



MATHEMATICS

N5

Unit 1
Part 1 of 2
Expressions and Formulae

Surds

Exercise 1

A Express each of the following in its simplest form:

1) $\sqrt{12}$

2) $\sqrt{18}$

3) $\sqrt{50}$

4) $\sqrt{28}$

5) $\sqrt{72}$

6) $\sqrt{180}$

7) $\sqrt{45}$

8) $\sqrt{96}$

9) $\sqrt{20}$

10) $\sqrt{27}$

11) $\sqrt{32}$

12) $\sqrt{45}$

13) $\sqrt{48}$

14) $\sqrt{63}$

15) $\sqrt{75}$

16) $\sqrt{98}$

17) $\sqrt{300}$

18) $\sqrt{147}$

19) $\sqrt{500}$

20) $\sqrt{288}$

21) $\sqrt{128}$

22) $\sqrt{200}$

23) $5\sqrt{8}$

24) $3\sqrt{32}$

25) $10\sqrt{40}$

26) $2\sqrt{98}$

27) $3\sqrt{18}$

28) $4\sqrt{200}$

29) $5\sqrt{112}$

30) $10\sqrt{1000}$

B Simplify the following:

1) $4\sqrt{2} + \sqrt{2}$

2) $7\sqrt{3} + 3\sqrt{3}$

3) $\sqrt{5} + 4\sqrt{5}$

4) $\sqrt{2} + \sqrt{2} + \sqrt{2}$

5) $2\sqrt{7} + 3\sqrt{7} - \sqrt{7}$

6) $\sqrt{8} + \sqrt{2}$

7) $\sqrt{3}\sqrt{48}$

8) $\sqrt{7} - \sqrt{63}$

9) $\sqrt{250} - \sqrt{10} - 6\sqrt{27}$

10) $\sqrt{11} - \sqrt{99} + \sqrt{44}$

11) $2\sqrt{5} + 3\sqrt{2} - 2\sqrt{5} - \sqrt{2}$

12) $-4\sqrt{11} + 8\sqrt{10} - 2\sqrt{11} - 2\sqrt{10}$

13) $6\sqrt{5} + \sqrt{125}$

14) $4\sqrt{3} - \sqrt{12} - 6\sqrt{27}$

15) $\sqrt{28} + 2\sqrt{7} - 3\sqrt{112}$

16) $\sqrt{45} + 2\sqrt{80}$

17) $2\sqrt{45} - \sqrt{20}$

18) $3\sqrt{48} - \sqrt{75}$

- 19)** $2\sqrt{28} + \sqrt{175} - 5\sqrt{63}$ **20)** $3\sqrt{8} - 5\sqrt{32} - \sqrt{72} + 2\sqrt{18}$
- 21)** $6\sqrt{20} - 3\sqrt{27}$ **22)** $4\sqrt{32} + 2\sqrt{96}$
- 23)** $5\sqrt{54} - 3\sqrt{300}$ **24)** $4\sqrt{98} + 5\sqrt{8} - 6\sqrt{40}$
- 25)** $3\sqrt{75} - 2\sqrt{40} + 6\sqrt{48}$ **26)** $\sqrt{90} + 3\sqrt{250}$
- 27)** $2\sqrt{63} + \sqrt{96} - \sqrt{175} - 4\sqrt{24}$
- 28)** $6\sqrt{108} - \sqrt{243} + 4\sqrt{27} + \sqrt{20}$
- 29)** $5\sqrt{90} + \sqrt{125} - 3\sqrt{180} - 2\sqrt{20}$
- 30)** $2\sqrt{112} + 3\sqrt{567} - \sqrt{60} - 3\sqrt{175}$

Exercise 2

Simplify:

- 1)** $\sqrt{3} \times \sqrt{3}$ **2)** $\sqrt{5} \times \sqrt{5}$
- 3)** $\sqrt{7} \times \sqrt{7}$ **4)** $\sqrt{4} \times \sqrt{4}$
- 5)** $\sqrt{2} \times \sqrt{10}$ **6)** $\sqrt{3} \times \sqrt{15}$
- 7)** $\sqrt{3} \times \sqrt{27}$ **8)** $\sqrt{6} \times \sqrt{8}$
- 9)** $\sqrt{5}(1 + \sqrt{5})$ **10)** $\sqrt{2}(1 + \sqrt{2})$
- 11)** $\sqrt{3}(1 + \sqrt{3})$ **12)** $\sqrt{7}(5 + \sqrt{7})$
- 13)** $(3 - 2\sqrt{2})\sqrt{2}$ **14)** $\sqrt{5}(3\sqrt{5} - 2)$
- 15)** $\sqrt{2}(5 + 4\sqrt{2})$ **16)** $\sqrt{3}(1 - 6\sqrt{3})$
- 17)** $\sqrt{2}(3\sqrt{10} - 5\sqrt{2})$ **18)** $\sqrt{7}(2\sqrt{7} - 4\sqrt{3})$
- 19)** $(3\sqrt{3} + 2\sqrt{5})\sqrt{5}$ **20)** $\sqrt{11}(5\sqrt{11} - 2\sqrt{6})$
- 21)** $\sqrt{2}(\sqrt{20} + 2\sqrt{6})$ **22)** $\sqrt{3}(\sqrt{27} + 3\sqrt{5})$

$$23) \sqrt{5}(\sqrt{15} + \sqrt{20})$$

$$25) (4\sqrt{18} + 3\sqrt{6})\sqrt{2}$$

$$27) \sqrt{7}(5\sqrt{20} - 2\sqrt{5})$$

$$29) (\sqrt{3} + 1)(\sqrt{3} - 1)$$

$$31) (2 + \sqrt{2})(3 - \sqrt{2})$$

$$33) (\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})$$

$$35) (\sqrt{7} - \sqrt{13})(\sqrt{7} + \sqrt{13})$$

$$37) (3\sqrt{5} + 2\sqrt{3})(3\sqrt{5} - 2\sqrt{3})$$

$$39) (3\sqrt{11} + 2\sqrt{10})(4\sqrt{11} - 2\sqrt{10})$$

$$40) (2\sqrt{3} + 3\sqrt{2})(2\sqrt{3} - 3\sqrt{2})$$

$$41) (\sqrt{3} - \sqrt{5})^2$$

$$43) (\sqrt{2} + \sqrt{7})^2$$

$$45) (5\sqrt{10} - 2\sqrt{6})(3\sqrt{5} + 2\sqrt{6})$$

$$47) (6\sqrt{8} + 2\sqrt{5})(3\sqrt{2} + 3\sqrt{5})$$

$$49) (4\sqrt{6} + 3\sqrt{5})(7\sqrt{7} - 2\sqrt{3})$$

$$24) (3\sqrt{7} + \sqrt{12})\sqrt{2}$$

$$26) \sqrt{3}(2\sqrt{15} - 4\sqrt{12})$$

$$28) (6\sqrt{14} - 3\sqrt{8})\sqrt{2}$$

$$30) (\sqrt{5} - 2)(\sqrt{5} + 2)$$

$$32) (4 + \sqrt{3})(2 - \sqrt{3})$$

$$34) (\sqrt{6} + \sqrt{5})(\sqrt{10} - \sqrt{5})$$

$$36) (\sqrt{3} + \sqrt{11})(\sqrt{5} - \sqrt{11})$$

$$38) (4\sqrt{7} - 2\sqrt{6})(2\sqrt{7} + 2\sqrt{6})$$

$$42) (\sqrt{5} - 2)^2$$

$$44) (1 + \sqrt{3})^2$$

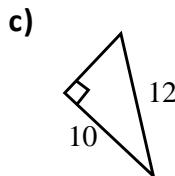
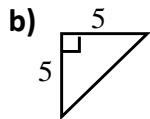
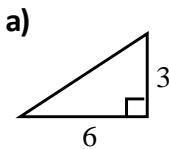
$$46) (3\sqrt{5} + 2\sqrt{7})(4\sqrt{10} - 2\sqrt{7})$$

