



# MATHEMATICS



## Unit 1

Part 2 of 2

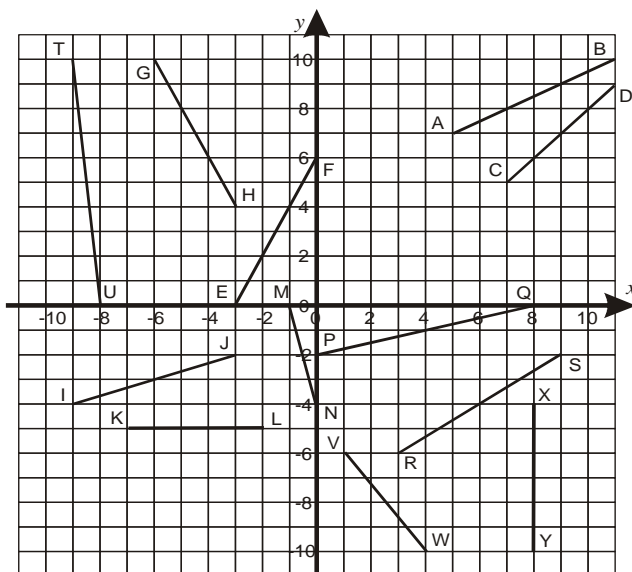
Expressions and Formulae

# Gradient

## Exercise 1

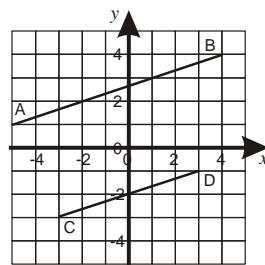
- 1) Work out the gradient of all the lines in the diagram. Write your answers in

the form  $m_{AB} = \frac{1}{2}$



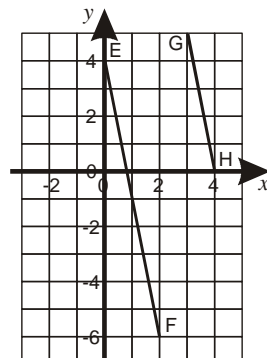
- 2) a) 2 lines have been drawn in this diagram.

Find the gradient of each line.



- b) 2 lines have been drawn in this diagram.

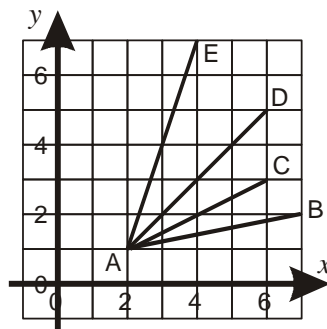
Find the gradient of each line.



- c) Now look carefully at parts **a** and **b** above.

What do you notice about parallel lines?

- 3) a) In this diagram calculate the gradient of the lines AB, AC, AD and AE.



- b) Look at your answers to part a.

What conclusion can you reach about the steepness of a line compared with the number which represents its gradient?

- 4) Use the gradient formula,  $m = \frac{y_2 - y_1}{x_2 - x_1}$ , to calculate the gradient of the following lines.

a) A(3, 2) B(5, 10)    b) C(0, -1) D(4, 1)    c) E(-1, 4) F(3, 2)

d) G(0, 2) H(6, -2)    e) I(-3, -2) J(2, 1)    f) K(-5, -1) L(-1, -5)

g) M(7, -2) N(-5, -4)    h) P(4, -3) Q(-2, -9)    i) R(-2, -34) S(-17, -4)

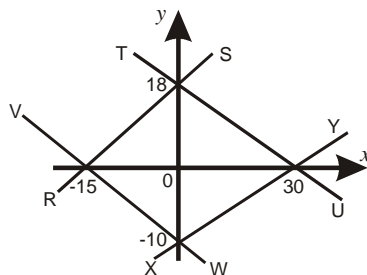
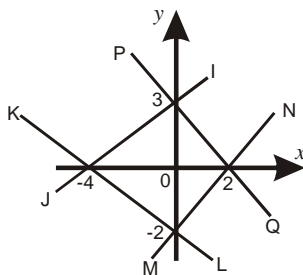
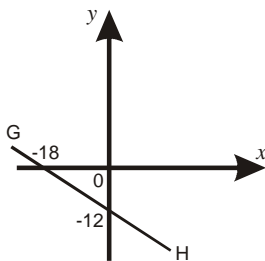
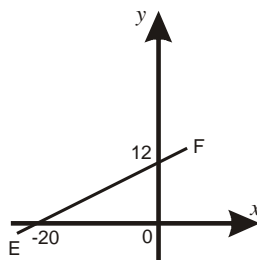
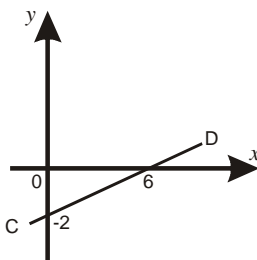
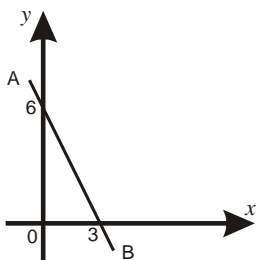
j) T(-7, -2) U(1, -8)    k) V(4, -11) W(-8, 5)    l) X(3, -8) Y(-7, 7)

m) P(4, -3) W(-8, 5)    n) Q(-2, -3) V(4, -11)    o) T(-7, -2) W(-8, 5)

p) U(1, -8) V(4, -11)    q) J(2, 1) X(3, -8)    r) H(6, -2) U(1, -8)

s) U(1, -8) X(3, -8)    t) C(0, -1) V(4, -11)    u) L(-1, -5) S(-17, -9)

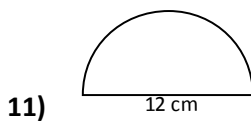
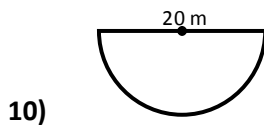
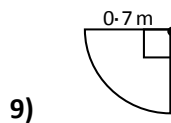
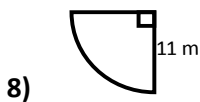
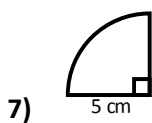
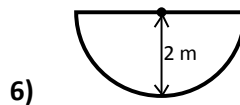
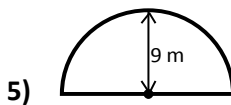
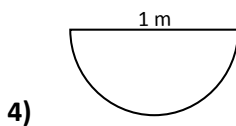
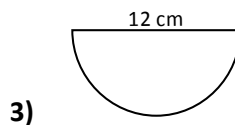
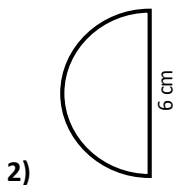
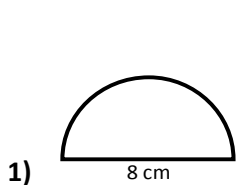
5) Write down the gradient of the sloping lines in each diagram below.



