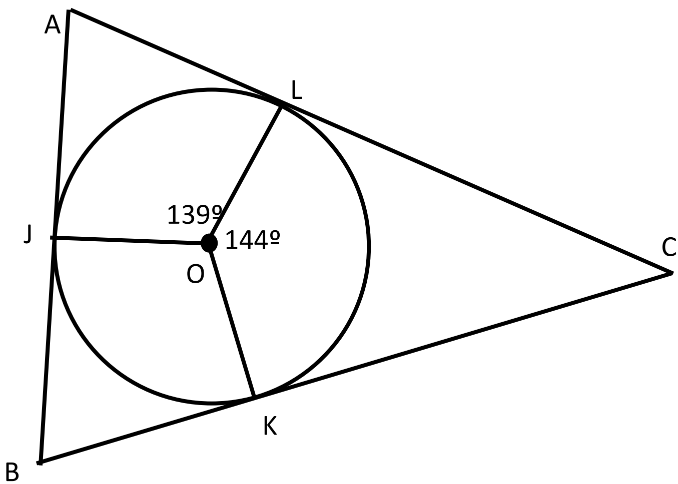


i)



j)



Exercise 5

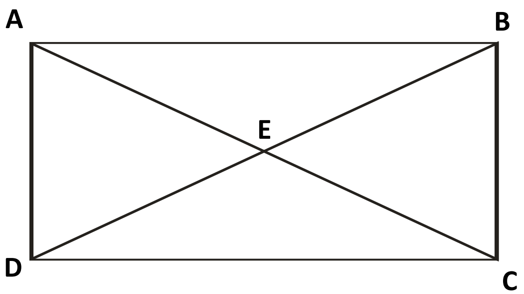
1) The rectangle ABCD below has angle BAC = 27^o. State the size of the following angles:

a) angle DAB **b)** angle DAC

c) angle BED **d)** angle ABD

e) angle AEB **f)** angle BEC

g) angle BCE **h)** the <u>reflex</u> angle BCE.



2) This diagram shows **two** squares ABCD and PQRS with a common centre Z.

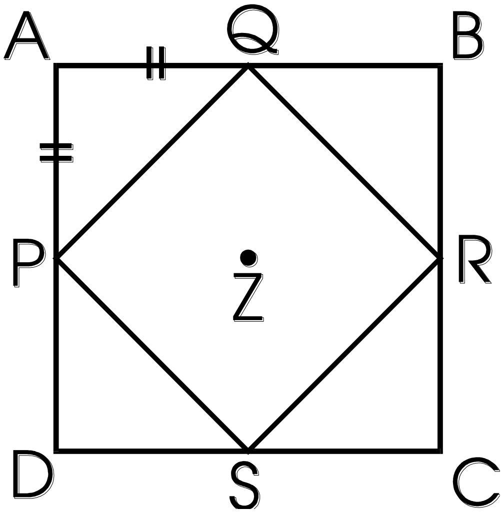
State the size of:

a) angle APQ

b) angle DPS

c) angle QPS

d) angle APS.



3) The rhombus KLMN below has angle KNL = 23°. State the size of the following angles:

a) angle MZL

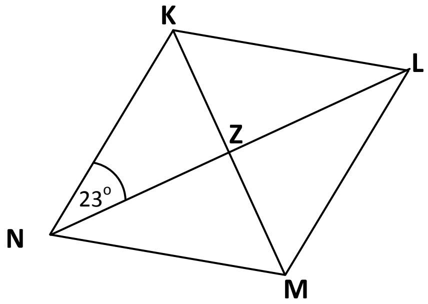
b) angle NKZ

c) angle ZLM

d) angle KLM

e) angle NML

f) angle <u>reflex</u> angle KNM.



4) The rhombus PQRS below has angle QPS = 28°. State the size of the following angles:

a) angle QZR.

b) angle OPZ.

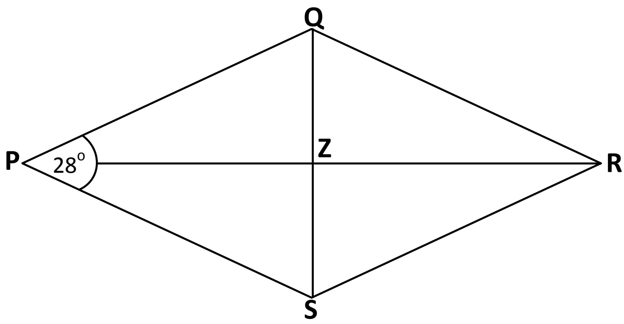
c) angle PQZ.

d) angle PQR.

e) angle PSZ.

f) angle PSR.

g) angle ZRS.



5) The kite PQRS below has angle SPQ = 64^o. Notice carefully what you are told about triangle QZR!
State the size of the following angles:

a) angle PZS.

b) angle SPZ.

c) angle PQZ.

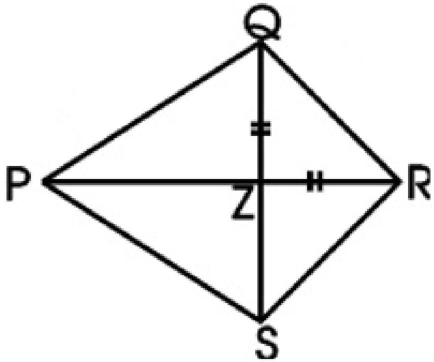
d) angle ZRS.

e) angle ZRQ.

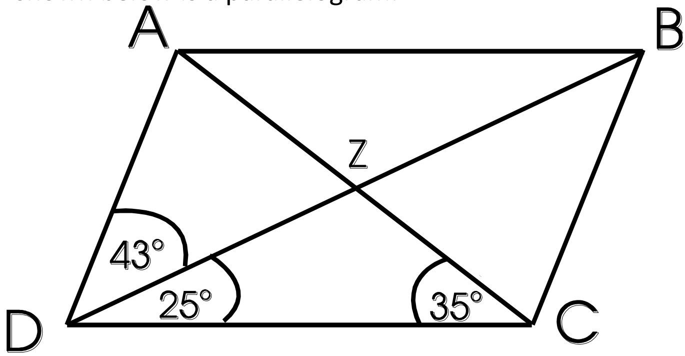
f) angle PQR.

g) the reflex angle PQR.

h) the reflex angle QPS.



6) ABCD shown below is a parallelogram.



State the following angle sizes:

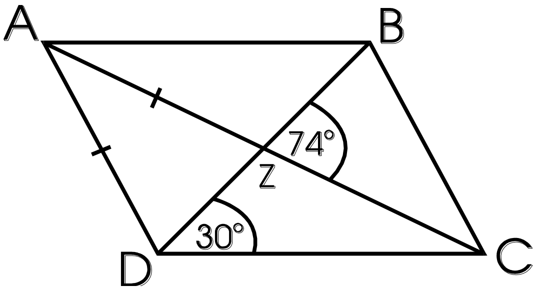
a) angle ABC **b)** angle DAB

c) angle ABD **d)** angle DBC

e) angle BAZ **f)** angle AZB

g) angle BZC **h)** angle DZC

7) (HINT: Look carefully at triangle AZD) ABCD is a parallelogram.



State the following angle sizes:

a) angle AZD.

b) angle ADZ.

c) angle DAZ.

d) angle ABC.

e) angle ZAB.

f) angle DAB.

g) angle DZC.

h) angle ZCD.

8) State the sizes of the following angles:

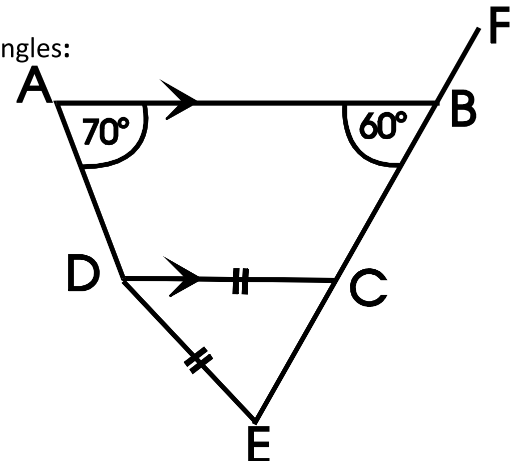
a) angle DCE

b) angle DCB

c) angle ADC

d) angle ABF

g) angle EDC

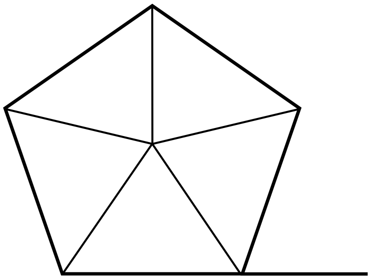


Exercise 5

For each regular polygon calculate

- **a)** central angles
- **b)** internal angles
- **c)** external angles

1)



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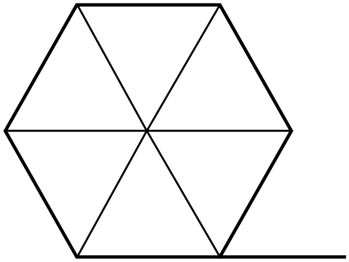

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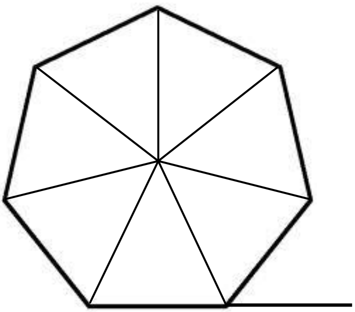
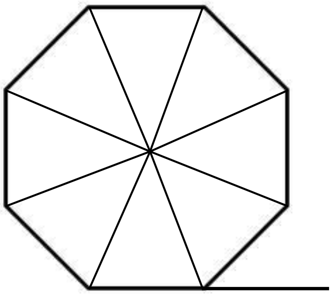
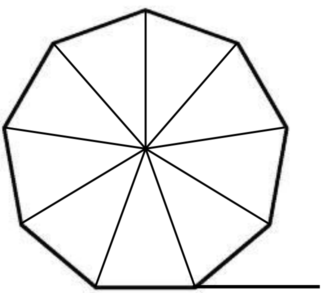


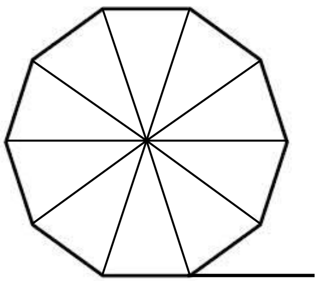
4)



5)

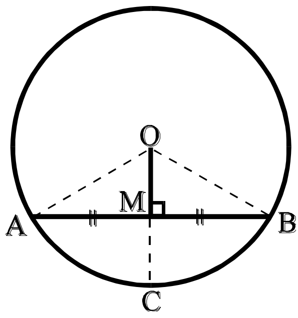


6)



Chords and perpendicular bisectors

1) Chord AB has length 18 cm and is 4 cm from the centre of the circle.



a) Make a copy of this diagram and fill the above information on your sketch.

b) Calculate the length of the radius of the circle.

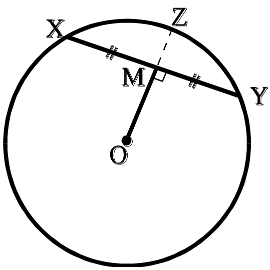
c) Thus state the length of the diameter.

d Write down the length of MC.

e Calculate the area of triangle AOB.

f What kind of shape is OACB?

2) Chord XY has length 12 cm. It is 10 cm from the centre O of the circle..



a) Calculate the length of the diameter of the circle.

b) Calculate the area of triangle XOY.

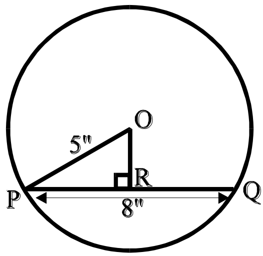
c) Write down the length of MZ.

3) A chord of length 6 inches is 6 inches from the centre of a circle.

a) Draw a diagram to illustrate the above information.

b) Calculate the lengths of the radius and diameter of the circle.

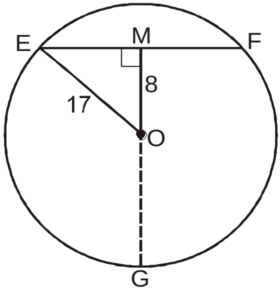
4) The diagram shows a circle of diameter 10" (radius 5").
Chord PQ within this circle has length 8".



a) Calculate the distance of the chord from the centre.

b) Calculate the area of triangle POQ.