$$48) (3\sqrt{5} - 2\sqrt{7})(2\sqrt{5} + 5\sqrt{6})$$

$$50) (4\sqrt{8} - 2\sqrt{12})(3\sqrt{10} + 6\sqrt{7})$$

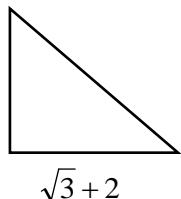
Exercise 3

- 1) Calculate the exact length of the unknown side. Write each answer as a surd in its simplest form.

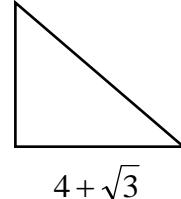


- 2)** Find the exact length of the hypotenuse in each of the following triangles.

a) $\sqrt{3} + 2$



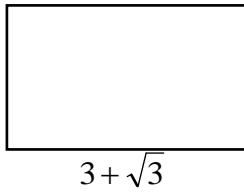
b) $3 + \sqrt{5}$



- 3)** Find the area and the length of the diagonal in the rectangles below. Give your answers as surds in their simplest form. Lengths are in centimetres.

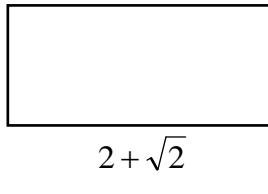
a)

$3 - \sqrt{3}$

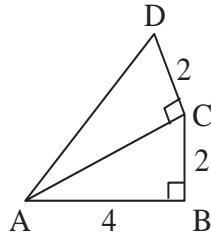


b)

$2 - \sqrt{2}$



- 4)** Find the length of AC and AD as surds in their simplest form.



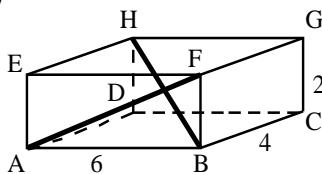
- 5)** The surface area of a cube is 120 cm^2 . Find the length of the side as a surd.

- 6)** Calculate the length of the following, expressing your answer as surds in their simplest form

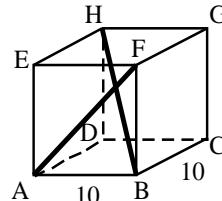
i) face diagonal AF

ii) space diagonal BH

a)



b)



Exercise 4

Rationalise the denominator and simplify:

$$1) \frac{3}{\sqrt{5}}$$

$$2) \frac{10}{\sqrt{12}}$$

$$3) \frac{4}{\sqrt{8}}$$

$$4) \frac{2}{\sqrt{3}}$$

$$5) \frac{8}{\sqrt{2}}$$

$$6) \frac{6}{\sqrt{3}}$$

$$7) \frac{5}{\sqrt{20}}$$

$$8) \frac{3}{\sqrt{48}}$$

$$9) \frac{4}{\sqrt{50}}$$

$$10) \frac{\sqrt{8}}{\sqrt{2}}$$

$$11) \frac{\sqrt{3}}{\sqrt{5}}$$

$$12) \frac{\sqrt{5}}{\sqrt{2}}$$

$$13) \frac{\sqrt{20}}{\sqrt{5}}$$

$$14) \frac{\sqrt{68}}{\sqrt{3}}$$

$$15) \frac{\sqrt{189}}{\sqrt{3}}$$

$$16) \frac{2\sqrt{20}}{\sqrt{5}}$$

$$17) \frac{3\sqrt{216}}{\sqrt{6}}$$

$$18) \frac{5\sqrt{44}}{\sqrt{10}}$$

$$19) \frac{\sqrt{3}}{\sqrt{18}}$$

$$20) \frac{\sqrt{5}}{\sqrt{20}}$$

$$21) \frac{\sqrt{6}}{\sqrt{12}}$$

$$22) \frac{2\sqrt{7}}{\sqrt{98}}$$

$$23) \frac{3\sqrt{5}}{\sqrt{192}}$$

$$24) \frac{4\sqrt{3}}{\sqrt{32}}$$

$$25) \frac{\sqrt{27}}{\sqrt{8}}$$

$$26) \frac{\sqrt{18}}{\sqrt{80}}$$

$$27) \frac{\sqrt{98}}{\sqrt{12}}$$

$$28) \frac{4\sqrt{125}}{\sqrt{75}}$$

$$29) \frac{5\sqrt{12}}{\sqrt{50}}$$

$$30) \frac{6\sqrt{40}}{\sqrt{8}}$$

$$31) \frac{10}{7\sqrt{32}}$$

$$32) \frac{3}{5\sqrt{15}}$$

$$33) \frac{4}{3\sqrt{48}}$$

$$34) \frac{\sqrt{10}}{3\sqrt{28}}$$

$$35) \frac{\sqrt{12}}{4\sqrt{20}}$$

$$36) \frac{\sqrt{18}}{3\sqrt{6}}$$

$$37) \frac{5\sqrt{5}}{4\sqrt{72}}$$

$$38) \frac{6\sqrt{3}}{5\sqrt{8}}$$

$$39) \frac{7\sqrt{2}}{3\sqrt{12}}$$

$$40) \frac{\sqrt{300}}{6\sqrt{40}}$$

$$41) \frac{\sqrt{98}}{7\sqrt{200}}$$

$$42) \frac{\sqrt{405}}{6\sqrt{27}}$$

$$43) \frac{7\sqrt{54}}{5\sqrt{147}}$$

$$44) \frac{4\sqrt{32}}{3\sqrt{288}}$$

$$45) \frac{2\sqrt{63}}{\sqrt{500}}$$

$$46) \frac{10}{\sqrt{5}} - \sqrt{75}$$

$$47) \frac{9}{2\sqrt{3}} - \sqrt{192}$$

$$48) \frac{10}{\sqrt{5}} - \sqrt{180}$$

$$49) \frac{7}{\sqrt{2}} + \sqrt{98}$$

$$50) \frac{8}{\sqrt{7}} + \sqrt{112}$$

Exercise 5

- 1) Express the following as surds in simplest form with rational denominators:

$$a) \frac{6}{\sqrt{3}}$$

$$b) \frac{2}{\sqrt{3}}$$

$$c) \frac{8}{\sqrt{2}}$$

- 2) For the function $f(x) = \frac{2}{\sqrt{x}}$, $x \neq 0$, find the exact value of $f(6)$, giving your answer with a rational denominator.