# Arcs and Sectors

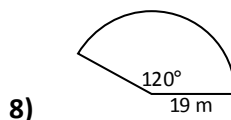
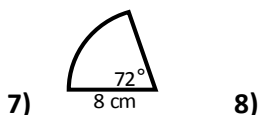
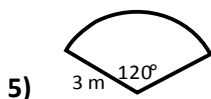
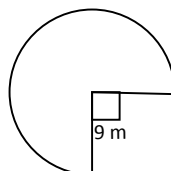
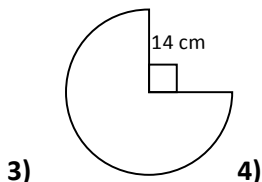
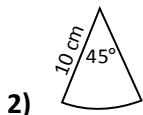
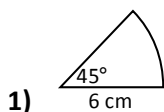
## Exercise 1

Find the **area** of the following sectors



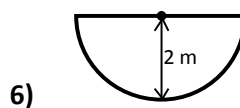
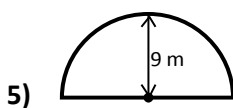
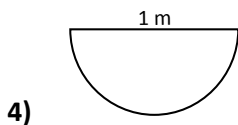
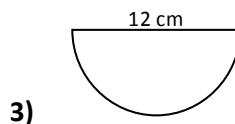
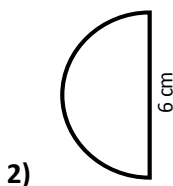
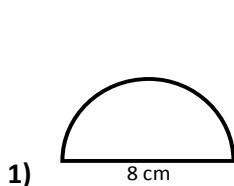
## Exercise 2

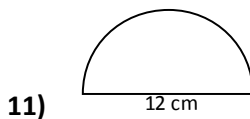
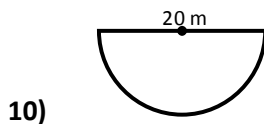
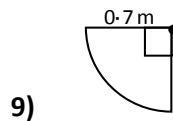
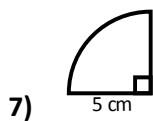
Find the **area** of the following sectors



## Exercise 3

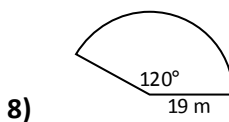
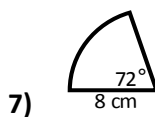
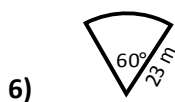
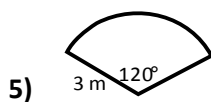
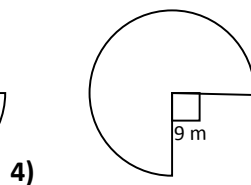
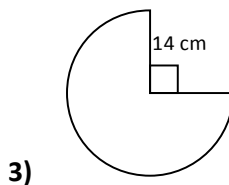
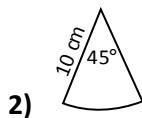
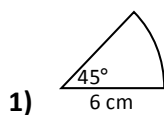
Find the **length of the arc** of the following shapes





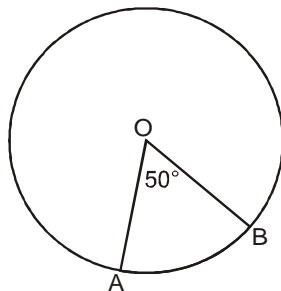
## Exercise 4

Find the **length of the arc** of the following shapes

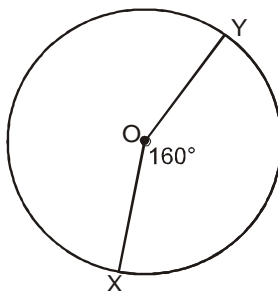


## Exercise 5

- 1) In the diagram O is the centre of a circle of **radius 10 cm**. A and B are points on the circumference such that  $\hat{AOB} = 50^\circ$ .

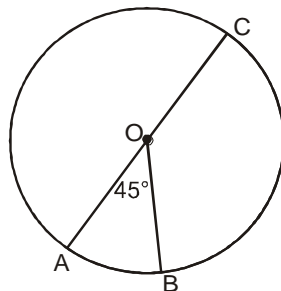


- Calculate the length of the minor arc AB.
  - Calculate the length of the major arc AB. [2 methods possible]
  - Calculate the area of the sector AOB.
- 2) O is the centre of a circle of **diameter 15 cm**. X and Y are points on the circumference such that  $\hat{XOY} = 160^\circ$ .



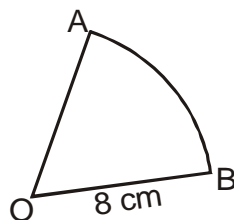
- Calculate the length of the minor arc XY.
- Calculate the length of the major arc XY.
- Calculate the area of the sector XOY.
- Copy the diagram and complete triangle XOY. Mark the sizes of angles  $\hat{OXY}$  and  $\hat{OYX}$
- What kind of triangle is XOY?

- 3) AC is a diameter of the circle with centre O.  
AC = 35 cm.



- If  $\hat{AOB} = 45^\circ$ , write down the size of  $\hat{BOC}$
- Calculate the length of minor arc AB.
- Calculate the area of sector AOB.

- 4) AOB is a sector of a circle of radius 8 cm.  
The area of this sector is  $40.2 \text{ cm}^2$ .



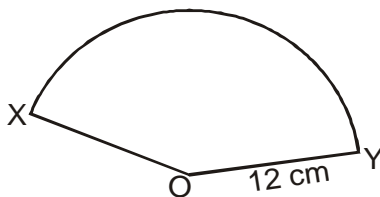
- Calculate the size of angle AOB.
- Hence calculate the length of arc AB.



- 5) XOY is a sector of a circle of radius 12 cm.

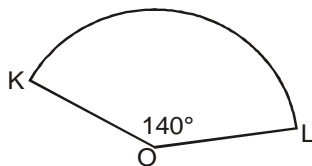
The area of this sector is  $150.7 \text{ cm}^2$ .

- a) Calculate the size of angle XOY.  
b) Hence calculate the length of arc XY.



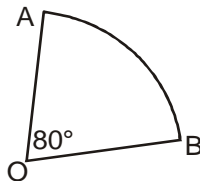
- 6) The area of sector KOL is  $312.6 \text{ cm}^2$  and angle KOL is  $140^\circ$ .

- a) Calculate the radius of the circle.  
b) Hence calculate the length of arc KL.