5) EF is a chord 8 cm from the centre of a circle of radius 17 cm.



a) Calculate the length of the chord EF.

b) MO is produced until it meets the circle at point G.

Calculate the area of triangle EFG.

6) A chord 6 ins long is drawn in a circle of diameter 10 ins.

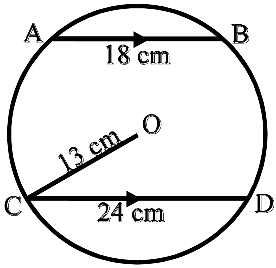
Calculate the distance of the chord from the centre.

7) A chord is drawn in a circle of radius 5·2 cm at a distance of 2 cm from the centre. Calculate the length of the chord.

8) A chord 6 ins long is at a distance of $1\frac{3}{5}$ ins from the centre of a circle.

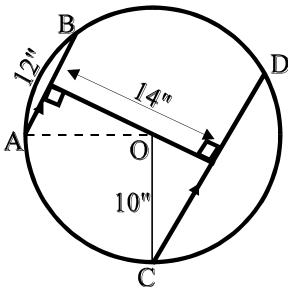
Calculate the length of the diameter of the circle.

9) AB and CD are parallel chords situated on opposite sides of the centre as shown in the diagram. AB has length 18 cm, CD has length 24 cm and the radius of the circle is 13 cm.



a) Calculate the distance between the 2 chords.

b) What would their distance apart have been had they been situated on the same side of the centre?



10) The parallel chords AB and CD are 14" apart. The chord AB has length 12", and the circle has radius 10".

a) Calculate the length of the chord CD.

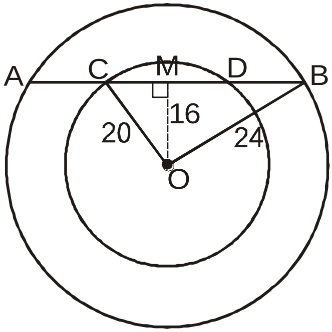
b) Calculate the area of the trapezium ABCD.

11) 2 parallel chords of a circle of radius 14 cm are 15 cm and 18 cm in length. How far are the chords apart? (There are 2 different answers)

12) In a circle of radius 2.5", two parallel chords are placed 3.1" apart. If the length of one chord is 4.8", find the length of the other.

13) Two equal and parallel chords in a circle of radius 6.5" are 7.8" apart. Calculate the length of each chord.

14) The two circles shown in this diagram are **concentric** (same centre, O).



The chord AB is 16 cm away from the centre and the radii are 20 cm and 24 cm.

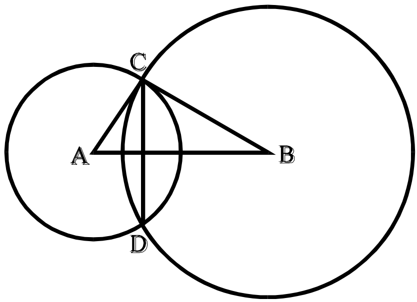
a) Calculate the length of CM.

b) Calculate the length of MB.

c) Write down the length of AC.

d) Calculate the area of triangle BCO.

15) A and B are the centres of the 2 circles in the accompanying diagram.



The circles have radii 7 cm and 12 cm. The common chord CD has length 10 cm.

a) Calculate the length of the line joining the 2 centres AB.

b) What is the maximum possible distance between 2 points which are on the circumferences of either circle?

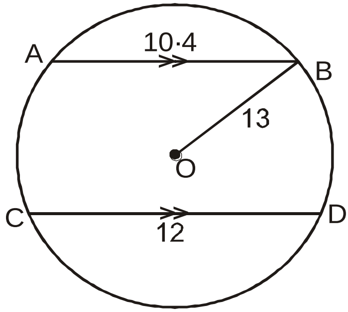
c) The line AB meets the circles at E and F.

Calculate the length of EF.

16) AB and CD are two parallel chords of this circle whose radius is 13 cm.

If AB = 10·4 cm and CD = 12 cm. calculate the distance between them.

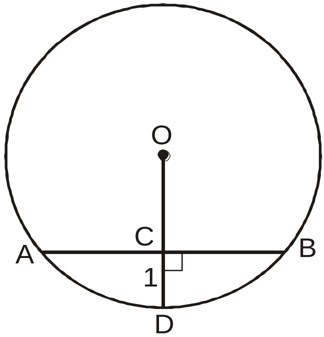
(There are two possible answers – another diagram may help)



17) AB is a chord of a circle centre O.

The perpendicular from O to the chord cuts the chord at C and meets the circle at D.

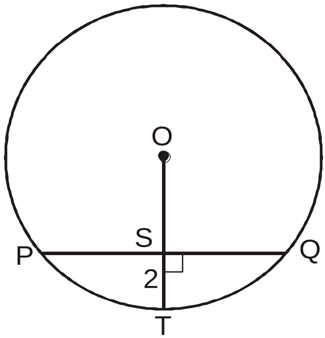
If AB = 8 cm and CD = 1 cm, find the radius of the circle.



18) PQ is a chord of a circle centre O.

The perpendicular from O to the chord cuts the chord at S and meets the circle at T.

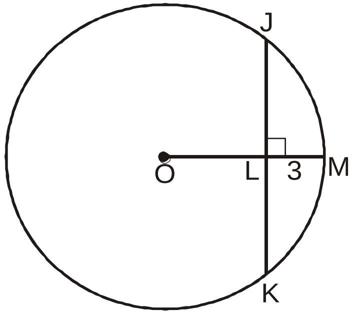
If PQ = 12 cm and ST = 2 cm, find the radius of the circle.



19) JK is a chord of a circle centre O.

The perpendicular from O to the chord cuts the chord at L and meets the circle at M.

If JK = 14 cm and LM = 3 cm, find the diameter of the circle.



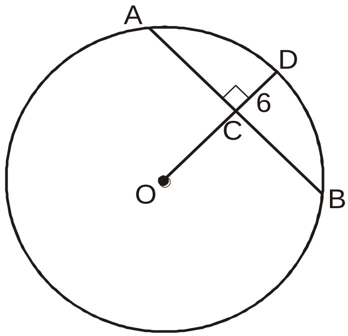
20) AB is a chord of a circle centre O.

The perpendicular from O to the chord cuts the chord at C and meets the circle at D.

If AB = 20 cm and CD = 6 cm, find

a) the radius of the circle

b) the area of triangle AOB.



21) A sheep shelter is part of a cylinder as shown in Figure 1.

It is 6 metres wide and 2 metres high.

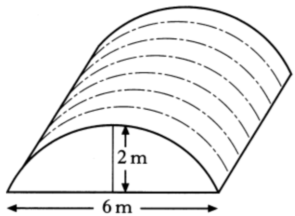


Figure 1

The cross section of the shelter is a segment of a circle with centre O, as shown in Figure 2.

OB is the radius of the circle.

Calculate the length of OB.

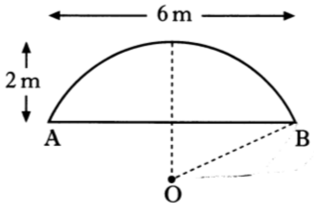
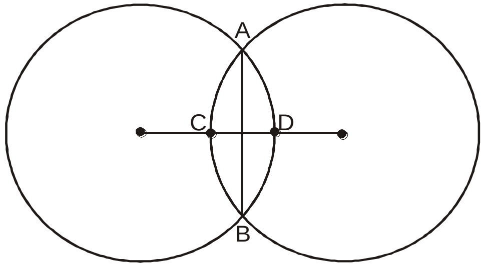


Figure 2

22) Two equal circles cut each other at A and B. The line joining the two centres meets the circles at C and D.

If AB = 14 cm and CD = 4 cm, calculate the length of the radius.



23) Water is flowing through a pipe with a circular cross-section.

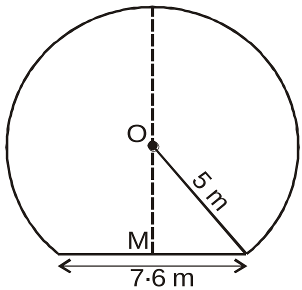
If the pipe has a radius of 2 metres and the width of the water surface is 3·5 metres, how deep is the water? (2 possible answers)

24) The entrance to a tunnel is an arc of radius 5 metres.

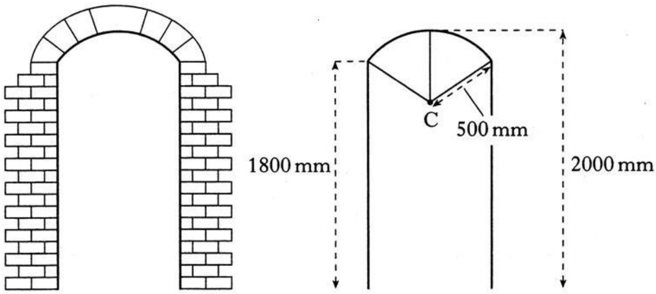
The width of the road is 7·6 metres. Calculate the

a) distance OM

b) height of the tunnel.



25) The curved part of a doorway is an arc of a circle with radius 500 mm and centre C.



The height of the doorway to the top of the arc is 2000 mm.

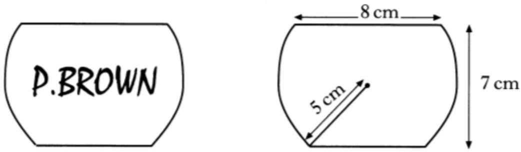
The vertical edge of the doorway is 1800 mm.

Calculate the width of the doorway.

26) A badge is made from a circle of radius 5 cm.

Segments are taken off the top and bottom of the circle as shown.

The straight edges are parallel.



The badge measures 7 cm from the top to the bottom.

The top is 8 cm wide.

Calculate the width of the base.

27) An oil tanker has a circular cross-section of diameter 4 metres.

The surface of the oil is 2·8 metres from the top of the tank.

a) What is the height of OM?

b) What is the width of the oil's surface? (see fig 1)

c) The tanker is filled to a depth of 3·5 metres.

What is now the width of the oil's surface?

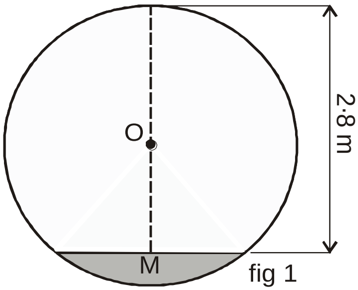


fig 1

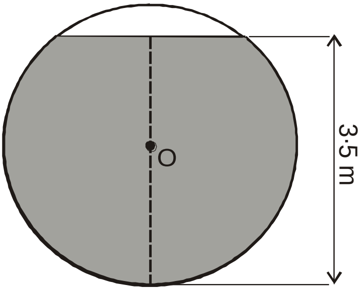
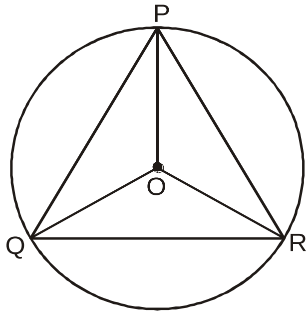


fig 2

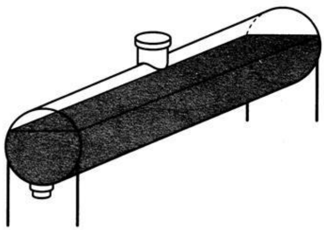
28) The design of a circular medallion is based on an equilateral triangle PQR.

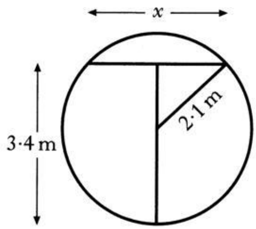
The sides of the triangle are 3 cm long.

Calculate the radius of the circle.



29 An oil tank has a circular cross-section of radius 2·1 m.





It is filled to a depth of 3·4 metres.

a) Calculate *x*, the width in metres of the oil surface.

b) What other depth of oil would give the same surface width?