- 3) Express $\sqrt{32} + \sqrt{8}$ as a surd in simplest form.

- 4) Express the following as surds in simplest form with rational denominators:

$$a) \frac{6}{\sqrt{3}}$$

$$b) \frac{2}{\sqrt{3}}$$

$$c) \frac{8}{\sqrt{2}}$$

- 5) Find the value of $(p+q)(r+s)$ when $p+q = \frac{1}{\sqrt{5}}$, $r+s = \frac{1}{\sqrt{10}}$.

Give your answer with a rational denominator.

- 6)** Find the area of the rectangle shown.



7) **a)** Prove that when $\frac{12}{\sqrt{6}}$ is expressed in simplest surd form with a rational denominator, $\frac{12}{\sqrt{6}}$ is equal to $2\sqrt{6}$.

b) Hence express $\frac{6}{\sqrt{3}}$ as a surd in simplest form with a rational denominator.

8) Simplify the following, expressing your answer with a rational denominator

a) $\frac{\sqrt{48}}{\sqrt{3}}$

b) $\frac{\sqrt{48}}{\sqrt{2}}$

9) Simplify $\sqrt{12} + 5\sqrt{3} - \sqrt{27}$

Indices

Exercise 1

Simplify

$$1) \ x^3 x^5$$

$$2) \ y^4 y^{10}$$

$$3) \ a^7 a^9$$

$$4) \ b^6 b$$

$$5) \ t^8 t^3 t^9$$

$$6) \ k^{10} k k^{13}$$

$$7) \ q^7 q^{14} q^{23}$$

$$8) \ u^7 u^7 u$$

$$9) \ z^9 z^{32} z$$

$$10) \ f^{24} f^4 f^{11}$$

$$11) \ d^6 d^6 d^{66}$$

$$12) \ c c^7 c^{19} c$$

$$13) \ d d^{11} d^{21}$$

$$14) \ g^3 g^4 g^5 g$$

$$15) \ r^8 r^6 d^{44}$$

$$16) \ v v^2 v^3 v^4 v^5 v^6$$

Exercise 2

Simplify

$$1) \ \frac{x^8}{x^2}$$

$$2) \ \frac{y^{12}}{y^3}$$

$$3) \ \frac{k^9}{k}$$

$$4) \ \frac{a^{14}}{a^4}$$

$$5) \ \frac{b^{20}}{b^5}$$

$$6) \ \frac{l^{22}}{l}$$

$$7) \ \frac{m^6}{m^6}$$

$$8) \ \frac{q^{30}}{q^6}$$

$$9) \ \frac{n^{28}}{n^{19}}$$

$$10) \ \frac{z^{100}}{z}$$

$$11) \ \frac{c^5 c^6}{c^7}$$

$$12) \ \frac{d^2 d^3}{d^4}$$

$$13) \ \frac{e^8 e^{11}}{e^6}$$

$$14) \ \frac{v^{10}}{v^4 v^5}$$

$$15) \ \frac{k^{15}}{k^5 k^{10}}$$

$$16) \ \frac{f^{11} f^{14}}{f^6 f^{16}}$$

$$17) \ \frac{m m^8}{m^3 m^5}$$

$$18) \ \frac{q^{13} q^{18}}{q^{30} q}$$

$$19) \ \frac{r^{100}}{r^{20} r^{35}}$$

$$20) \ \frac{s s^6 s^8}{s^4 s^9}$$

Exercise 3

Simplify

$$1) \left(y^3\right)^6$$

$$2) \left(t^4\right)^9$$

$$3) \left(k^7\right)^3$$

$$4) \left(z^5\right)^5$$

$$5) \left(m^8\right)^4$$

$$6) \left(x^9\right)^8$$

$$7) \left(b^6\right)^{10}$$

$$8) \left(q^{15}\right)^5$$

$$9) \left(n^2\right)^{22}$$

$$10) \left(v^{11}\right)^{11}$$

$$11) \left(a^7\right)^{20}$$

$$12) \left(s^5\right)^9$$

$$13) \left(c^6\right)^7$$

$$14) \left(f^4\right)^{13}$$

$$15) \left(g^{30}\right)^{40}$$

Exercise 4

Rewrite the following without brackets

$$1) (ab)^3$$

$$2) (kl)^6$$

$$3) (pq^3)^5$$

$$4) (x^2y^4)^4$$

$$5) (3yz^5)^3$$

$$6) (4x^7y^8)^2$$

$$7) (ab^3c^8)^6$$

$$8) (2e^2f)^5$$

$$9) (4a^8b^6)^4$$

$$10) 4(a^8b^6)^4$$

$$11) (2s^7t^5)^6$$

$$12) 2(s^7t^5)^6$$

$$13) (5k^5l^9)^3$$

$$14) 5(k^5l^9)^3$$

$$15) (ab^2c^3)^4$$

$$16) a(b^2c^3)^4$$

Exercise 5 (Miscellaneous Exercise)

Simplify

$$1) x^{12}x^4$$

$$2) \frac{x^{12}}{x^4}$$

$$3) (x^4)^{12}$$

$$4) (x^4y^{12})^4$$

$$5) 2a^2 \times 3a^3$$

$$6) \frac{16b^{16}}{8b^8}$$

$$7) \frac{d(d^5)^7}{d^5d^7}$$

$$8) \frac{3a^3 \times 4a^4}{6a^6}$$

$$9) \frac{(3m^2n^6)(6m^5n^4)}{2m^3n^9}$$

$$10) \frac{(4x^5y^7)(3x^4y^3)}{6x^9y^9}$$

$$11) \frac{\left(x^5\right)^5}{x^5x^5}$$

$$12) \left(2a^2b\right)^3 \times 2ab^2$$

$$13) \frac{2a^4b \times \left(2ab^3\right)^3}{8a^5b^9}$$

$$14) \left(\frac{x^2\left(x^3\right)^4}{x^5}\right)^6$$

$$15) \frac{\left(3a^2b\right)^3 \times 2\left(ab^3\right)^4}{18\left(a^5b\right)^2}$$

$$16) \frac{\left(8x^2y^3\right)^3\left(2x^4y\right)^5}{\left(4x^4y^2\right)^6}$$

$$17) \left(\frac{2a^5}{3a^2}\right)^4$$

Exercise 6 (Negative Indices)