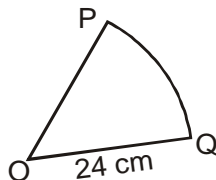


- 7) The area of sector AOB is  $157 \text{ cm}^2$  and angle AOB is  $80^\circ$ .

- a) Calculate the radius of the circle.  
b) Hence calculate the length of arc AB.



- 8) If the length of arc PQ is 16.8 cm and the radius of the circle is 24 cm, calculate the size of angle POQ.

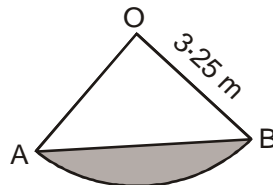


- 9) The sector AOB has a radius of 3.25 m and an angle of  $90^\circ$ .

AB is joined to make a triangle AOB.

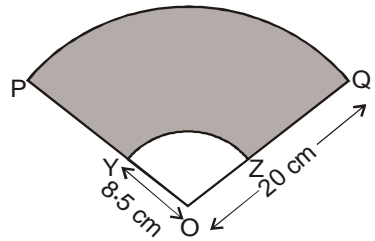
Find the area of:

- a) sector AOB  
b) triangle AOB  
c) the shaded segment to the nearest  $\text{m}^2$ .



- 10)** The blade PY of a car windscreen wiper turns about O through an angle of  $135^\circ$ . Find the:

- area of sector POQ
- area of sector YOZ
- area of windscreen which the wiper covers (answer to the nearest  $\text{cm}^2$ ).



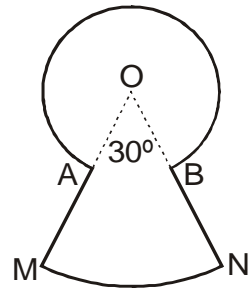
- 11)** A keyhole has a shape formed by two sectors as shown.

The larger sector, OMN, has an angle of  $30^\circ$  and radius of  $8.6 \text{ cm}$ .

The radius, OA, of the smaller sector is  $3.2 \text{ cm}$ .

Find the:

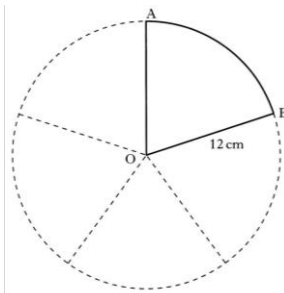
- area of sector OMN.
- angle of the other sector.
- area of the other sector.
- total area of the keyhole.



## Exercise 6

- 1) A circle, with centre O and radius 12 cm, is cut into 5 equal sectors.

Calculate the perimeter of sector AOB



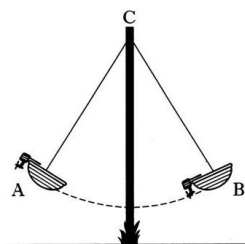
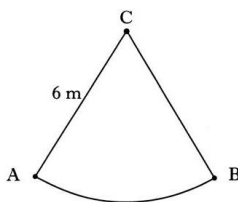
- 2) The boat on a carnival ride travels along an arc of a circle, centre C.

The boat is attached to C by a rod 6 metres long.

The rod swings from position CA to position CB.

The length of the arc AB is 7 metres.

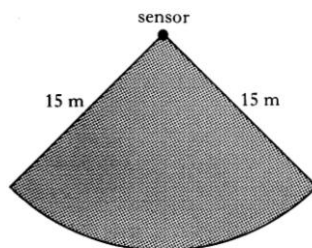
Find the angle through which the rod swings from position A to position B.



- 3) A sensor in a security system covers a horizontal area in the shape of a sector of a circle of radius 15 m.

The area of the sector is  $200 \text{ m}^2$ .

Find the size of the angle at the centre of the arc.

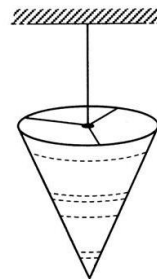
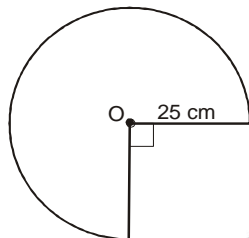


- 4) A lamp shade is made in the shape of a cone, as shown.

The shape of the material used for the lampshade is a sector of a circle.

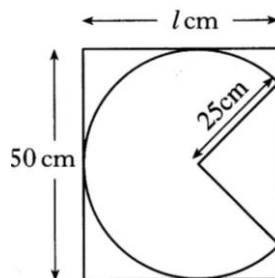
The circle has radius 25 cm and the angle of the sector is  $270^\circ$ .

- a) Find the area of the sector of the circle.



Each sector is cut from a rectangular piece of material, 50 cm wide.

- b) Find, to the nearest centimetre, the **minimum** length,  $l$ , required for the piece of material.

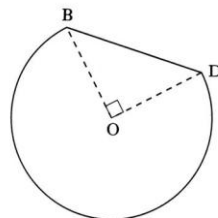


- 5) The diagram shows a table whose top is in the shape of part of a circle with centre,  $O$ , and radius 60 centimetres.

$BD$  is a straight line.

Angle  $BOD$  is  $90^\circ$ .