Trigonometric Graphs

Exercise 1

a) Draw a rough sketch of each of the following functions.

b) State the period of the function.

c) State the amplitude of the function

---

1) $y=\sin x$ **2)** $y=\cos x$ **3)** $y=3\cos x$

4) $y=10\sin x$ **5)** $y=\frac{1}{2}\cos x$ **6)** $y=\sin x+1$

7) $y=\cos x-1$ **8)** $y=\sin x+2$ **9)** $y=\cos x-3$

10) $y=\sin 2x$ **11)** $y=\cos 2x$ **12)** $y=\sin 3x$

13) $y=\cos 4x$ **14)** $y=3\sin 2x$ **15)** $y=2\cos 3x$

16) $y=3\cos x+1$ **17)** $y=2\sin x-1$ **18)** $y=3\cos x-2$

19) $y=4\sin x-2$ **20)** $y=2\sin x+3$ **21)** $y=\sin 2x+1$

22) $y=\cos 2x-1$ **23)** $y=\sin 3x+2$ **24)** $y=\cos 4x-2$

25) $y=3\cos 2x+1$ **26)** $y=2\sin 2x-1$ **27)** $y=4\cos 2x+3$

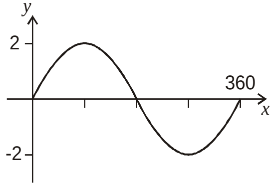
28) $y=2\sin 3x+1$ **29)** $y=\sin\left(x-90^0\right)$ **30)** $y=\cos\left(x+90^0\right)$

31) $y=\sin\left(x+180^0\right)$ **32)** $y=\sin\left(x-270^0\right)$

Exercise 2 Identification of Trigonometric graphs

1) The diagram shows the graph of

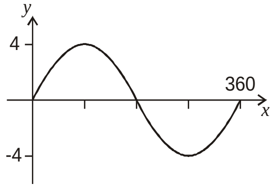
$y = k\sin x, \quad 0 \le x < 360$.



a) Find the value of *k*
b) State the period
c) State the amplitude

2) The diagram shows the graph of

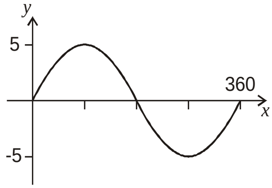
$y = k\sin x, \quad 0 \le x < 360$.



a) Find the value of *k*
b) State the period
c) State the amplitude

3) The diagram shows the graph of

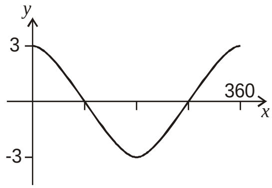
$y = k\sin x, \quad 0 \le x < 360$.



a) Find the value of *k*
b) State the period
c) State the amplitude

4) The diagram shows the graph of

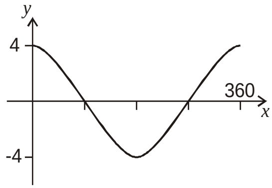
$y = k\cos x, \quad 0 \le x < 360$.



Find the value of *k*.

5) The diagram shows the graph of

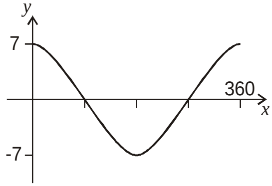
$y = k\cos x, \quad 0 \le x < 360$.



a) Find the value of *k*
b) State the period
c) State the amplitude

6) The diagram shows the graph of

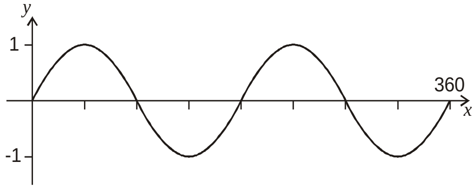
$y = k\cos x, \quad 0 \leq x < 360$.



a) Find the value of *k*
b) State the period
c) State the amplitude

7) The diagram shows the graph of

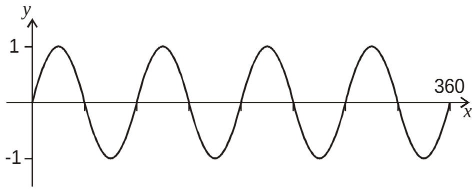
$y = \sin ax, \quad 0 \leq x < 360$.



a) Find the value of *a*
b) State the period
c) State the amplitude

8) The diagram shows the graph of

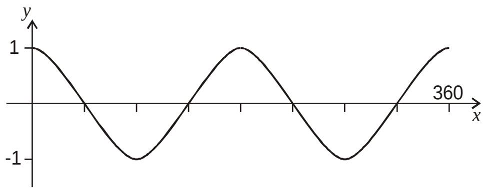
$y = \sin ax, \quad 0 \leq x < 360$.



a) Find the value of *a*
b) State the period
c) State the amplitude

9) The diagram shows the graph of

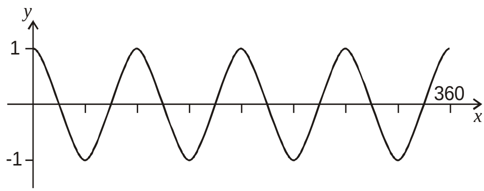
$y = \cos ax, \quad 0 \leq x < 360$.



a) Find the value of *a*
b) State the period
c) State the amplitude

10) The diagram shows the graph of

$y = \cos ax, \quad 0 \leq x < 360$.

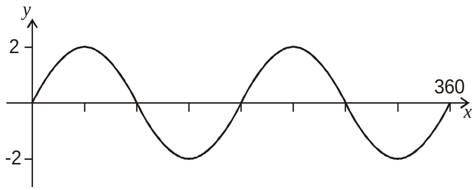


a) Find the value of *a*
b) State the period
c) State the amplitude

11) The diagram shows the graph of

$$y = k \sin ax, \quad 0 \le x < 360.$$

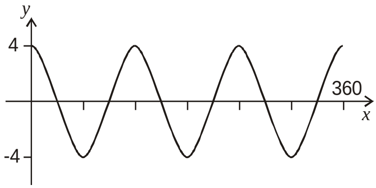
a) Find the values of *a and k*
b) State the period
c) State the amplitude



12) The diagram shows the graph of

$$y = k \cos ax, \quad 0 \le x < 360.$$

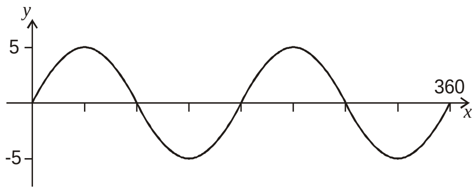
a) Find the values of *a and k*
b) State the period
c) State the amplitude



13) The diagram shows the graph of

$$y = k \sin ax, \quad 0 \le x < 360.$$

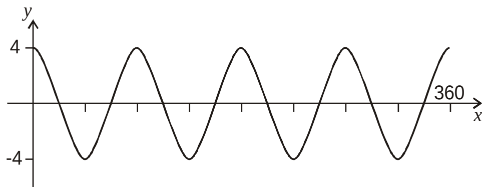
a) Find the values of *a and k*
b) State the period
c) State the amplitude



14) The diagram shows the graph of

$$y = k \cos ax, \quad 0 \le x < 360.$$

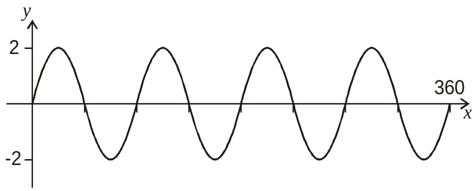
a) Find the values of *a and k*
b) State the period
c) State the amplitude



15) The diagram shows the graph of

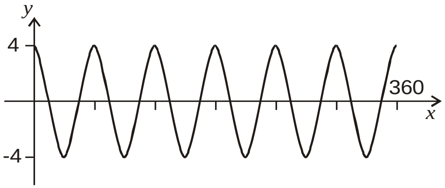
$$y = k \sin ax, \quad 0 \le x < 360.$$

a) Find the values of *a and k*
b) State the period
c) State the amplitude



16) The diagram shows the graph of

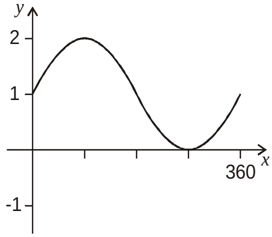
$$y = k\cos ax, \quad 0 \le x < 360.$$



a) Find the values of *a and k*
b) State the period
c) State the amplitude

17) The diagram shows the graph of

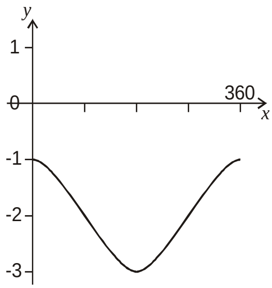
$$y = k + \sin x, \quad 0 \le x < 360.$$



a) Find the value of *k*
b) State the period
c) State the amplitude

18) The diagram shows the graph of

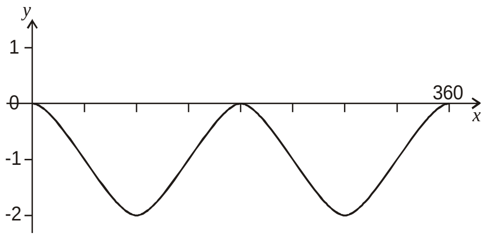
$$y = k + \cos x, \quad 0 \le x < 360.$$



a) Find the value of *k*
b) State the period
c) State the amplitude

19) The diagram shows the graph of

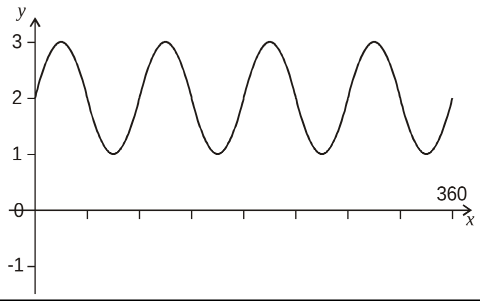
$$y = k + \cos ax, \quad 0 \le x < 360.$$



a) Find the values of *a and k*
b) State the period
c) State the amplitude

20) The diagram shows the graph of

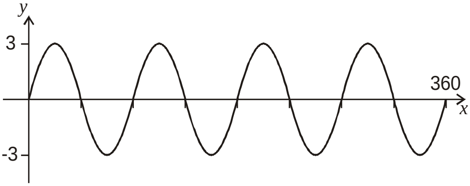
$$y = k + \sin ax, \quad 0 \le x < 360.$$



a) Find the values of *a and k*
b) State the period
c) State the amplitude

21) The diagram shows the graph of

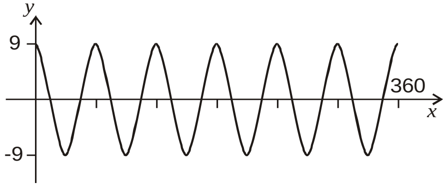
$y = k\sin ax, \quad 0 \le x < 360$.



a) Find the values of *a and k*
b) State the period
c) State the amplitude

22) The diagram shows the graph of

$y = k\cos ax, \quad 0 \le x < 360$.



a) Find the values of *a and k*
b) State the period
c) State the amplitude

Exercise 3

For each function, find

a) the maximum and minimum values of the function.

b) a corresponding replacement for x for each.

1) $y = 3 + \sin x$ **2)** $y = \cos x + 7$ **3)** $y = 2\sin x$

4) $y = 4\cos x$ **5)** $y = 25\cos x$ **6)** $y = \sin x - 3$

7) $y = \cos x - 1$ **8)** $y = 2\sin x + 4$ **9)** $y = 3\cos x + 5$

10) $y = -2 + \cos x$ **11)** $y = -8 + 3\sin x$ **12)** $y = \cos 3x$

13) $y = \sin 2x$ **14)** $y = 1 + \sin 2x$ **15)** $y = 1 + 2\sin x$

16) $y = 5 + \cos 2x$ **17)** $y = 4 + \sin 2x$ **18)** $y = -1 + \cos 2x$

19) $y = -1 + 2\cos x$ **20)** $y = 4\cos x - 3$ **21)** $y = \cos 4x - 3$

22) $y = 3\cos 4x - 2$ **23)** $y = 4\cos 3x - 2$

24) On a certain day the depth, D metres, of water in an estuary, t hours after midnight, is given by the formula:

$$D = 15\cdot 2 + 7\cdot 8\sin(30t)^0$$

a) find the depth of the water at 2.00 pm.

b) what is the time after midnight when the water is 20 m?

c) find the maximum depth of the estuary and at what time after midnight it first occurs.

d) find the minimum depth of the estuary and at what time after midnight this first occurs.

e) how long does it take for the depth to change from maximum to minimum depth?

25) Repeat question **24** with formula:

$$D = 12 + 8\cdot 6\cos(30t)^0$$

26) A toy is hanging by a spring from the ceiling.