Evaluate

$$1) 3^{-2}$$

$$2) 2^{-3}$$

$$3) 5^{-1}$$

$$4) 2^{-4}$$

$$5) 5^{-3}$$

$$6) 7^{-1}$$

$$7) 2^{-6}$$

$$8) 3^{-5}$$

$$9) 6^{-3}$$

$$10) 8^{-1}$$

$$11) 9^{-2}$$

$$12) 10^{-4}$$

$$13) 5^{-4}$$

$$14) 9^0$$

$$15) 11^{-2}$$

$$16) 2^{-7}$$

$$17) 13^{-1}$$

$$18) 3^{-6}$$

$$19) 14^0$$

$$20) 20^{-1}$$

$$21) 30^{-3}$$

$$22) 2^{-9}$$

$$23) 17^0$$

$$24) 7^{-3}$$

$$25) \frac{1}{3^{-2}}$$

$$26) \frac{1}{6^{-1}}$$

$$27) \frac{4}{2^{-4}}$$

$$28) \frac{10}{5^{-2}}$$

$$29) \left(\frac{3}{2}\right)^{-3}$$

$$30) \left(\frac{4}{5}\right)^{-2}$$

$$31) \left(\frac{1}{6}\right)^{-1}$$

$$32) \left(\frac{3}{7}\right)^{-2}$$

$$33) \frac{8}{100^{-1}}$$

$$34) \left(\frac{3}{2}\right)^{-5}$$

$$35) \frac{2^{-4}}{4^{-2}}$$

$$36) \frac{9^{-3}}{3^{-4}}$$

Express each of the following in simplest form with positive indices.

$$37) a^{20}a^{-6}$$

$$38) s^8t^{-11}$$

$$39) 7k^{-1}$$

$$40) \frac{c}{d^{-4}}$$

$$41) \left(v^5\right)^{-9}$$

$$42) d^{-35}d^{15}$$

$$43) f^{-6}g^{-9}$$

$$44) \frac{1}{2}m^{-3}$$

$$45) \frac{x}{y^{-9}}$$

$$46) \left(z^{-6}\right)^{-8}$$

$$47) \frac{x^{-6}}{x^9}$$

$$48) \frac{a^{-18}}{a^{-6}}$$

$$49) \frac{p^4}{p^9}$$

$$50) \frac{z^{-4}}{z^9}$$

$$51) \frac{n^{-4}}{n^{-9}}$$

$$52) x^{-3}x^2$$

$$53) x^{-3}y^2$$

$$54) (xy)^{-3}$$

$$55) xy^{-3}$$

$$56) \frac{y^{-3}}{x}$$

$$57) \frac{(2x)^{-1}}{2x^2}$$

$$58) \frac{(2x)^4}{8x^{-3}}$$

$$59) (2x)^{-2}x^2$$

$$60) (2x)^{-2}y^2$$

$$61) \frac{(2x)^{-1}y}{3x^2y^{-2}}$$

Exercise 7 (Fractional Indices)

Evaluate

$$1) 16^{\frac{1}{2}}$$

$$2) 121^{\frac{1}{2}}$$

$$3) 8^{\frac{1}{3}}$$

$$4) 32^{\frac{1}{5}}$$

$$5) 81^{\frac{1}{4}}$$

$$6) 1000^{\frac{1}{3}}$$

$$7) 9^{\frac{3}{2}}$$

$$8) 8^{\frac{2}{3}}$$

$$9) 27^{\frac{4}{3}}$$

$$10) 16^{\frac{3}{4}}$$

$$11) 49^{\frac{3}{2}}$$

$$12) 243^{\frac{2}{5}}$$

$$13) 64^{\frac{5}{6}}$$

$$14) 49^{-\frac{1}{2}}$$

$$15) 125^{-\frac{1}{3}}$$

$$16) 4^{-\frac{3}{2}}$$

$$17) \ 64^{-\frac{2}{3}}$$

$$18) \ 1000^{-\frac{4}{3}}$$

$$19) \ 32^{-\frac{3}{5}}$$

$$20) \ 81^{-\frac{3}{4}}$$

$$21) \ 144^{-\frac{3}{2}}$$

$$22) \ (-8)^{\frac{1}{3}}$$

$$23) \ (-32)^{\frac{2}{5}}$$

$$24) \ (-125)^{-\frac{1}{3}}$$

$$25) \left(\frac{1}{4}\right)^{-\frac{1}{2}}$$

$$26) \left(\frac{1}{8}\right)^{\frac{5}{3}}$$

$$27) \left(\frac{4}{9}\right)^{\frac{3}{2}}$$

$$28) \left(\frac{16}{81}\right)^{-\frac{1}{4}}$$

$$29) \left(\frac{125}{64}\right)^{\frac{2}{3}}$$

$$30) \left(\frac{1}{32}\right)^{\frac{1}{5}}$$

$$31) \left(\frac{16}{625}\right)^{-\frac{3}{4}}$$

$$32) \left(\frac{243}{1024}\right)^{-\frac{4}{5}}$$

$$33) \left(1 \frac{9}{16}\right)^{\frac{3}{2}}$$

$$34) \left(-3 \frac{3}{8}\right)^{-\frac{2}{3}}$$

35) Evaluate $2a^{\frac{1}{2}}$ when $a=49$.

36) Evaluate $5x^{\frac{3}{4}}$ when $x=81$.

37) Evaluate $10x^0$ when $x=9$.

38) Evaluate $x^{\frac{2}{3}} + x^{\frac{3}{2}}$ when $x=64$.

39) Evaluate $k^{-\frac{1}{4}} \times l^{\frac{1}{3}}$ when $k=16$ and $l=125$.

40) Evaluate $x^{\frac{1}{2}} \times y^{-\frac{1}{4}}$ when $x=144$ and $y=256$.

41) Evaluate $(x+y)^{-\frac{3}{2}}$ when $x=9$ and $y=16$.

42) Evaluate $(2xy)^{-\frac{1}{3}}$ when $x=4$ and $y=27$.

43) Evaluate $\left(\frac{12x}{y}\right)^{\frac{1}{2}}$ when $x=4$ and $y=27$.

44) Evaluate $x^{-\frac{1}{3}} + y^{-\frac{1}{2}}$ when $x=8$ and $y=\frac{1}{9}$.

45) Evaluate $\left(\frac{1}{3}xy\right)^{\frac{2}{3}}$ when $x=8$ and $y=\frac{1}{9}$.

Solve the following equations

$$46) \quad 7^e = 343 \quad 47) \quad 2^f = 512 \quad 48) \quad 10^g = 10000 \quad 49) \quad 3^h = 729$$

$$50) \quad 4^k = \frac{1}{64} \quad 51) \quad 3^l = \frac{1}{81} \quad 52) \quad 5^m = \frac{1}{125} \quad 53) \quad 10^n = \frac{1}{100\,000}$$

$$54) \quad 8^p = 16 \quad 55) \quad 9^q = 27 \quad 56) \quad 125^r = 25 \quad 57) \quad 216^s = 6$$

$$58) \quad 64^t = \frac{1}{16} \quad 59) \quad 243^u = \frac{1}{81} \quad 60) \quad 8^{-v} = 64 \quad 61) \quad 5^{-8w} = 625$$

$$62) \quad 256^{-3x} = \frac{1}{64} \quad 63) \quad 729^{-11y} = \frac{1}{81} \quad 64) \quad 100000^{13z} = \frac{1}{10000}$$

Exercise 8 (Various types of Simplifications involving Indices)

Simplify each of the following expressions giving your answers where necessary with positive indices.