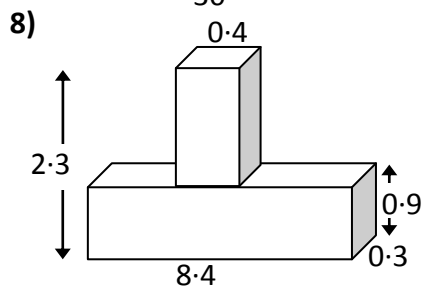
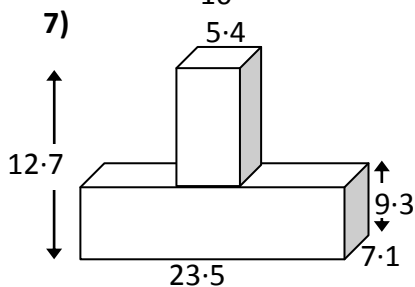
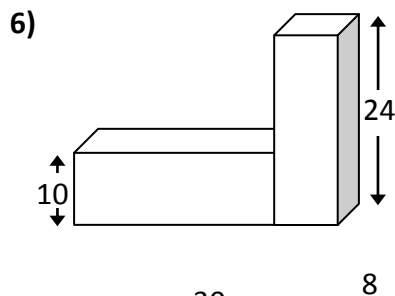
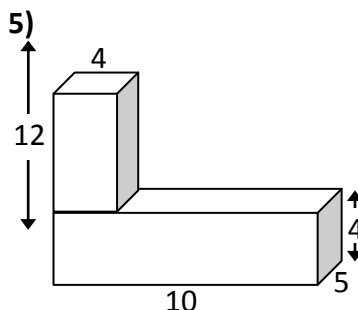
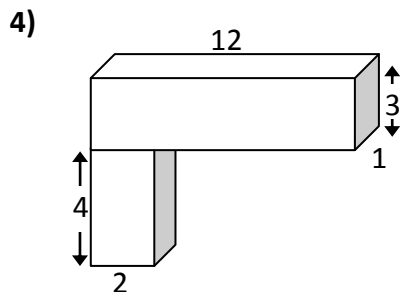
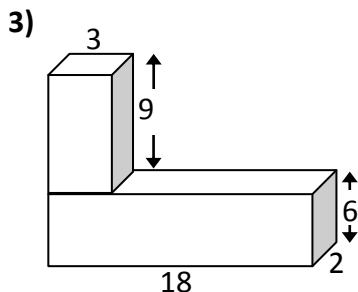
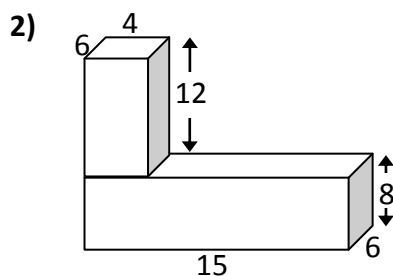
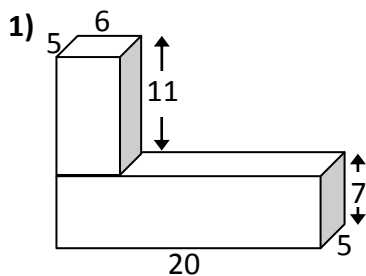
Calculate the perimeter of the table top.

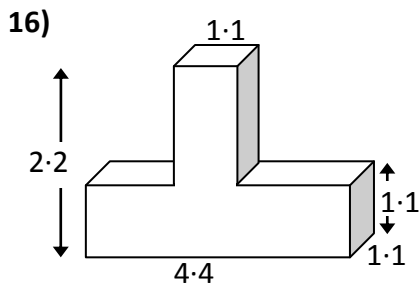
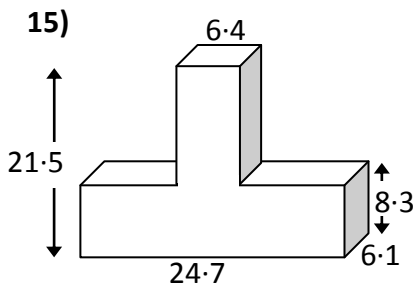
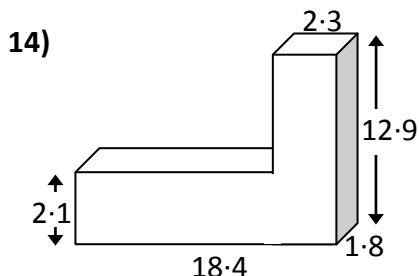
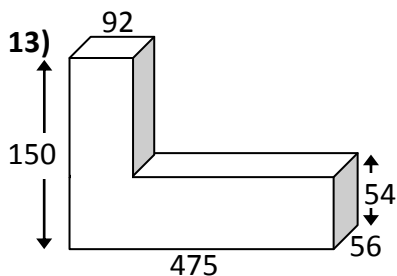
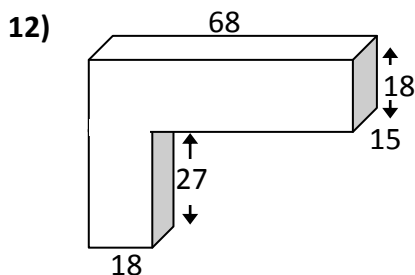
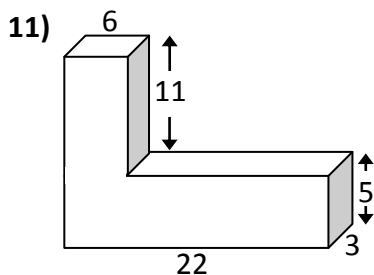
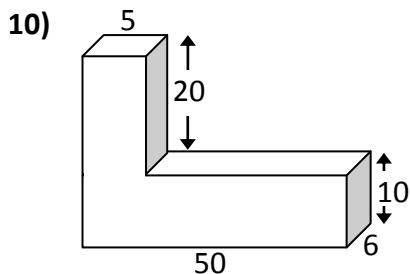
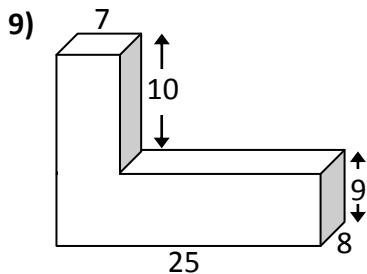


# Volume

## Exercise 1

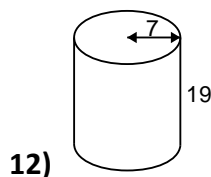
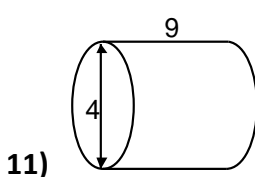
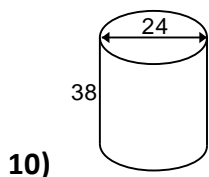
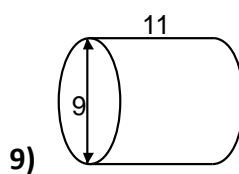
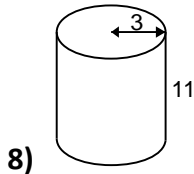
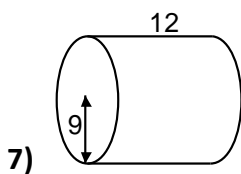
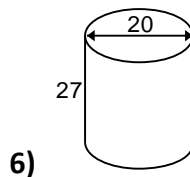
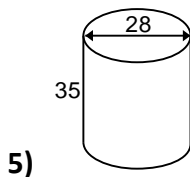
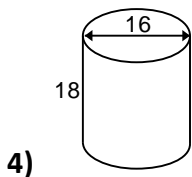
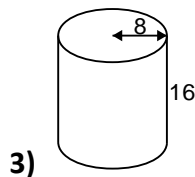
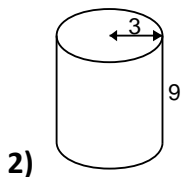
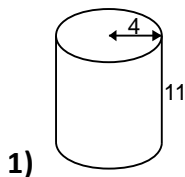
Work out the volume of the following cuboids (all sizes are in cm)





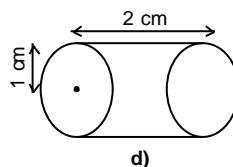
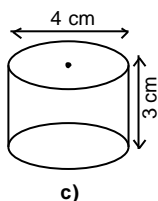
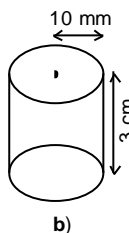
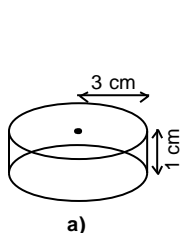
## Exercise 2

Calculate the volume of these cylinders (all sizes in cm).