Once the toy is set moving, the height H metres at time *t* seconds, of the toy above the floor is given by the formula:

$$H = 1\cdot 7 + 0\cdot 2\cos(30t)^0$$

a) state the minimum value of H.

b) find the height of the toy above the floor after 10 seconds.

c) find when the toy is first 1.6 m above the floor.

27) The number of hours of daylight h, in a British city is given by the formula;

$$h = 11\cdot 7 + 2\cdot 3\sin\left(\frac{360x}{365}\right)^0$$

where x is the number of days after 21 April.

a) what is the maximum number of hours of daylight?

b) what is the least number of hours of daylight,
and by how many days after 21 April does this occur?

28) The depth d metres, of water in a harbour entrance over a 24 hour period starting at midnight is given by:

$$d = 6 + 2\sin(30h)^0$$

where h is the number of hours after midnight.

a) find the maximum depth of the harbour and the first time after midnight this occurred.

b) a fishing boat needing a depth of 5 metres wants to dock in the harbour. If the depth of the harbour at midnight is 6 metres, how long is it before the depth reaches 5 metres and it becomes too shallow for the boat to dock?

c) what is the first time that the depth again goes above 5 metres and it becomes deep enough for the boat to leave the dock?

Exercise 4

Solve the following equations where $0^0 \le x \le 360^0$.

1) $3\cos x - 1 = 0$ **2)** $5\cos x - 2 = 0$ **3)** $6\sin x - 1 = 0$

4) $9\sin x - 2 = 0$ **5)** $8\sin x - 3 = 0$ **6)** $4\tan x - 3 = 0$

7) $3\tan x - 9 = 0$ **8)** $4\cos x + 1 = 0$ **9)** $7\cos x + 3 = 0$

10) $12\sin x + 1 = 0$ **11)** $8\tan x + 12 = 0$ **12)** $9\tan x + 5 = 0$

13) $4 + 3\tan x = 5$ **14)** $7 + \cos x = 2$ **15)** $2 + 3\sin x = 1$

16) $\cos 75° = 3\sin x + 1$ **17)** $\tan 35° = 2\cos x + 1$

18) $8\sin x + 2 = -3$ **19)** $2 - 11\sin x = 5$ **20)** $3(2\sin x + 1) = 4$

Exercise 5

Solve the following equations where $0^0 \le x \le 360^0$.

1) $2\sin x - 1 = 0$ **2)** $\sqrt{2}\cos x - 1 = 0$ **3)** $2\cos x - \sqrt{3} = 0$

4) $2\cos x - 1 = 0$ **5)** $\sqrt{3}\tan x + 1 = 0$ **6)** $\tan x + 1 = 0$

7) $2\cos x+\sqrt{3}=0$ 8) $\tan x-\sqrt{3}=0$ 9) $2\sin x+\sqrt{3}=0$

10) $\tan x+3=4$ 11) $\sqrt{2}\sin x+1=2$ 12) $2\sin x+1=0$

13) $\tan x+\sqrt{3}=0$ 14) $2+\sqrt{2}\cos x=1$ 15) $3-\sqrt{2}\sin x=4$

16) $1+2\sin x=3$

Exercise 6

Calculate where the following trigonometric graphs cut:

a) the x-axis **b)** the y-axis

1) $y=2\cos x+1$ 2) $y=4\sin x-3$ 3) $y=2\cos x-\sqrt{3}$

4) $y=4\tan x-1$ 5) $y=2\sin x+1$ 6) $y=6\sin x-5$

7) $y=2\cos x+4$ 8) $y=2\tan x-3$ 9) $y=3-7\cos x$

10) $y=\sqrt{3}\tan x-1$ 11) $y=4\cos x+3$ 12) $y=2-5\sin x$

Exercise 7

Prove the following identities:

1) $3\cos^2 x-2=1-3\sin^2 x$

2) $(\cos x+\sin x)(\cos x-\sin x)=2\cos^2 x-1$

3) $(1-\sin^2 x)\tan^2 x=\sin^2 x$ 4) $\dfrac{\sin x}{1+\cos x}+\dfrac{1+\cos x}{\sin x}=\dfrac{2}{\sin x}$

5) $3\cos^2 x+3\sin^2 x=3$ 6) $(\cos x-\sin x)^2+2\cos x\sin x=1$

7) $(1-\sin x)(1+\sin x)=\cos^2 x$ 8) $3-3\sin^2 x=3\cos^2 x$

9) $5\cos^2 x=5-5\sin^2 x$

10) $(\cos x-\sin x)(\cos x+\sin x)=1-2\sin^2 x$

11) $\cos x \tan x = \sin x$

12) $\tan x\sqrt{1-\cos^2 x} = \dfrac{\sin^2 x}{\cos x}$

13) $\dfrac{1-\cos^2 x}{\cos^2 x} = \tan^2 x$

14) $\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x} = \dfrac{1}{\cos x \sin x}$

15) $4\sin^2 x - 3 = 1 - 4\cos^2 x$

16) $2\cos^2 x - 1 = 1 - 2\sin^2 x$

17) $(\cos x - \sin x)^2 = 1 - 2\sin x \cos x$

18) $\cos^4 x - \sin^4 x = \cos^2 x - \sin^2 x$

19) $2\cos^2 x + 2\sin^2 x - 2 = 0$

20) $\dfrac{1}{1+\sin x} + \dfrac{1}{1-\sin x} = \dfrac{2}{\cos^2 x}$

21) $\cos^2 x(1+\tan^2 x) = 1$

22) $(\cos x + \sin x)^2 = 1 + 2\sin x \cos x$

23) $2\sin^2 x + 3\cos^2 x = 2 + \cos^2 x$

24) $\cos^4 x + \cos^2 x \sin^2 x = \cos^2 x$

25) $\dfrac{\sin x - 2\sin^3 x}{2\cos^3 x - \cos x} = \tan x$

26) $\sin^4 x = 1 - 2\cos^2 x + \cos^4 x$

27) $1 + 3\tan^2 x = \dfrac{1+2\sin^2 x}{\cos^2 x}$

28) $\tan^2 x - \sin^2 x = \dfrac{\sin^4 x}{\cos^2 x}$

29) $\sqrt{1-\sin^2 x}.\tan^2 x = \dfrac{\sin^2 x}{\cos x} = \dfrac{1}{\cos x} - \cos x$

30) $3\sin^2 x + 2\cos^2 x = 3\cos^2 x$

31) $\sin x \cos x \tan x = 1 - \cos^2 x$

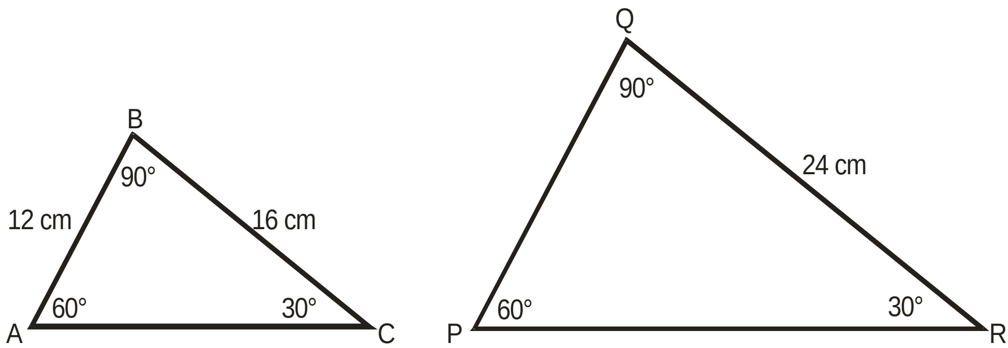
32) $\tan x + \dfrac{1}{\tan x} = \dfrac{1}{\sin x \cos x}$

33) $\dfrac{\cos x}{1+\sin x} + \dfrac{1+\sin x}{\cos x} = \dfrac{2}{\cos x}$

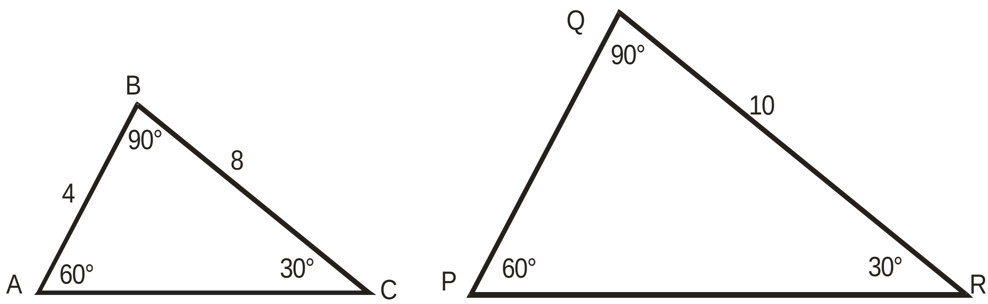
Similar Triangles

Exercise 1

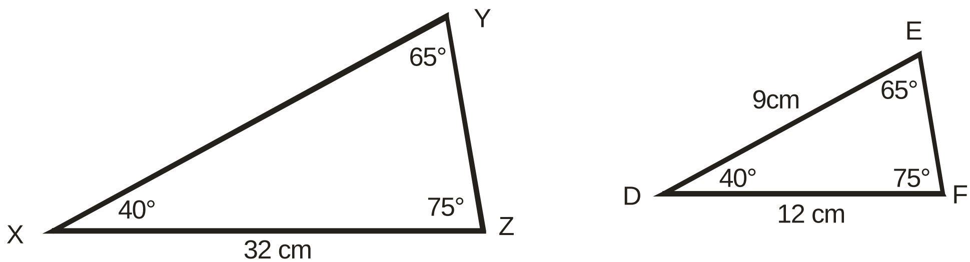
1) Find the length of the side PQ.



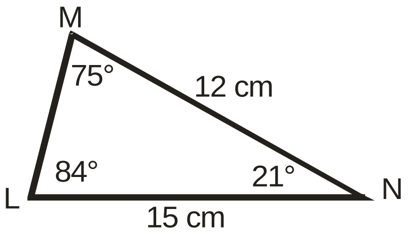
2) Find the length of the side PQ.

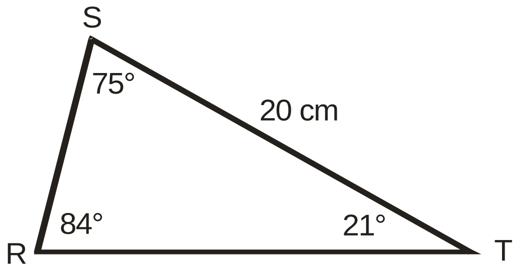


3) Find the length of the side XY.

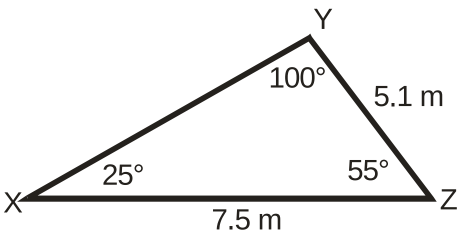


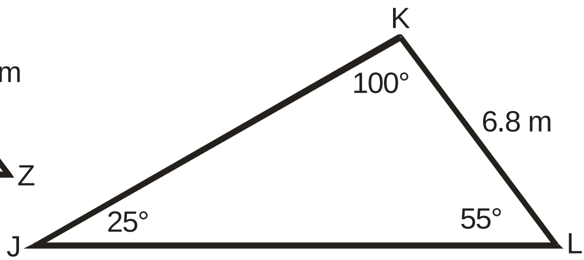
4) Find the length of the side RT.



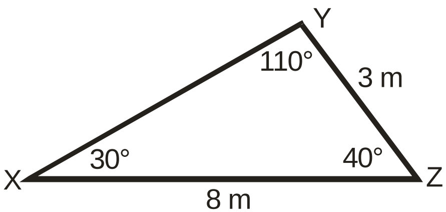


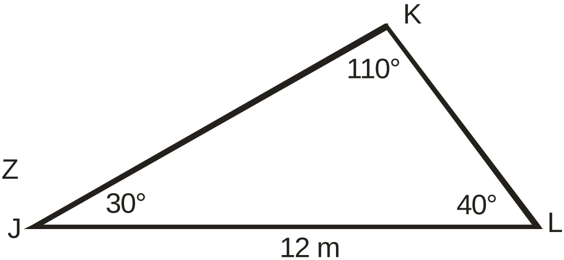
5) Find the length of the side JL.