$$1) \quad \left(t^{12}\right)^{\frac{2}{3}} \quad 2) \quad \left(x^{15}\right)^{-\frac{1}{3}} \quad 3) \quad \left(z^{-2}\right)^{\frac{1}{8}} \quad 4) \quad \left(n^{-\frac{3}{4}}\right)^{-\frac{2}{5}}$$

$$5) \quad \left(b^{\frac{2}{3}}\right)^0 \quad 6) \quad \left(e^{\frac{5}{2}}\right)^{-2} \quad 7) \quad 4u^{-\frac{1}{2}} \times 2u^{\frac{1}{2}} \quad 8) \quad 3m^{\frac{2}{3}} \times 4m^{\frac{1}{3}}$$

$$9) \quad 6h^{-\frac{5}{4}} \times 3h^{\frac{1}{4}} \quad 10) \quad 4l^{-1} \times \frac{1}{8}l^3 \quad 11) \quad p^4 \times p^{-\frac{1}{2}} \quad 12) \quad 2w^{\frac{1}{2}} \times \frac{1}{4}w^{-\frac{5}{2}}$$

$$13) \quad \frac{6x^{\frac{3}{2}}}{2x^{\frac{1}{2}}} \quad 14) \quad \frac{8z^{\frac{1}{4}}}{2z^{-\frac{1}{2}}} \quad 15) \quad \frac{12b^{\frac{1}{2}}}{6b^{\frac{3}{2}}} \quad 16) \quad \frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{4}}}$$

$$17) \quad \frac{18q^{-\frac{1}{3}}}{9q^{-\frac{1}{3}}} \quad 18) \quad \frac{2u^{\frac{1}{3}}}{10u^{-\frac{2}{3}}} \quad 19) \quad \frac{(4x)^{-\frac{1}{2}}}{x} \quad 20) \quad \left(9x^2\right)^{-\frac{1}{2}}$$

$$21) \quad \frac{(8x)^{\frac{2}{3}}}{x^{-\frac{2}{3}}} \quad 22) \quad \frac{2x^{-\frac{1}{3}}\left(x^{-\frac{1}{3}}\right)^{-1}}{\left(x^{\frac{1}{3}}\right)^2}$$

Express each of the following expressions without brackets in simplest form with positive indices where necessary.

23) $x^{\frac{1}{2}} \left(x^{\frac{1}{2}} + 1 \right)$

24) $x^{-\frac{1}{2}} \left(x^{\frac{3}{2}} - 1 \right)$

25) $x^{\frac{1}{2}} \left(x^{\frac{1}{2}} + x^{-\frac{1}{2}} \right)$

26) $x^{-2} \left(x^3 - x \right)$

27) $x^{-1} \left(x + x^{-1} \right)$

28) $x^{\frac{2}{3}} \left(x^{\frac{4}{3}} - x^{\frac{1}{3}} \right)$

29) $x^{-1} \left(x + 1 \right)$

30) $x^{\frac{1}{2}} \left(x - 2 \right)$

31) $u^{\frac{1}{3}} \left(u^{\frac{2}{3}} + u^{-\frac{2}{3}} \right)$

32) $a^{\frac{1}{2}} \left(a^{\frac{1}{2}} + 1 \right)$

33) $b^{-\frac{1}{3}} \left(b^{\frac{1}{3}} - b^{\frac{2}{3}} \right)$

34) $3a^{-\frac{1}{2}} \left(a^{\frac{1}{2}} + a^{-\frac{3}{2}} \right)$

35) $2k^{-\frac{3}{4}} \left(3k^{\frac{1}{4}} - 5k^{-\frac{1}{4}} \right)$

36) $\frac{1}{2} y^{-\frac{1}{2}} \left(y^{\frac{3}{2}} + 4y^{-\frac{3}{2}} \right)$

37) $\frac{1}{3} t^{-\frac{4}{3}} \left(t - 6t^{\frac{7}{3}} \right)$

38) $\frac{1}{6} x^{-\frac{5}{3}} \left(2x^{-\frac{1}{3}} + 12x^{\frac{2}{3}} \right)$

39) $\frac{2}{3} z^{\frac{3}{4}} \left(6z - \frac{1}{2} z^{-\frac{7}{4}} \right)$

40) $\frac{3}{5} y^{-5} \left(2y + 10y^{\frac{11}{2}} \right)$

Express each of the following expressions without brackets in simplest form with positive indices where necessary.

41) $(3x + 5y)^2$

42) $(7a - 3b)^2$

43) $\left(x^{\frac{1}{2}} + 1 \right)^2$

44) $\left(y^{\frac{1}{2}} - 3 \right)^2$

45) $\left(z^{\frac{3}{2}} - 5 \right)^2$

46) $\left(t^{\frac{5}{2}} + 6 \right)^2$

47) $\left(x^{\frac{3}{2}} - x^{\frac{1}{2}} \right)^2$

48) $\left(y + y^{-1} \right)^2$

49) $\left(b + b^{\frac{3}{2}} \right)^2$

50) $\left(2x - 3x^{\frac{1}{2}} \right)^2$

51) $\left(3y^{\frac{1}{2}} + 4y^{-\frac{1}{2}} \right)^2$

52) $\left(c^{-1} - 2c \right)^2$

53) $\left(3t^{-3} + \frac{1}{3} \right)^2$

54) $\left(x^{\frac{1}{4}} - 5 \right)^2$

55) $\left(4x^{\frac{1}{2}} - 3x^{-1} \right)^2$

56) $\left(a - a^{\frac{1}{2}} \right) \left(a + a^{\frac{1}{2}} \right)$

57) $\left(b^{\frac{3}{2}} - b^{\frac{1}{2}} \right) \left(b^{\frac{3}{2}} + b^{\frac{1}{2}} \right)$

$$58) \left(c - c^{\frac{5}{2}}\right)\left(c + c^{\frac{5}{2}}\right)$$

$$59) \left(d^{\frac{1}{3}} - d\right)\left(d^{\frac{1}{3}} + d\right)$$

$$60) \left(2e - e^{\frac{1}{2}}\right)\left(2e + e^{\frac{1}{2}}\right)$$

$$61) \left(3f - 2f^{-\frac{3}{2}}\right)\left(3f + 2f^{-\frac{3}{2}}\right)$$

$$62) \left(3c - 2c^{\frac{1}{2}}\right)\left(3c + 5c^{\frac{1}{2}}\right)$$

$$63) \left(2d^{\frac{1}{3}} - d\right)\left(7d^{\frac{1}{3}} + 3d\right)$$

Exercise 9

(Miscellaneous Harder Questions including some Examination Questions)

1) Simplify a) $\left(y^{\frac{1}{2}}\right)^2 \times \left(a^{-2}\right)^0$ b) $\left(x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right)^2$ c) $3x^{-1} - (3x)^{-1}$.

2) Express $y^8 \times \left(y^3\right)^{-2}$ in its simplest form.

- 3) The intensity of light, I , emerging after passing through a liquid with concentration, c , is given by the equation

$$I = \frac{20}{2^c} \quad c \geq 0.$$

a) Find the intensity of light when the concentration is 3.

b) Find the concentration of the liquid when the intensity is 10.

c) What is the maximum possible intensity?

4) Express $a^{\frac{1}{2}}\left(a + \frac{1}{a}\right)$ without brackets in its simplest form.