## Exercise 3

1) Find the volume ( $\text{cm}^3$ ) of these closed cylinders



- 2) A tin of lentil soup has a radius of 4 cm and a height of 15 cm. The label on the tin gives the volume as  $\frac{3}{4}$  of a litre.

Is this correct and, if not, what is the error?

- 3)** A tub of oil is advertised as holding one litre. It has a height of 12 cm and a diameter of 10.4 cm. Find

- a) its radius                                      b) its volume
- c) whether you should complain about the advertisement.

- 4)** An oil drum has a radius of 20 cm and a height of 1 metre.

Calculate its volume

- a)** in  $\text{cm}^3$                       **b)** in litres

- 5) A jar of jam has a volume of  $240 \text{ cm}^3$  and a diameter of 6 cm.

Find it's height.

- 6) A cylindrical box of vitamin supplements is 8 cm high with a volume of  $905 \text{ cm}^3$ . Calculate its diameter.

- 7) Find the height of a cylinder of volume  $231 \text{ cm}^3$  and radius  $1.75 \text{ cm}$ .

- 8) Calculate the radius of a cylindrical rod of volume  $484 \text{ cm}^3$  and length  $1.4 \text{ m}$ .

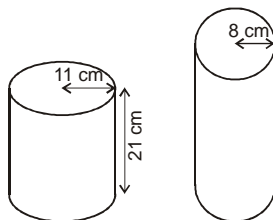
- 9)** It is required to make a cylindrical can which will hold 1 litre of liquid.

- a)** If the diameter is to be 10 cm, what must the height of the can be?
- b)** If the height is to be 10 cm, what must the diameter of the can be?

- 10)** What area of cardboard is needed to make a closed cylinder of height 20 cm and radius of base 14 cm?

- 11)** The two cylinders shown have equal volumes.

Find the height of the second cylinder.

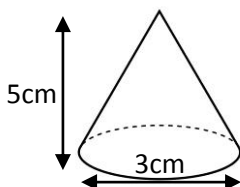




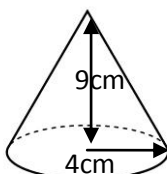
## Exercise 4

Calculate the volume of these cones.

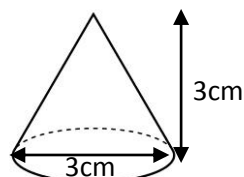
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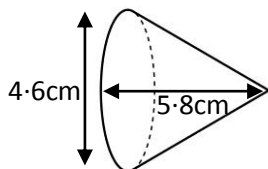
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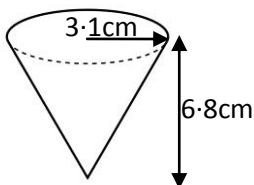
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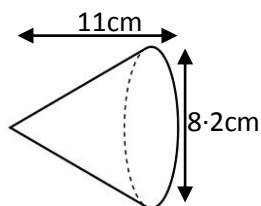
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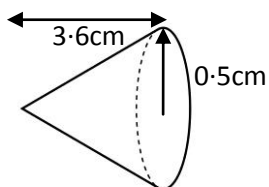
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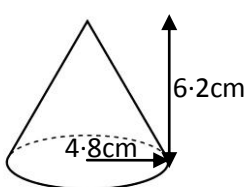
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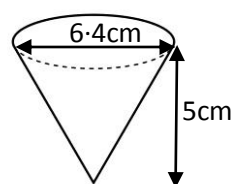
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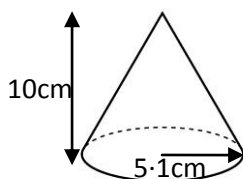
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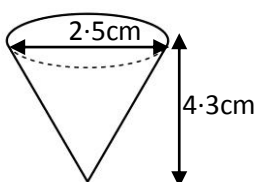
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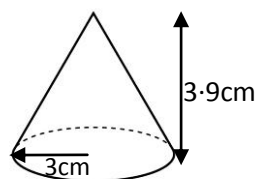
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11)



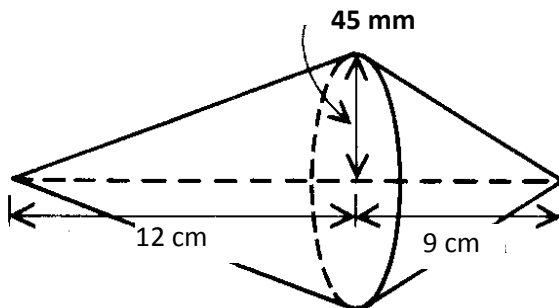
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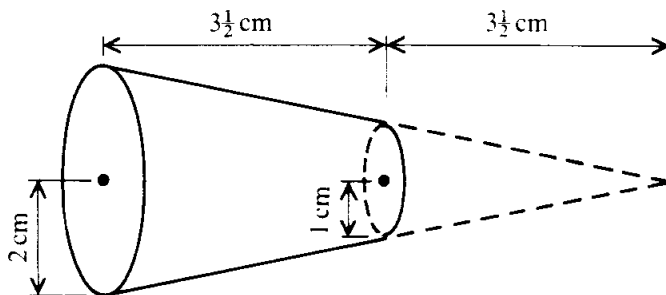
## Exercise 5

- 1) Calculate the volume of a cone with height 12 m and diameter of base 12 m.
- 2) A closed cone has a circular base of radius 7 cm and slant height of 25 cm.  
Calculate:
  - a) the area of the base
  - b) the height
  - c) the volume of the cone.
- 3) A closed cone has a circular base of diameter 10 cm and a slant height of 13 cm.  
Find the volume of the cone.
- 4) Find the depth of a conical flask which holds 200 ml of liquid, the diameter of the base being 12 cm.
- 5) A fire extinguisher, which is approximately conical in shape, has a capacity of 7 litres and a base diameter of 22 cm. Calculate its height.
- 6) Find the volumes of the cones with these radii and heights:
  - a)  $r = 2.25$  cm,  $h = 4.6$  m
  - b)  $r = 2$  m,  $h = 2.67$  m
  - c)  $r = 0.5$  cm,  $h = 5.2$  mm
  - d)  $r = 7.5$  m,  $h = 26.4$  cm

- 7) A child's toy is made from two cones of the same 45 mm radius placed base to base as shown. If their heights are 12 cm and 9 cm, find the volume of the toy.