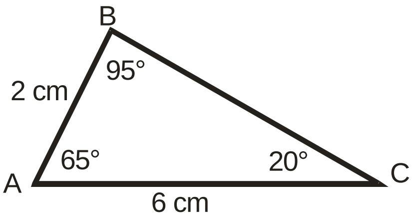


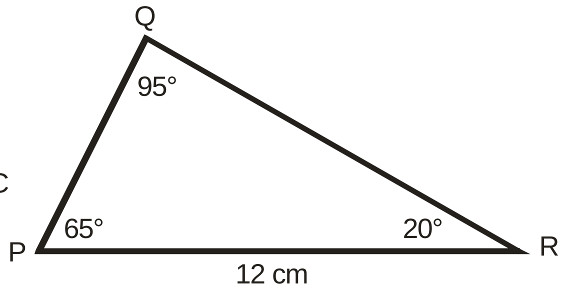
6) Find the length of the side KL.



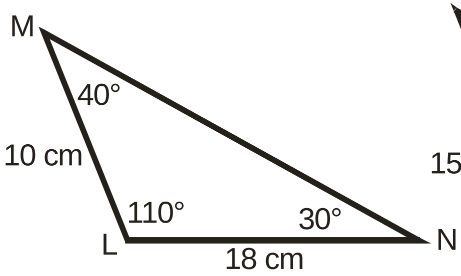


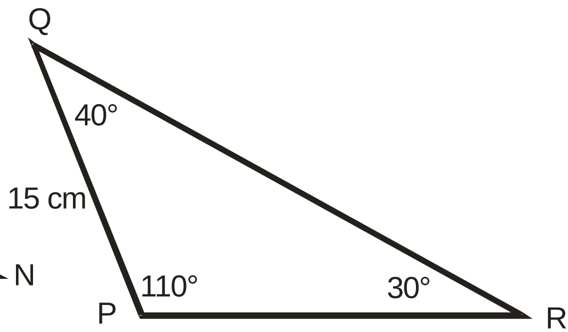
7) Find the length of side PQ



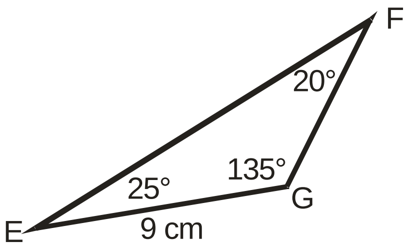


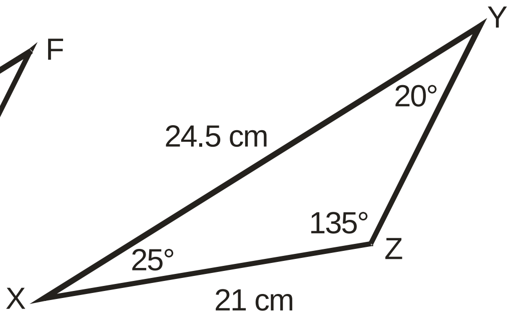
8) Find the length of PR.



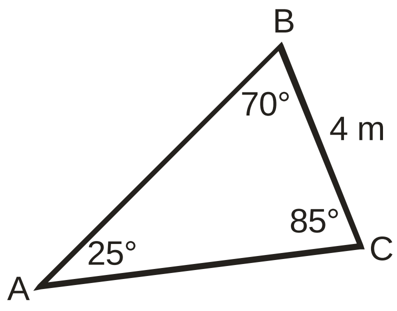


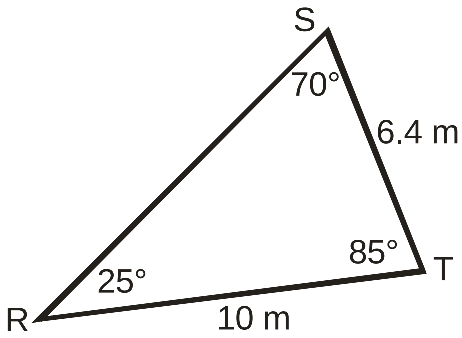
9) Find the length of EF.



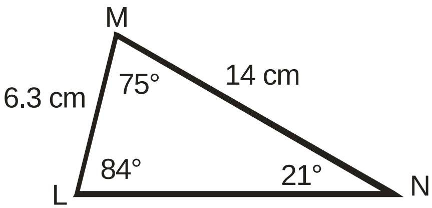


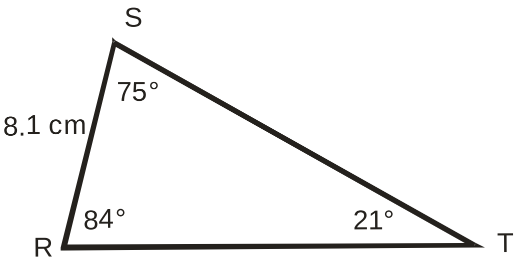
10) Find the length of the side AC



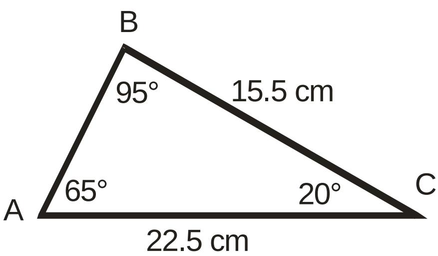


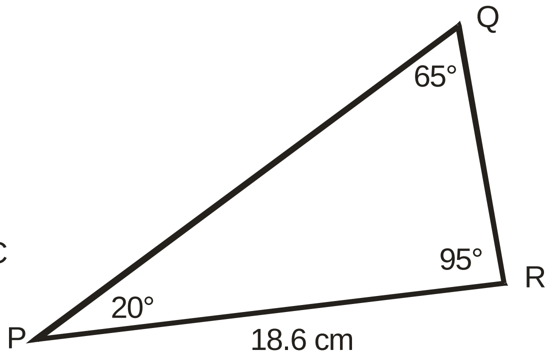
11) Find the length of the side ST.



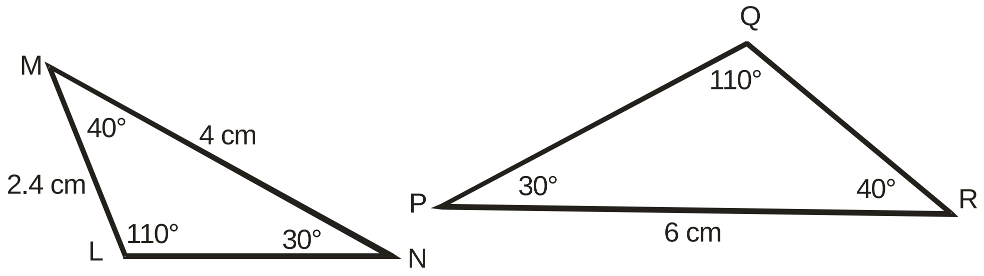


12) Find the length of side PQ.

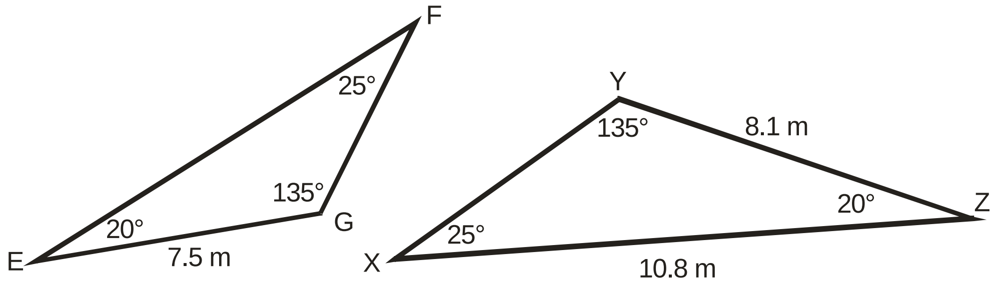




13) Find the length of QR.

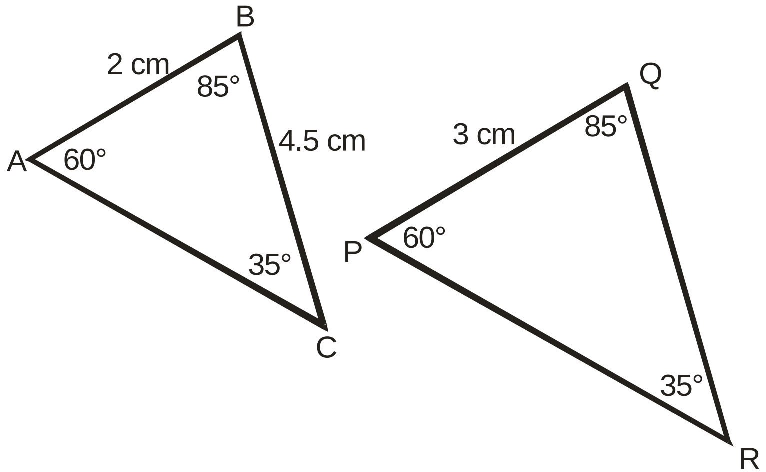


14) Find the length of side EF.



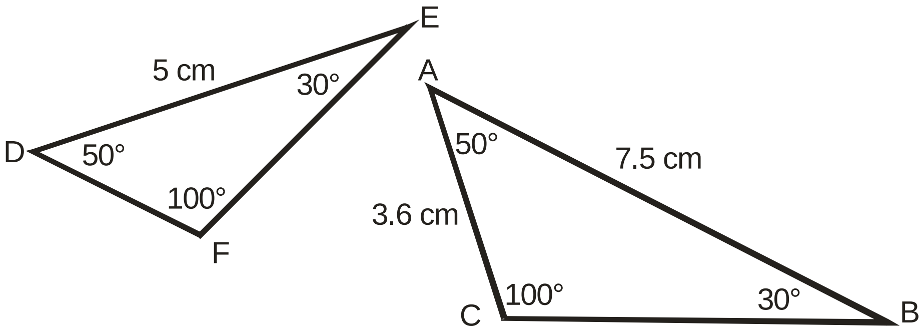
15) **a)** Find the length of side QR.

b) Find the ratio area of triangle ABC : area of triangle PQR.



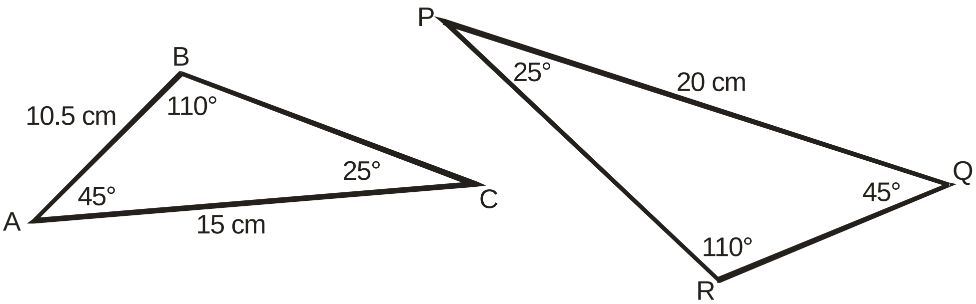
16) a) Find the length of DF.

b) Find the ratio of area of triangle DEF : area of triangle ABC.



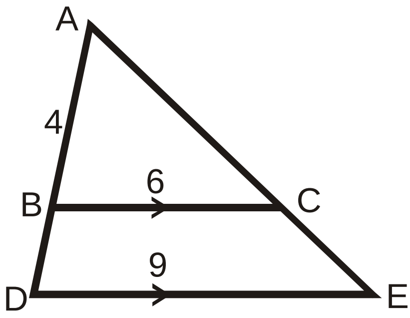
17) a) Find the length of RQ.

b) Find the ratio of area of triangle PQR : area of triangle ABC.