5) Express $a^2\left(a^{-5} + 4\right)$ without brackets in its simplest form.

6) $f(x) = 3^x$.

a) Find $f(4)$.

b) Given that $f(x) = \sqrt{27}$, find x .

7) Express a) $\frac{b^{\frac{1}{2}} \times b^{\frac{3}{2}}}{b^3}$ b) $\frac{t^4 \times t}{t^{-2}}$ c) $\frac{3y^5 \times 4y^{-1}}{6y}$

in simplest form.

8) a) Simplify $b^{\frac{1}{3}} \left(b^{\frac{1}{3}} - b^{\frac{5}{3}} \right)$.

b) If $b = -8$, hence evaluate the expression.

9) Simplify $\frac{5a^3 b^{-\frac{2}{3}}}{3ab^{\frac{1}{3}}}$.

Scientific Notation

Exercise 1

1) Write each of the following numbers in standard form:

- | | | |
|----------------|-----------------------|------------|
| a) 8 500 | b) 62 000 000 | c) 430 000 |
| d) 5 400 000 | e) 87 000 000 000 000 | f) 123 000 |
| g) 675 000 000 | h) 87 600 | i) 508 000 |
| j) 9 462 | k) 1 134 000 | |

2) Write each of the following numbers in the form $a \times 10^n$ ($1 \leq a < 10$):

- | | | |
|------------------|----------------------|-------------|
| a) 0.08 | b) 0.000 02 | c) 0.000 5 |
| d) 0.000 097 | e) 0.000 008 4 | f) 0.000 62 |
| g) 0.000 000 752 | h) 0.000 000 000 803 | i) 0.005 24 |
| j) 0.000 000 022 | k) 0.502 | l) 0.080 88 |

Exercise 2

Write each of the following as decimals (or whole numbers):

1) 3×10^2

2) 4×10^5

3) 6.9×10^3

4) 4.56×10^7

5) 8.06×10^6

6) 4×10^{-3}

7) 5×10^{-1}

8) 8.2×10^{-4}

9) 9.75×10^{-6}

10) 3.0678×10^3

11) 3.0678×10^{-3}

12) 4.98×10^6

13) 2.406×10^2

14) 2.406×10^{-2}

15) 2.406×10^8

16) 2.406×10^{-5}

Exercise 3

- 1) It costs £10 for an adult and £5 for a child to visit the Motor Show.

325 000 adults and 282 000 children visited the show.

Calculate the total ticket money in standard form.

- 2) Mercury has a circular orbit of radius 3.6×10^7 .

Calculate the length of the orbit of planet Mercury giving your answer in scientific notation.

- 3) A large company made $\text{£}3.5 \times 10^6$ profit in one year.

The next year they made eleven times that amount.

Give the profit for the next year in standard form.

- 4) Light travels at 3×10^{10} cm per second.

There are 86 400 seconds in a day.

How far does light travel in one day (answer in scientific notation)?

- 5)** The population of the UK is 5.5×10^7 .

The area of the UK is 244 000 km².

Find the number of people per square kilometre to the nearest whole number.

- 6)** A crystal is rectangular in shape and its dimensions are 4.2×10^{-3} mm by 5.1×10^{-4} mm.

Calculate its area, expressing the answer in standard form.

- 7)** The mass of a protein molecule is approximately 4×10^{-9} g.

How many molecules are in a sample weighing 5 g (answer in scientific notation)?

- 8)** Mars is in a circular orbit of diameter 4.54×10^8 km.

Calculate the length of the orbit (answer in scientific notation).

- 9)** The distance between two cities is 3.75×10^3 km.

The time taken to travel between them is 36 hours.

Calculate the speed of the vehicle.

- 10)** A manufacturer makes 4.85×10^7 nails. He puts them in bags of 50.

How many full bags will he have?

- 11)** How many seconds are in a decade.

Give your answer in standard form (ignore leap years).

- 12)** The nearest star is 5×10^{13} km from the Earth.

Light travels at 3×10^5 km per second.

How long will it take light from the star to reach the Earth?
(give your answer in hours correct to 2 decimal places)

- 13)** A camera on a space telescope photographs a galaxy, a distance of 50 million light years away.

One light year is approximately 9.46×10^{12} km.

Calculate the distance of the galaxy from the space telescope.
(give your answer in scientific notation)

- 14)** A planet takes 94 days to travel round the sun. The path of the planet round the sun is a circle with diameter 1.2×10^7 km.

Find the speed of the planet in kilometres per hour as it travels.
(give your answer in km per hour correct to 1 decimal place)

- 15)** There are 5×10^9 red blood cells in 1 millilitre of blood.

The average person has 5.5 litres of blood in their body.

How many red blood cells does the average person have?
Give your answer in scientific notation.

- 16)** A cyclotron produces high speed particles. A particle moving inside the cyclotron takes 9.7×10^{-10} seconds to travel 2.3×10^{-1} metres.

Calculate the speed of the particle in metres per second.

Simplification

Exercise 1

Simplify

- 1) $9x^2 + 4x^2$ 2) $15y^2 - 4y^2$ 3) $3ab + 5ab$
4) $6cd - 2dc$ 5) $8p + 3p + p$ 6) $2x + 3y + 5x$
7) $4x + 3y - 2x + y$ 8) $5 \times g \times g$ 9) $3p \times 4q$
10) $x^2 - xy + 3x^2 - 2xy$ 11) $5x + 3x^2 + 4x - x^2$ 12) $3y^2 + 5y + 3y - 3y^2$
13) $m \times m^2 \times m^3$ 14) $n \times 2n \times 3n$ 15) $4x^2 + x^3 - 4x^2 - x^3$
16) $-5x + 7x - 3x - 2x$ 17) $-4x^2 - 3x^2 + 2x^2 - x^2$
18) $3x - 2y + 4z - 2x - 3y + 5z + 6x + 2y - 3z$
19) $3a^2b + 2ab^3 + 4a^2b^2 - 5ab^3 + 11b^4 + 6a^2b$
20) $1 \cdot 2x^3 - 3 \cdot 4x^2 + 2 \cdot 6x + 3 \cdot 7x^2 + 3.6x - 2.8$
21) $pq + 2 \cdot 1qr - 2 \cdot 2rq + 8qp$ 22) $\frac{1}{4}q \times 16p$
23) $w \times (-y)$ 24) $12x \div 6$ 25) $4ab \div 2a$
26) $8a^2bc^2 \div 4ac^2$ 27) $8mn \times (-3m^2n^3)$ 28) $5a^2 \times (-3b) \times 5ab$
29) $7a^2b^2 \div 3ab$ 30) $m^2n \times (-mn) \times 5m^2n^2$

Exercise 2

Simplify

- 1) $3(x + 1) + 2(x + 4)$ 2) $5(2a + 4) - 3(4a + 2)$ 3) $3(x + 4) - (2x + 5)$
4) $4(1 - 2x) - 3(3x - 4)$ 5) $5(2x - y) - 3(x + 2y)$ 6) $y - 2(y - z)$