- 8) A steel plug is machined into the shape of a truncated cone with end radii of 1 cm and 2 cm.



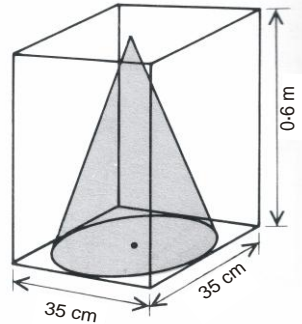
Calculate the volume of

- the large cone shown
- the small cone removed
- the plug.

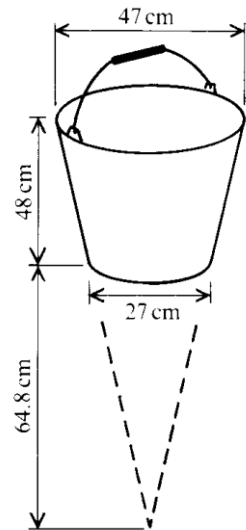
- 9) A closed cone of diameter 35 cm is packed into a square-based box so that it just fits.

If the box is 0.6 m high, find

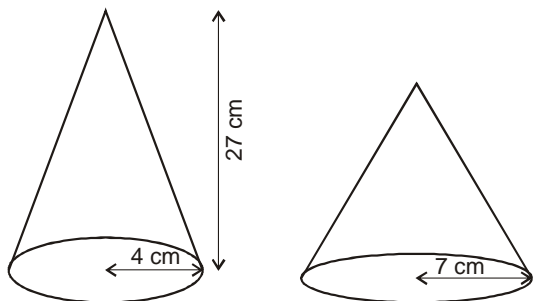
- the volume of the box
- the volume of the cone.



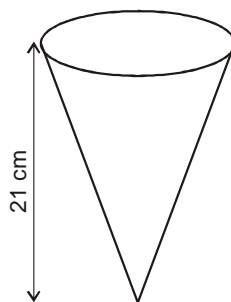
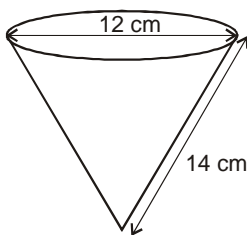
- 10) A bucket has a base diameter of 27 cm and a top diameter of 47 cm. If it has a height of 48 cm as shown in the diagram, what is its volume in litres?



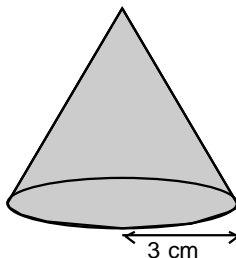
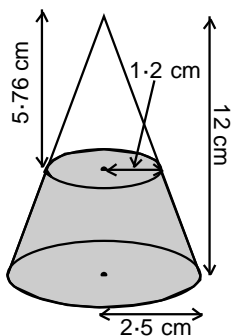
- 11) The cones shown have equal volumes. Calculate the height of the smaller cone.



- 12)** The cones shown have the same volume. Calculate the radius of the second cone.



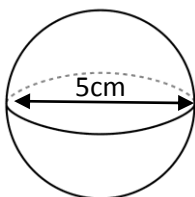
- 13)** The cones shown contain the same amount of sand. Calculate the height of the sand in the smaller cone.



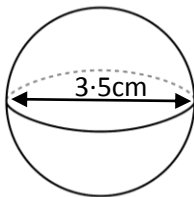
## Exercise 6

Calculate the volume of these spheres.

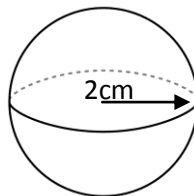
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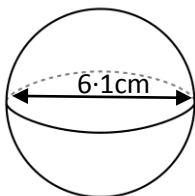
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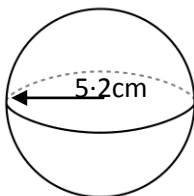
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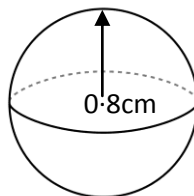
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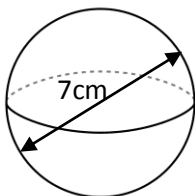
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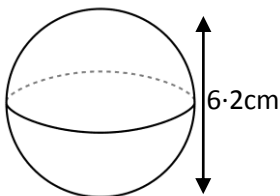
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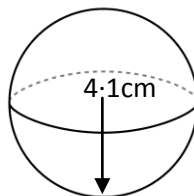
7)



8)



9)



## Exercise 7

- 1 Find the volume of the spheres with the dimensions given.  
Give your answer in  $\text{cm}^3$ .

a)  $r = 2.5 \text{ cm}$

b)  $d = 3.2 \text{ cm}$

c)  $r = \frac{1}{2} \text{ m}$

d)  $d = 0.3 \text{ m}$

e)  $d = 14.65 \text{ cm}$

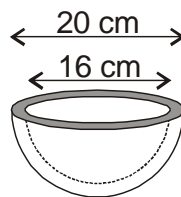
f)  $r = 30 \text{ mm}$

- 2) Calculate the volume of

a) a solid hemisphere of diameter  $34.6 \text{ cm}$

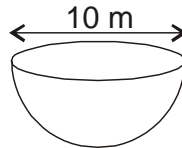
b) a solid hemisphere of diameter  $31.6 \text{ cm}$

- 3) Calculate the volume of the material used to make the hemispherical bowl shown here.

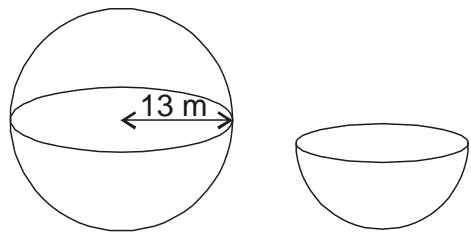


- 4) a) Find the volume of the earth (in standard form) in  $\text{km}^3$  if its radius is  $6370 \text{ km}$ .  
b) The average density of the earth is  $5.52 \times 10^9 \text{ tonnes per km}^3$ , find the total mass of the earth in standard form.
- 5) What is the volume of the largest sphere that can be put in a cubical box of edge  $6 \text{ m}$ ?
- 6) Calculate the radius of a sphere of volume  $12 \text{ cm}^3$ .
- 7) A football has a diameter of  $35 \text{ cm}$ , what volume of air is required to fill it?