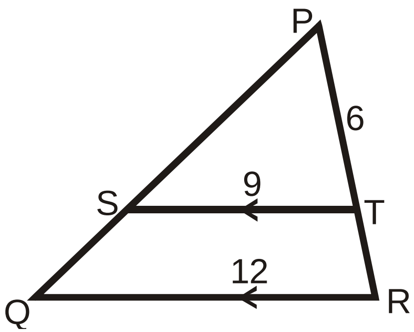


Exercise 2 (All sizes in centimetres)

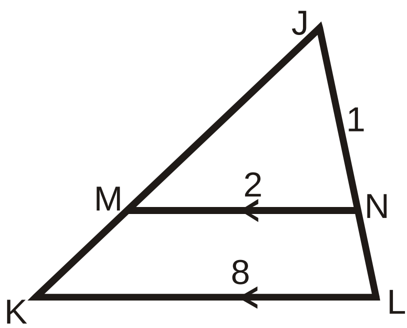
1) Find AD



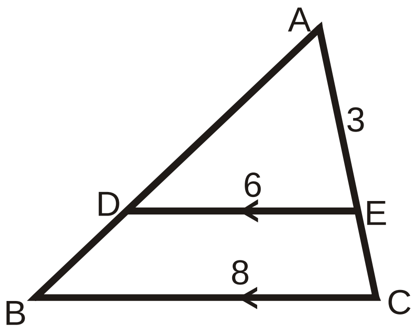
2) Find PR



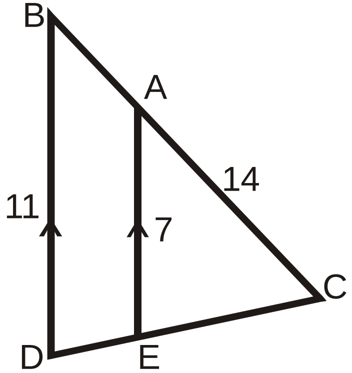
3) Find JL



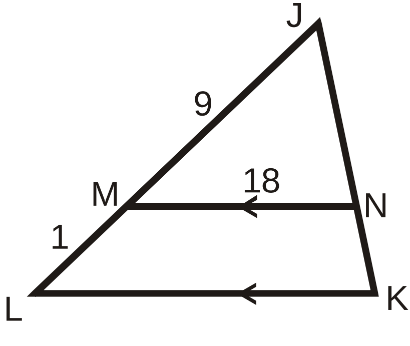
4) Find AC



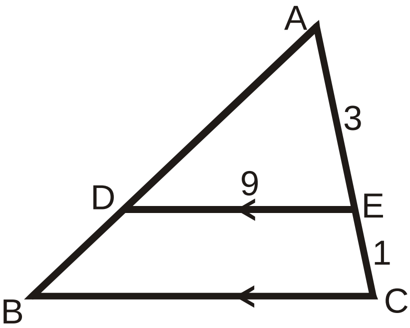
5) Find BC



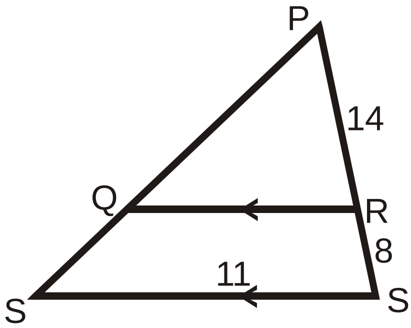
6) Find LK



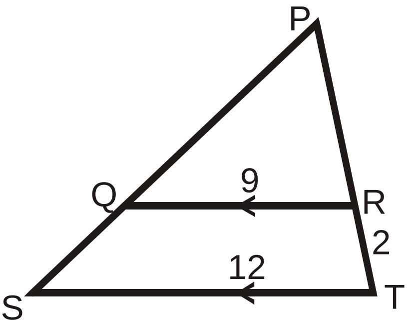
7) Find BC



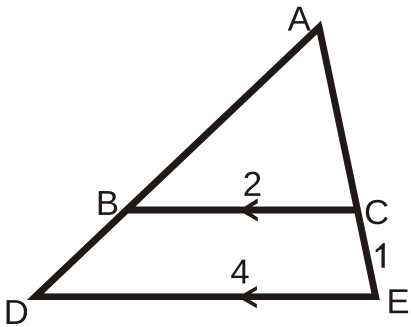
8) Find QR



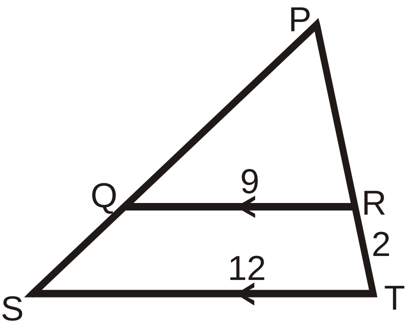
9) Find PR



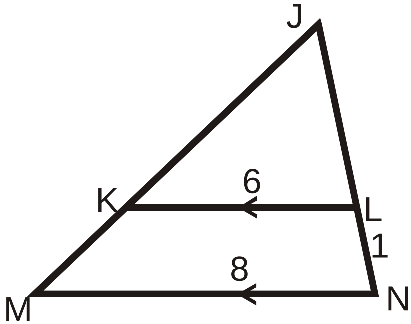
10) Find AC



11) Find PR

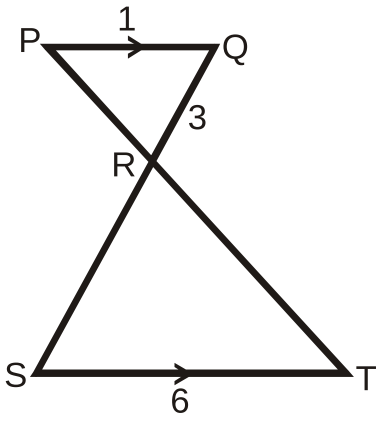


12) Find JL

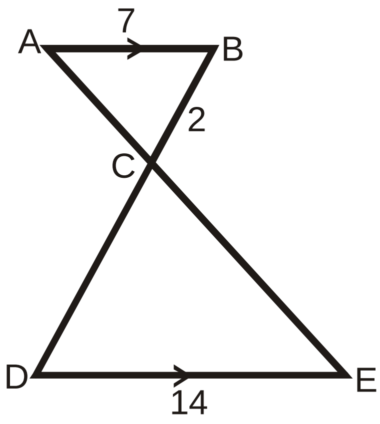


Exercise 3 (All sizes in centimetres)

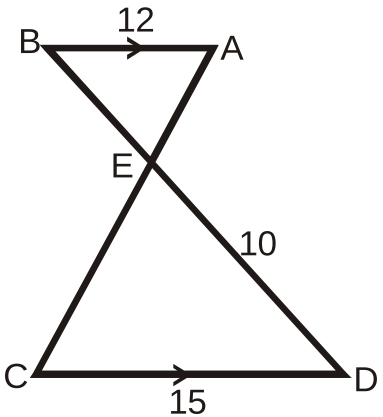
1) Find RS



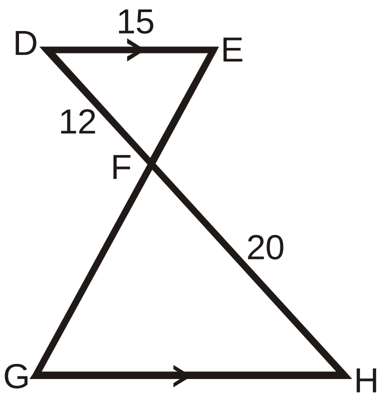
2) Find DC



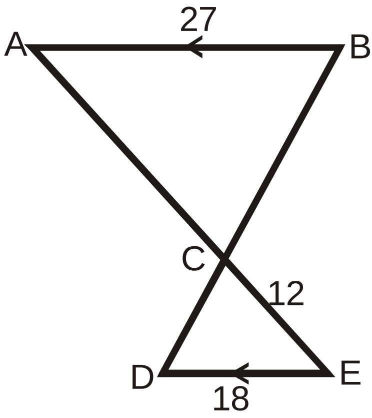
3) Find BE



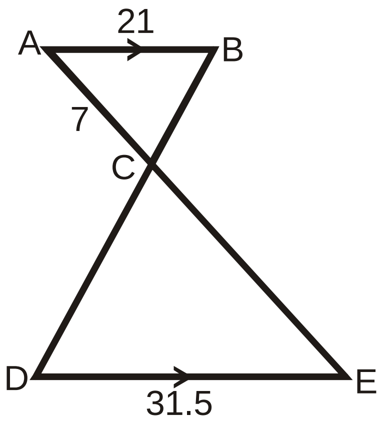
4) Find GH



5) Find AC



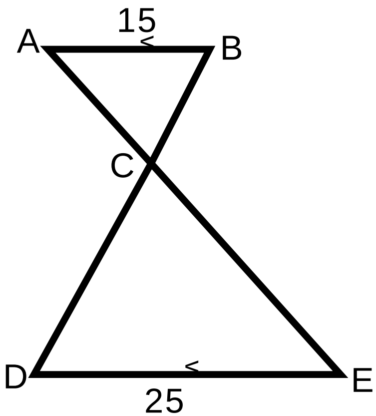
6) Find CE



7) If AE = 32, find

a) AC

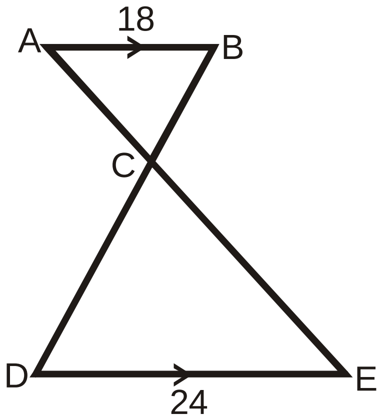
b) CE



8) If AE = 21, find

a) AC

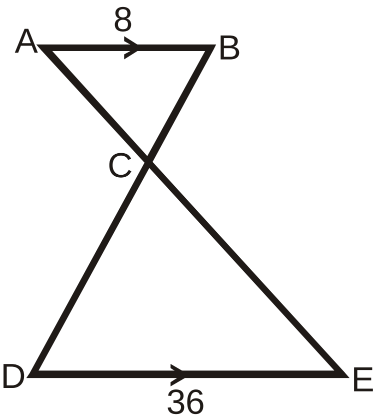
b) CE



9) If AE = 55, find

a) AC

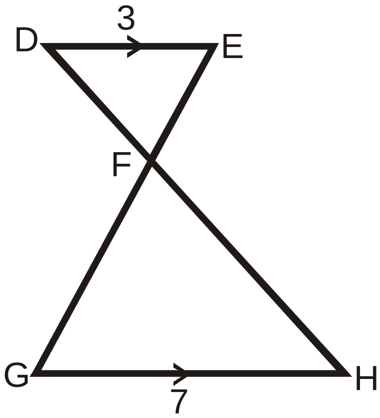
b) CE



10) If DE = 30, find

a) FE

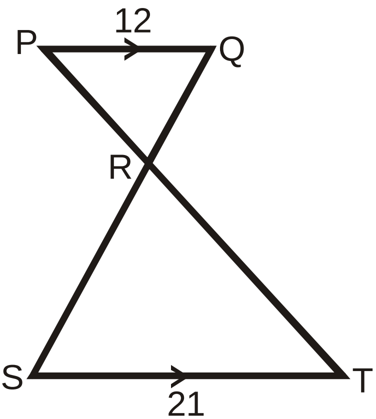
b) DF



11) If PT = 55, find

a) PR

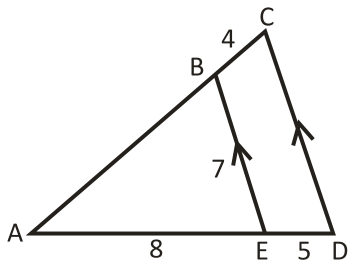
b) RT



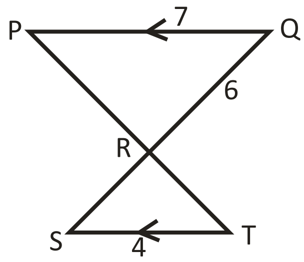
Exercise 4 (All sizes in centimetres)