- 7)** $3(p + q) - 2(p + r)$ **8)** $2(a - b) - (b - a)$ **9)** $2x - (x + y)$
- 10)** $5x + 7(x - 1)$ **11)** $9 - 2(3x - 1)$ **12)** $5x - 2x(x - 1)$
- 13)** $4(x - 1) - 3x$ **14)** $3x(x - 1) - 7x^2$ **15)** $4a - 3(a - 3)$
- 16)** $3y - y(2 - y)$ **17)** $a(a + b) - 2(a - b)$ **18)** $w(w + x) - x(w - x)$
- 19)** $x(x - y) + y(y - x)$ **20)** $3(m + n) - 5(m - n)$ **21)** $7i - 2j + 3k - (2i + k - 2j)$
- 22)** $a^2 - 2a - 5 - (2a^2 + a - 3)$ **23)** $2(2x + 3y) - 3(x - 4y) + 2(x + y + 3)$
- 24)** $a(b + c) - b(c + a) - c(a + b)$ **25)** $a^2 + \frac{1}{3}a - 2 - (\frac{1}{3}a^2 + \frac{1}{3}a + 2)$
- 26)** $p(q - r) + q(r - p) + r(p - q)$ **27)** $3x(x - 3) - 2x(x - 1) + x(2x - 2)$
- 28)** $\frac{1}{2}(y - 1) + \frac{1}{3}(2y - 3)$ **29)** $-(4a + 5b - 3c) - 2(2a + 3b - 4c)$
- 30)** $2x(x - 5) - x(x - 2) - 3x(x - 5)$ **31)** $3(a - b) - 2(2a - 3b) + 4(a - 3b)$
- 32)** $3x(x^2 + 7x - 1) - 2x(2x^2 + 3) - 3(x^2 + 5)$
- 33)** $3x(x + 4) - x(x - 2)$ **34)** $7(2x + 2) - (2x + 2)$
- 35)** $7b(a + 2) - a(3b + 3)$ **36)** $2(x + 3) + 2x(x + 3) + 2x^2(x + 3)$
- 37)** $a(a^2 + 1) + a^2(a + 1) - 1(1 - a^2)$ **38)** $a(b + 3) + b(a + 3) - ab(2a + 3b)$

Exercise 3

Expand

- 1)** $(x + 3)(x + 2)$ **2)** $(x - 2)(x + 3)$ **3)** $(x - 4)(x - 5)$
- 4)** $(x + 4)(x - 3)$ **5)** $(x + 4)(x + 5)$ **6)** $(a - 2)(a - 5)$
- 7)** $(t + 5)(t - 12)$ **8)** $(x + 5)(x - 5)$ **9)** $(2x + 3)(x - 4)$
- 10)** $(2x + 3)(3x + 2)$ **11)** $(3x + 5)(4x - 3)$ **12)** $(4x - 5)(5x - 4)$

- 13)** $(2x + 3)(2x + 3)$ **14)** $(2x + 1)(x + 1)$ **15)** $(x + 2)(5x + 2)$
- 16)** $(7x + 2)(x + 3)$ **17)** $(3x + 2)(2x + 3)$ **18)** $(3x - 2)(4x + 1)$
- 19)** $(3b + 5)(2b - 5)$ **20)** $(4x - 7)(4x + 5)$ **21)** $(5a + 4)(4a - 3)$
- 22)** $(5y - 2)(5y + 2)$ **23)** $(4y + 3)(7y - 4)$ **24)** $(3x + 4)(5x - 2)$
- 25)** $(10x + 1)(x - 3)$ **26)** $(2x + 5)(5x - 3)$ **27)** $(6x - 5)(3x - 2)$
- 28)** $(4p + 3)(3p - 4)$ **29)** $(2q - 11)(5q - 3)$ **30)** $(3 - x)(4 - x)$
- 31)** $(5 - 2x)(3 + 4x)$ **32)** $(6 - 5x)(3 - 2x)$ **33)** $(1 - 12a)(3 + a)$
- 34)** $(4 - 3m)(2 - m)$ **35)** $(5 + 3x)(2 - 4x)$ **36)** $(6 - 7x)(4 - 3x)$
- 37)** $(1 + 15c)(1 - 15c)$ **38)** $(13 - m)(4 + 3m)$ **39)** $(12 - p)(5 - 4p)$
- 40)** $(2a + 3b)(3a + 2b)$ **41)** $(3m - 4n)(4m + 3n)$ **42)** $(2x - 5y)(3x + 4y)$
- 43)** $(4r - 7s)(3r + 2s)$ **44)** $(1 - y)(15 + y)$ **45)** $(2x + 1)(1 + 3x)$

Exercise 4

Expand

- 1)** $(x + 3)(x^2 + 2x + 1)$ **2)** $(x + 5)(x^2 - 2x - 4)$
- 3)** $(x - 3)(x^2 + 5x - 4)$ **4)** $(x - 5)(x^2 - 4x - 7)$
- 5)** $(2x + 3)(x^2 - 4x - 5)$ **6)** $(3x - 4)(2x^2 - 5x + 3)$
- 7)** $(2x + 1)(2x^2 + 5x - 7)$ **8)** $(3x - 4)(2x^2 + 3x + 4)$
- 9)** $(4x - 1)(4x^2 - 4x - 1)$ **10)** $(5x + 2)(x^2 + 3x + 4)$
- 11)** $(4x - 3)(5x^2 - 7x + 2)$ **12)** $(3x + 4)(4x^2 + 5x + 6)$
- 13)** $(5x + 1)(3x^2 - 2x - 7)$ **14)** $(4x - 3)(5x^2 + 3x + 8)$