- 8) A lead 'sinker' has the shape of a hemisphere topped by a cone. The diameter of the hemisphere and the height of the cone are each 1.4 cm. Find the mass of the sinker, given that  $1 \text{ cm}^3$  of lead weighs 11.4 g.
- 9) Calculate the mass of 500 ball bearings, each of diameter 0.7 cm, made of steel,  $1 \text{ cm}^3$  of which weigh 7.8 g.
- 10) A cylindrical measuring jar has diameter 5 cm, and contains water to a depth of 6 cm. A sphere of diameter 3 cm is dropped in and sinks to the bottom. What is the water depth now?
- 11) A sphere has a volume of  $325 \text{ cm}^3$ . Calculate its diameter.
- 12) A hemisphere has a volume of 5.6 litres. Calculate its radius.
- 13) The hemisphere has seven times the volume of the sphere.  
What is the diameter of the sphere?



- 14) If the sphere has five times the volume of the hemisphere, what will be the diameter of the hemisphere?

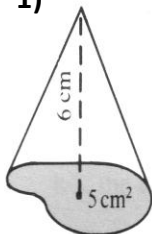




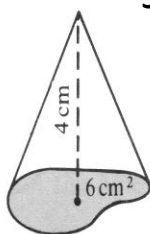
## Exercise 8

Calculate the volume of these pyramids.

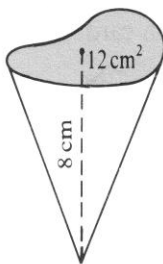
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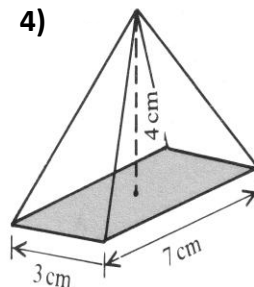
2)



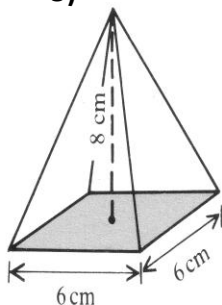
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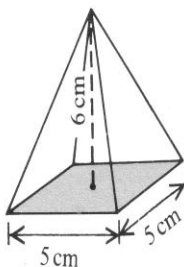
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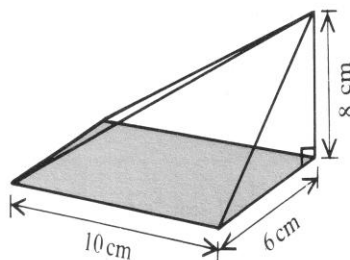
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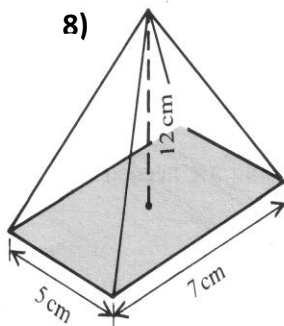
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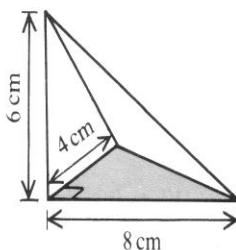
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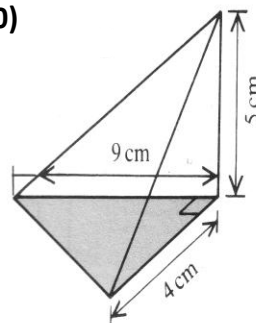
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9)



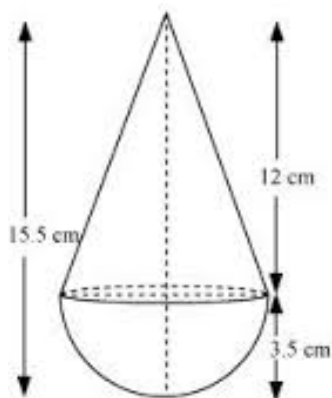
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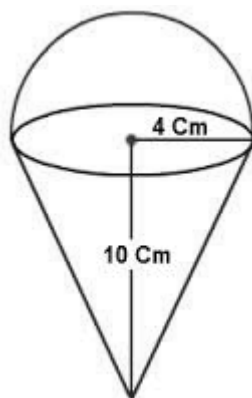
## Exercise 9

Calculate the volume of these composite shapes

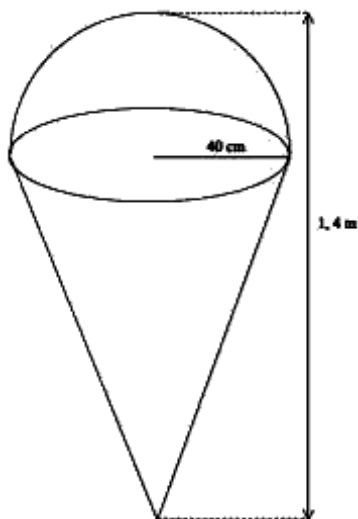
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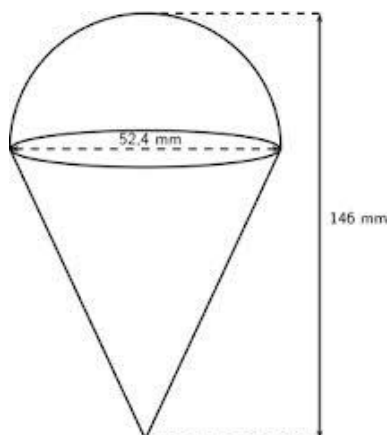
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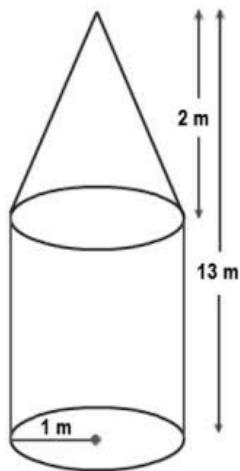
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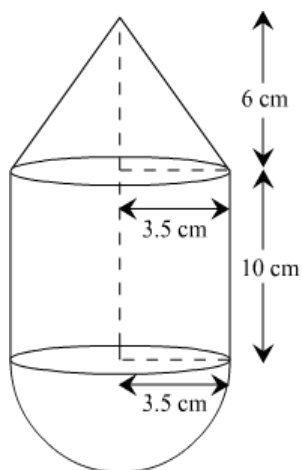
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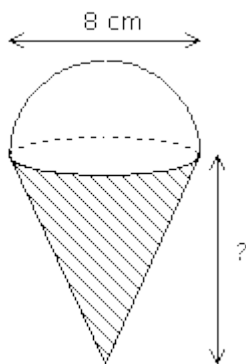
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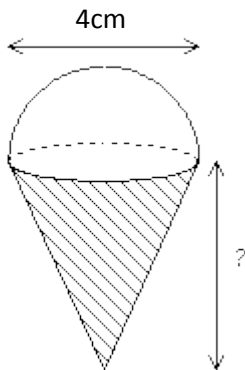
6)