1) **a)** Find CD

b) Find AB

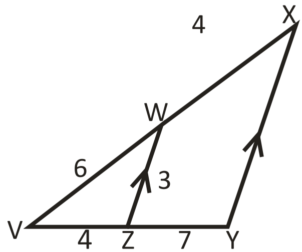


2) Find SR



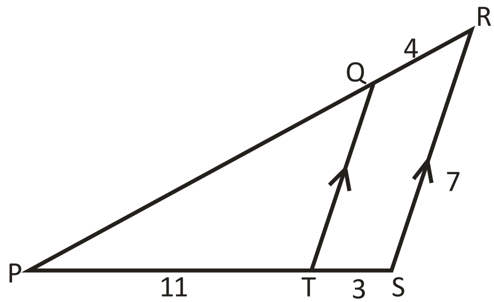
3) **a)** Find XY

b) Find WX



4) **a)** Find QT

b) Find PQ

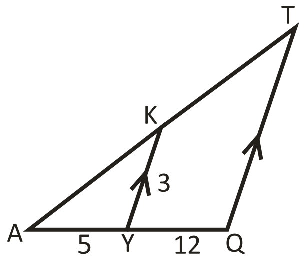


5) AT = 20

a) Find AK

b) Find KT

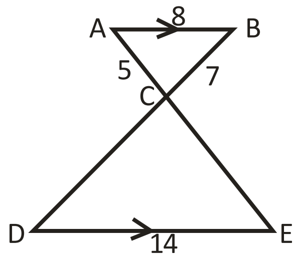
c) Find TQ



6) Calculate

a) CD

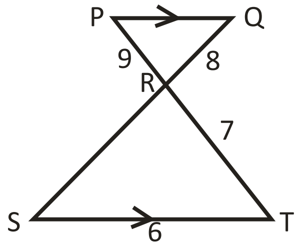
b) CE



7) Calculate

a) PQ

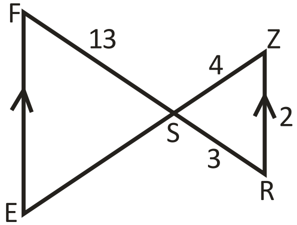
b) RS



8) Calculate

a) FE

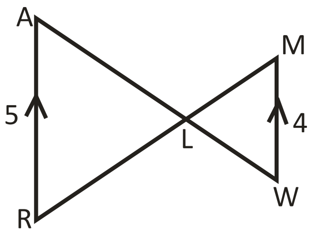
b) ES



9) AW =14

a) Calculate AL

b) Calculate LW

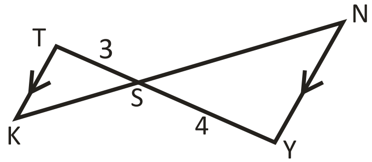


10) KN =12

a) Find KS and KN

b) If YN is 2 cm longer than TK,

find the lengths of YN and TK.

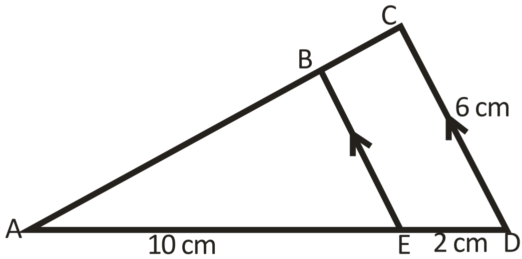


Exercise 5

1) Triangles ABE and ACD with some of their measurements, are shown opposite.

Triangle ABE is similar to triangle ACD.

Calculate the length of BE.



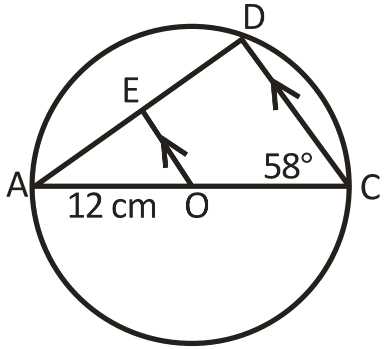
2) AC is the diameter of the circle with centre O and radius 12 cm.

AD is a chord of the circle.

OE is parallel to CD.

Angle ACD = 58^0.

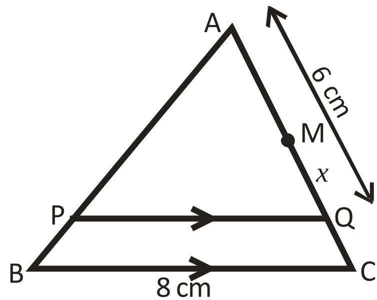
Calculate the length of ED.



3) In triangle ABC, BC = 8 cm, AC = 6 cm and PQ is parallel to BC.

M is the midpoint of AC.

Q lies on AC, x cm from M as shown in the diagram.



a) Write down an expression for the length of AQ.

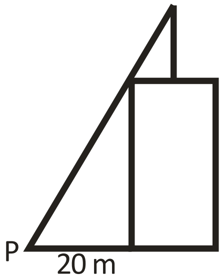
b) Show that PQ = $\left(4+\frac{4}{3}x\right)$ centimetres.

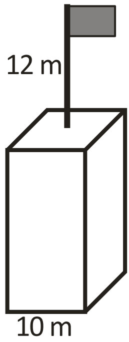
4) A vertical flagpole 12 metres high stands at the centre of the roof of a tower.

The tower is cuboid shaped with a square base of side 10 metres.

At a point P on the ground, 20 metres from the base of the tower, the top of the flagpole is just visible as shown.

Calculate the height of the tower.





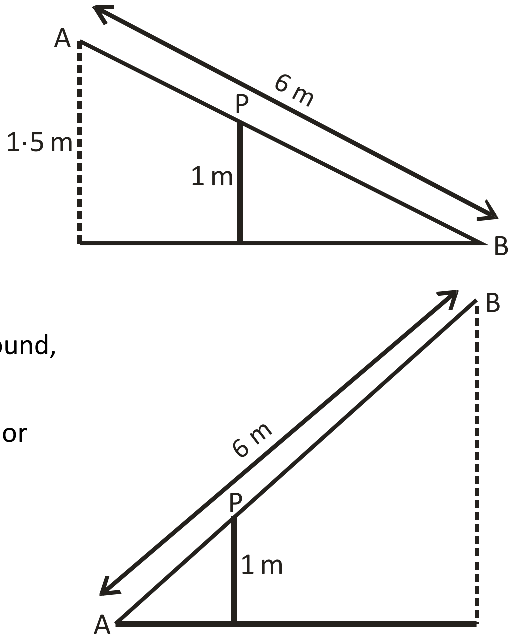
5) A metal beam AB is 6 metres long.

It is hinged at the top, P, of a vertical post 1 metre high.

When B touches the ground, A is 1·5 metres above the ground as shown.

When A comes down to the ground, B rises as shown.

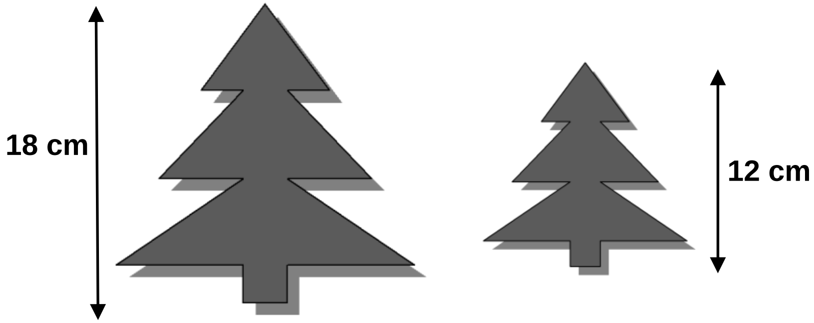
By calculating the length of AP, or otherwise, find the height of B above the ground.



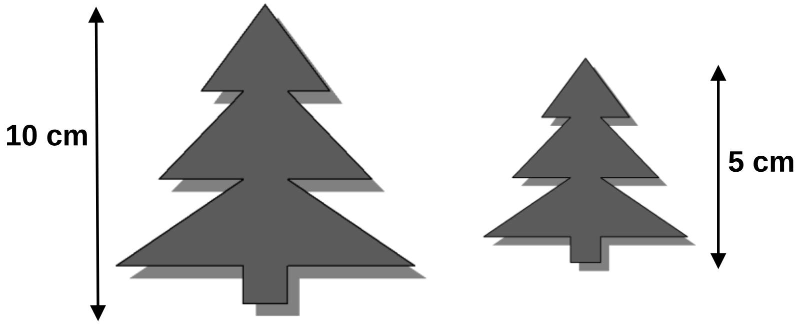
Exercise 6

In each question the shapes are similar. Find the ratio of their areas.

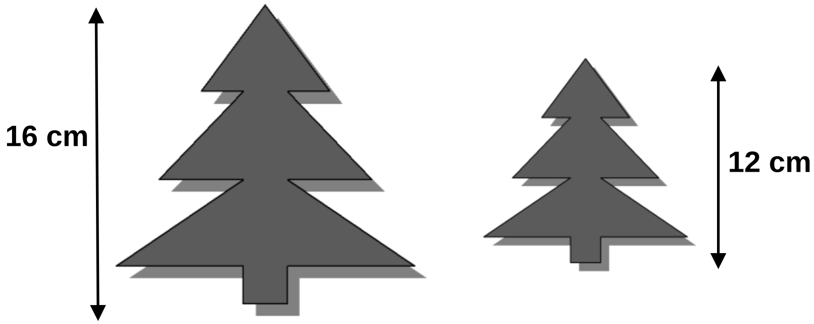
1)



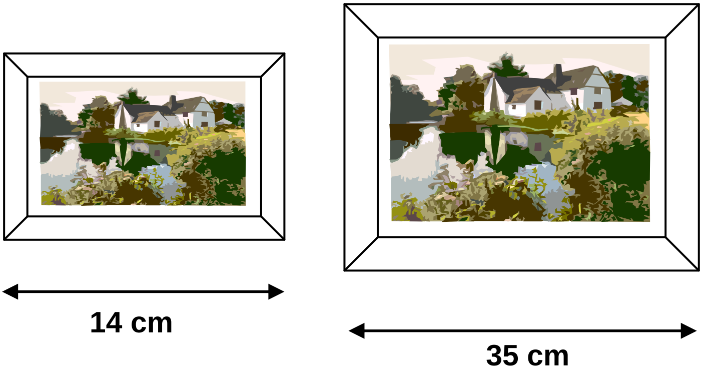
2)



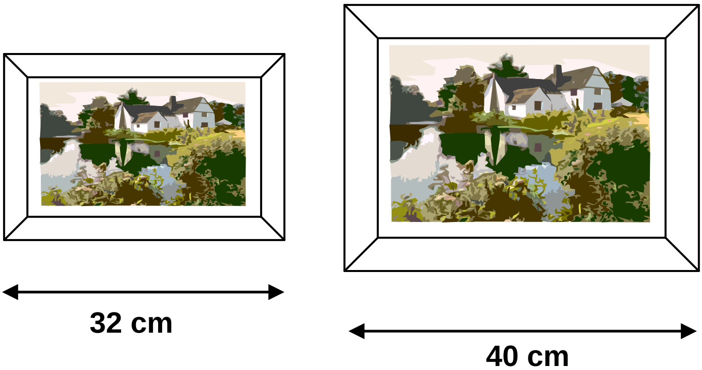
3)



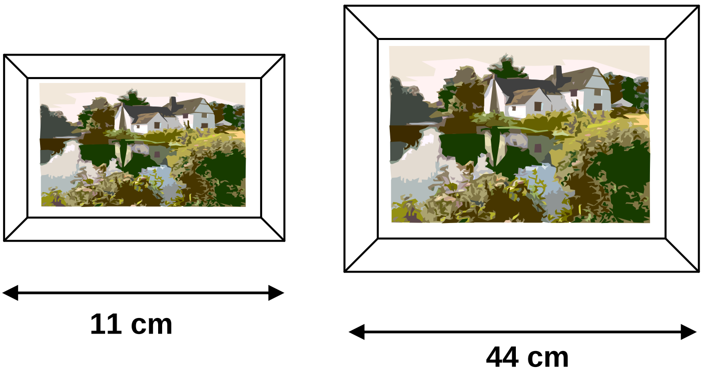
4)



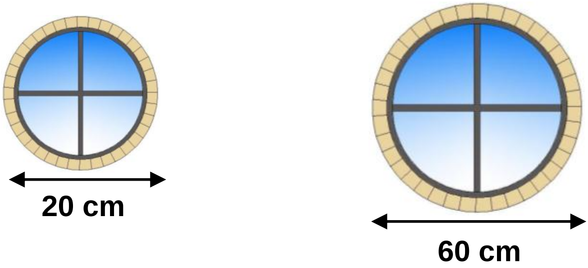
5)



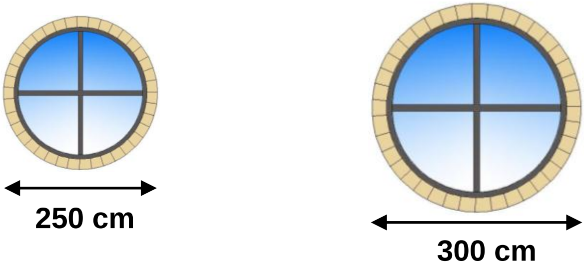
6)