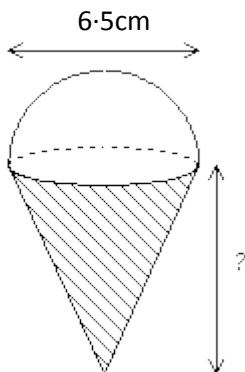
- 7) If the volume of the following shape is  $399 \text{ cm}^3$ , calculate the height of the cone



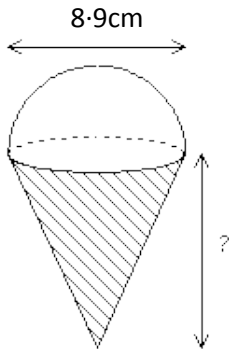
- 8) If the volume of the following shape is  $156 \text{ cm}^3$ , calculate the height of the cone



- 9) If the volume of the following shape is  $298.6 \text{ cm}^3$ , calculate the height of the cone



- 10) If the volume of the following shape is  $190 \text{ cm}^3$ , calculate the height of the cone



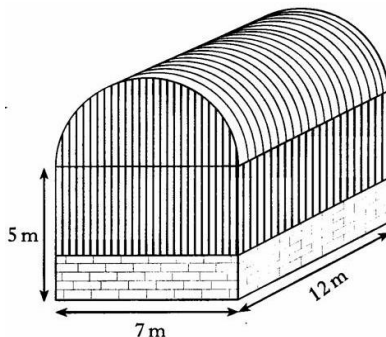
## Exercise 10

- 1) A storage barn is prism shaped, as shown below.

The cross-section of the storage barn consists of a rectangle measuring 7 m by 5 m and a semi-circle of radius 3.5 m.

- a) Find the volume of the storage barn.

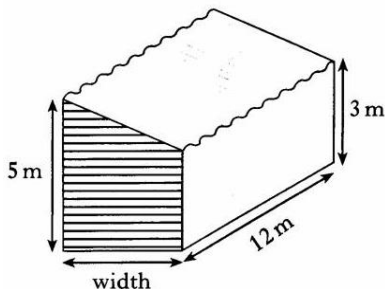
Give your answer in cubic metres,  
**correct to 2 significant figures.**



- b) An extension to the barn is planned to increase the volume by 200 cubic metres.

The uniform cross-section of the extension consists of a rectangle and a right angled triangle.

Find the width of the extension.

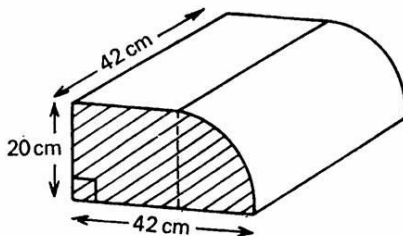


- 2) A metal sphere of diameter 8 cm is melted down. It is recast into a cone of height 11 cm. Calculate the radius of the base of the cone.

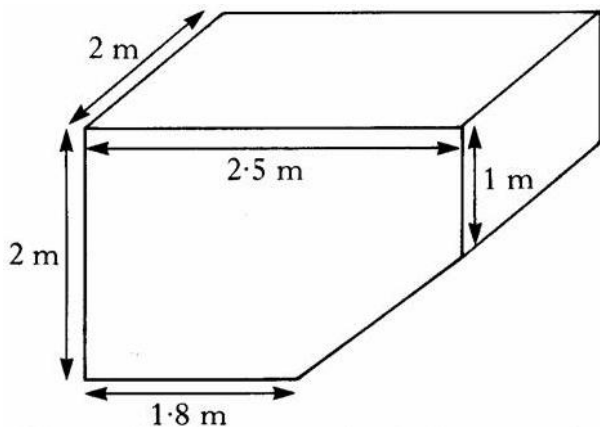
- 3) The dimensions of a bread-bin are marked on the diagram. The shaded side is made up of a rectangle and a quarter circle.

Calculate

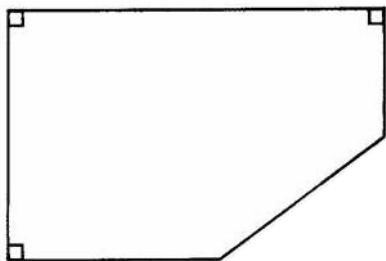
- a) the area of the shaded side of the bin.  
b) the volume of the bread bin.



- 4) A bottle bank is prism shaped, as shown.

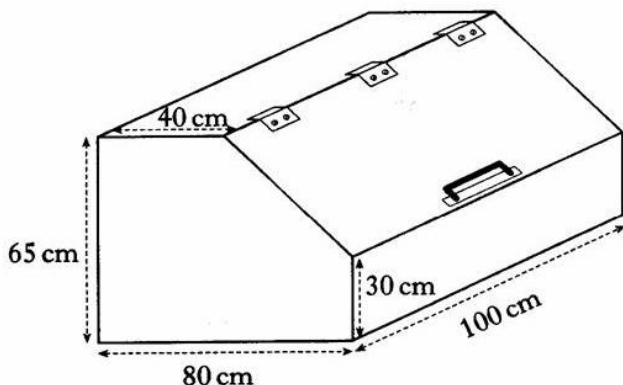


The uniform cross-section is shown below.

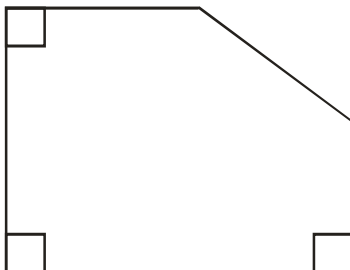


Find the volume of the bottle bank.

- 5) A wooden toy box is prism-shaped as shown below.



The uniform cross-section of the box is shown below.



Calculate the volume of the box in **cubic metres**.

- 6) A cylindrical soft drinks can is 15 centimetres in height and 6.5 centimetres in diameter.

A new cylindrical can holds the same volume but has a reduced height of 12 centimetres.

What is the diameter of the new can?

Give your answer to **1 decimal place**.

- 7) Calculate the slant height of a right circular cone whose base has a radius of 7 cm and which has a volume of  $1232 \text{ cm}^3$ .
- 8) A square based pyramid has a volume of  $1568 \text{ cm}^3$ . If the pyramid is 0.5 m high, find
- a) the length of a side of its base.
  - b) the slant height of the pyramid.
- 9) A cone and a square based pyramid are of equal height and equal volume. The side of the base of the pyramid is 6 cm long. Calculate correct to two decimal places the radius of the base of the cone.
- 10) A pencil is 18 cm long and its cross-section is a regular hexagon of side 4 mm. Find the volume of 100 of these pencils to the nearest cubic centimetre.