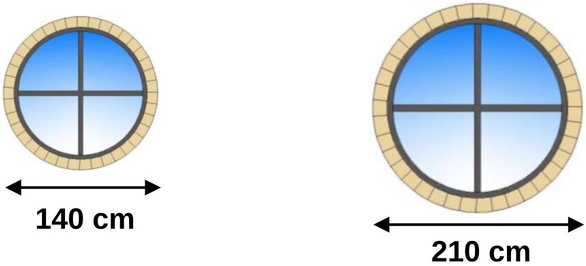
7)



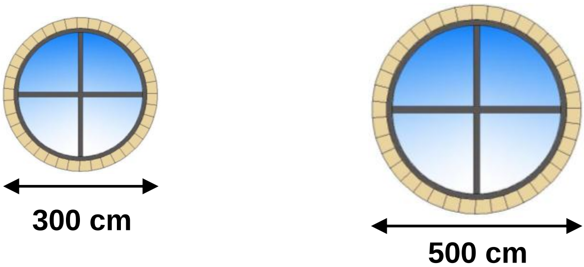
8)



9)



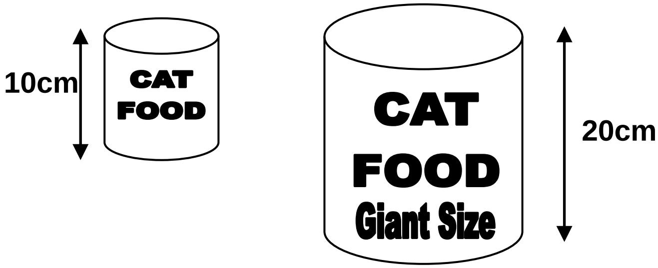
10)



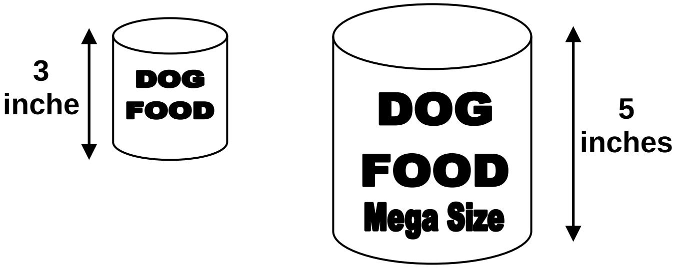
Exercise 7

In each question the shapes are similar. Find the ratio of their volumes.

1)

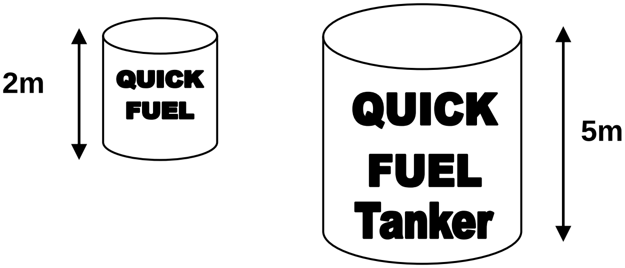


2)



3)

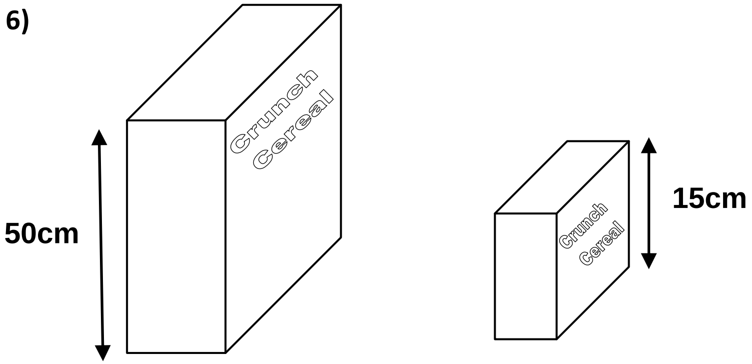
4)



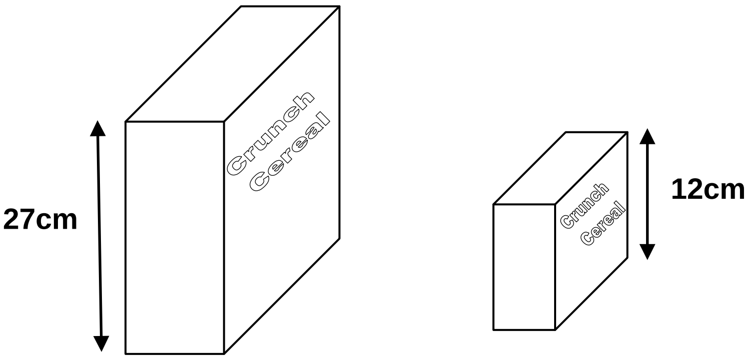
5)


7cm
SPORT
DRINK
SPORT
DRINK
Value Size
21cm

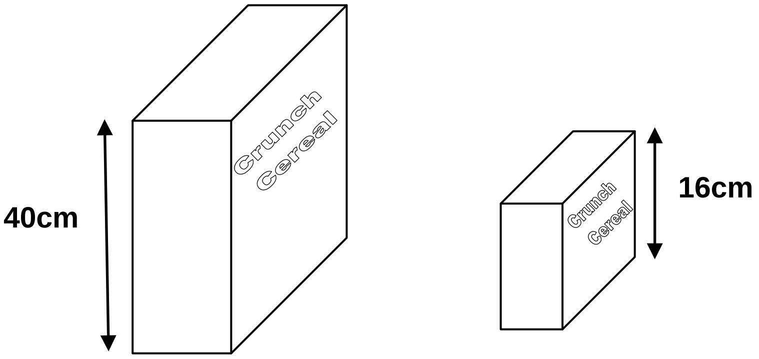
6)


Crunch
Cereal
50cm
Crunch
Cereal
15cm

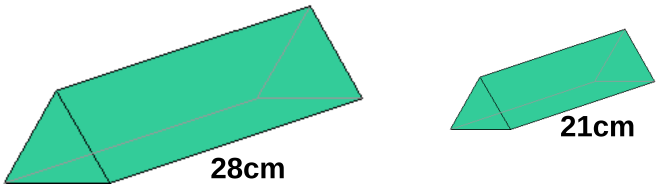
7)


Crunch
Cereal
27cm
Crunch
Cereal
12cm

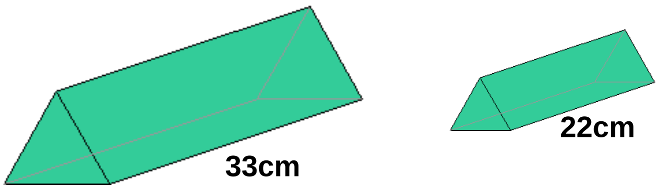
8)



9)

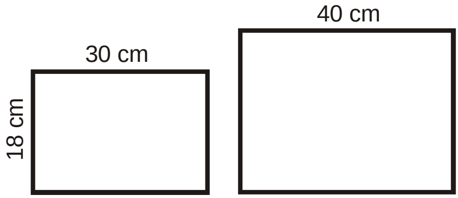


10)



Exercise 8

1) These 2 picture frames **are similar**.
The costs are directly related to their areas.

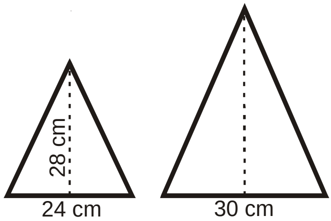


a) What is the breadth of the larger picture frame?

b) If the smaller frame costs £2·88, what is the cost of the larger frame?

c) If the larger frame costs £4·50, what is the cost of the smaller frame?

2) These 2 triangular boxes of chocolates are similar.
The costs are directly related to their areas.

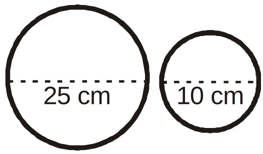


a) What is the "height" of the larger box?

b) If the smaller box costs £8, what is the cost of the larger box?

c) If the larger box costs £18·50, what is the cost of the smaller box?

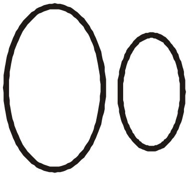
3) The costs of these 2 circular jigsaws are directly related to their areas.



a) If the cost of the larger jigsaw is £6, what is the cost of the smaller one?

b) If the cost of the smaller jigsaw is £1·40, what is the cost of the larger one?

4) These 2 mirrors **are similar**. The "heights" are 80 cm and 48 cm respectively.
The width of the larger mirror is 56 cm. The costs of the mirrors are directly related to their areas.

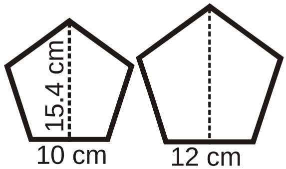


a) What is the width of the smaller mirror?

b) If the cost of the larger mirror is £36, find the cost of the smaller one.

c) The cost of the smaller mirror is £18·50, find the cost of the larger one.

5) These 2 pentagonal tiles **are similar**.
The costs are directly related to their areas.

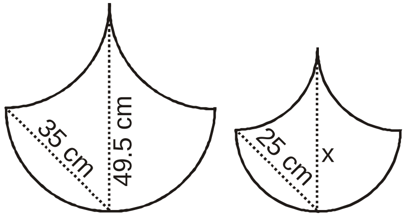


a) What is the length of the dotted line in the larger pentagon?

b) If the cost of the smaller tile is £1·50, what is the cost of the larger one?

c) If the cost of the larger tile is £3·50, find the cost of the smaller one.

6) These 2 chopping boards **are similar**.
The costs are directly related to their areas.



a) Calculate x.

b) If the cost of the large chopping board is £14, what is the cost of the smaller one?

c) If the cost of the small chopping board is £14, what is the cost of the large one?

7) 2 bottles **are similar** in shape. The heights of the bottles are 28 cm and 35 cm respectively. The width of the smaller bottle is 9 cm.
The costs of the 2 bottles are directly related to the volumes of the contents.

a) What is the width of the larger bottle?

b) If the cost of the smaller bottle is £1·40, what is the cost of the larger one?

c) If the cost of the large bottle is £1·83, find the cost of the small one.

8) The cost of 2 balls are directly related to their volumes. The diameters of the 2 balls are 10 cm and 15 cm.

a) If the cost of the smaller ball is £1·20, what is the cost of the larger one?

b) If the cost of the larger ball is £6·50, find the cost of the smaller one.

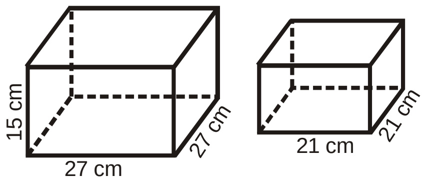
9) 2 jars are similar in shape. The heights of the jars are 15 cm and 12 cm. The width of the larger jar is 6 cm.
The costs of the 2 jars are directly related to their contents.

a) What is the width of the smaller jar?

b) If the cost of the larger jar is £1·10, what is the cost of the smaller one?

c) If the cost of the smaller jar is 68p, what is the cost of the larger one?

10) These 2 boxes of biscuits **are similar** in shape. The costs of the 2 boxes are directly related to their volumes.

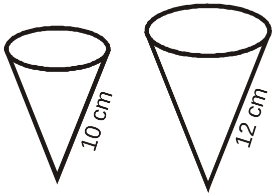


a) What is the height of the smaller box?

b) If the cost of the larger box is £9·54, find the cost of the smaller one.

c) If the cost of the smaller box is £3·25, find the cost of the larger one.

11) The 2 ice cream cones **are similar** in shape. The diameter of the smaller cone is 4 cm. The costs of the 2 cones are directly related to their volumes.



a) What is the diameter of the larger cone?

b) If the cost of the smaller cone is 48p, what is the cost of the larger one?

c) If the cost of the larger cone is £1, what is the cost of the smaller one?

12) Drinking chocolate is sold in 2 different sizes of cylindrical tins which **are similar** in shape. The diameters are 9 cm and 15 cm respectively. The height of the smaller cylinder is 12 cm. The costs are directly related to their volumes.

a) What is the height of the larger cylinder?

b) If the cost of the smaller tin is £1·20, what is the cost of the larger one?

c) If the cost of the larger tin is £8·60, what is the cost of the smaller one?