$$15) (2x - 1)(2x^2 - 2x - 1)$$

$$16) (x + y - 2)(2x - y - 1)$$

$$17) (x^2 - x - 1)(x^2 + x + 1)$$

$$18) (a^2 + 2a - 3)(2a^2 - a - 1)$$

$$19) (a - b - 2)(2a - b + 3)$$

$$20) (x + 2y - 5)(x - 3y + 2)$$

$$21) (x^2 - 3)(x^3 + 2x^2 - 3x - 4)$$

$$22) (1 - a + a^2)(1 + a - a^2)$$

$$23) (a^3 - a^2 - a - 1)(a^3 + a^2 + a - 1)$$

$$24) (x^3 - 2x^2 - x - 1)(x^3 + 3x^2 + 2x - 1)$$

Exercise 5

Expand

$$1) (x + 1)^2$$

$$2) (x + 4)^2$$

$$3) (x + 7)^2$$

$$4) (x - 4)^2$$

$$5) (x - 1)^2$$

$$6) (x - 3)^2$$

$$7) (y - 2)^2$$

$$8) (z + 9)^2$$

$$9) (t - 6)^2$$

$$10) (2x + 1)^2$$

$$11) (2x - 1)^2$$

$$12) (3x + 2)^2$$

$$13) (3x + 4)^2$$

$$14) (5x - 2)^2$$

$$15) (4x - 3)^2$$

$$16) (2x + 3)^2$$

$$17) (3x - 5)^2$$

$$18) (4x + 3y)^2$$

$$19) (x + 1)^3$$

$$20) (x - 2)^3$$

$$21) (3x + 2)^3$$

$$22) (a - 4b)^3$$

$$23) (2 - 5x)^3$$

$$24) (5 - 3x)^3$$

$$25) (x + 1)(x + 2)(x + 3)$$

$$26) (x + 2)(x - 2)(x + 4)$$

$$27) (x - 2)(x - 4)(x - 6)$$

$$28) (x - 1)(x - 3)(x + 4)$$

$$29) (x - 3)^3 - (x - 4)(x - 2)^2$$

$$30) (x - 1)(x + 2)(x - 3) - (x - 4)^3$$

$$31) (x - y - 1)^3$$

$$32) (x - 1)(x - 2)(x + 2)(x - 3)(x - 5)$$

Exercise 6

Expand

1) $(x + 3)(x + 4) + x(x + 2)$

2) $x(x + 6) + (x + 1)(x + 2)$

3) $(x + 4)(x + 5) + 6(x + 2)$

4) $(a - 5)(2a + 3) - 3(a - 4)$

5) $(2x + 1)(3x - 4) + (2x + 3)(5x - 2)$

6) $(5x - 2)(3x + 5) - (3x + 5)(x + 2)$

7) $(2x - 5)(3x + 4) - (6x - 5)(2x + 3)$

8) $(2x + 7)(3x - 2) + (4x + 3)(2x - 5)$

9) $(5x + 2)(2x + 5) - (2x - 3)(3x - 2)$

10) $2x(x - 3) - 3x(x - 2)$

11) $(2x + 3)(2x - 3) + (3x + 2)(3x - 2)$

12) $(5x - 4)(3x + 2) + (2x + 5)^2$

13) $(3x - 2)(2x + 5) - (2x - 3)^2$

14) $5x(x - 3) - (x + 2)^2$

15) $(2x - 3)^2 - (5x + 4)(3x + 2)$

16) $(3x - 4)^2 - (4x - 3)^2$

17) $(5x - 3)^2 + (3x + 4)^2$

18) $4x(5x + 2) - (3x - 5)^2$

19) $(4x + 5)^2 - x(x - 3)$

20) $(2x + 5)^2 - (3x + 2)^2$

Factorisation

Exercise 1

Factorise completely

1) $x^2 + 5x$

2) $x^2 - 6x$

3) $7x - x^2$

4) $y^2 + 8y$

5) $2y^2 + 3y$

6) $6y^2 - 4y$

7) $3x^2 - 21x$

8) $16a - 2a^2$

9) $6c^2 - 21c$

10) $15x - 9x^2$

11) $56y - 21y^2$

12) $ax + bx + 2cx$

13) $x^2 + xy + 3xz$

14) $x^2y + y^3 + z^2y$

15) $3a^2b + 2ab^2$

- 16)** $x^2y + xy^2$ **17)** $6a^2 + 4ab + 2ac$ **18)** $ma + 2bm + m^2$
- 19)** $2kx + 6ky + 4kz$ **20)** $ax^2 + ay + 2ab$ **21)** $x^2k + kx^2$
- 22)** $a^3b + 2ab^2$ **23)** $abc - 3b^2c$ **24)** $2a^2e - 5ae^2$
- 25)** $a^3b + ab^3$ **26)** $x^3y + x^2y^2$ **27)** $6xy^2 - 4x^2y$
- 28)** $3ab^3 + 3a^3b$ **29)** $2a^3b + 5a^2b^2$ **30)** $ax^2y - 2ax^2z$
- 31)** $ab - ac - a^2$ **32)** $x^2 + xy - xz$ **33)** $p^3 - p^2 + p$
- 34)** $xa + ya - za$ **35)** $3a^4 + 3a^3 - 6a^2$ **36)** $2x^3 - 4x^2 + 6x$
- 37)** $2p^2q^2 - 6pq$ **38)** $12a^2b^2 - 3ab$ **39)** $9a^2b + 3ab^2$
- 40)** $6x^2y + 8xy^2$ **41)** $2a^2 - 4ab + 6ac$ **42)** $3p^2 + 6pq - 12pr$
- 43)** $8x^2y^3 - 12x^3y^2$ **44)** $10r^4s^3 + 15r^2s^4$ **45)** $6a^2bc + 8abc^2$
- 46)** $9l^3mn^2 - 12lm^2n$ **47)** $2xy^2z + 4x^2yz - 6xyz^2$ **48)** $3p^2qr^3 - 6pq^2r^3 + 12p^3qr^2$
- 49)** $16x^4y^3 - 20x^2y^3z^2 + 24x^3y^2z$ **50)** $15a^5b^4 + 20a^6b^7 - 25a^7b^2$

Exercise 2

Factorise fully

- 1)** $x^2 - 1$ **2)** $y^2 - 4$ **3)** $z^2 - 16$
- 4)** $a^2 - 64$ **5)** $b^2 - 100$ **6)** $c^2 - 169$
- 7)** $4d^2 - 9$ **8)** $16e^2 - 49$ **9)** $64f^2 - 81$
- 10)** $25g^2 - 121$ **11)** $36 - 49h^2$ **12)** $49k^2 - 25$
- 13)** $l^2 - 4m^2$ **14)** $m^2 - 49n^2$ **15)** $4n^2 - 49p^2$
- 16)** $16p^2 - 81q^2$ **17)** $25q^2 - 36r^2$ **18)** $64r^2 - 225s^2$

$$19) \ 49s^2 - 121t^2$$

$$20) \ 9x^2 - 169y^2$$

$$21) \ x^2 - \frac{1}{4}$$

$$22) \ x^2 - \frac{1}{9}$$

$$23) \ y^2 - a^2$$

$$24) \ m^2 - n^2$$

$$25) \ x^2 - t^2$$

$$26) \ x^2 - \frac{y^2}{4}$$

$$27) \ 9m^2 - \frac{4}{9}n^2$$

$$28) \ 16t^2 - \frac{4}{25}s^2$$

$$29) \ 4x^2 - \frac{z^2}{100}$$

$$30) \ x^3 - x$$

$$31) \ 5p^2 - 5q^2$$

$$32) \ 45p^2 - 5q^2$$

$$33) \ 3p^2 - 12q^2$$

$$34) \ 27p^2 - 3q^2$$

$$35) \ 9a^2 - 36y^2$$

$$36) \ 3b^2 - 27$$

$$37) \ 4c^2 - 100$$

$$38) \ 5d^2 - 5$$

$$39) \ 8x^2 - 32y^2$$

$$40) \ a^4 - 1$$

$$41) \ c^4 - 16$$

$$42) \ d^4 - 81$$

$$43) \ 1 - x^4$$

$$44) \ a^4 - b^4$$

$$45) \ x^4 - y^4$$

$$46) \ p^4 - q^4$$

$$47) \ 1 - 16c^4$$

$$48) \ 3x^4 - 48$$

$$49) \ 4y^4 - 4$$

$$50) \ 2z^4 - 162$$

$$51) \ 80 - 5a^4$$

Exercise 3

Factorise

$$1) \ x^2 + 3x + 2$$

$$2) \ a^2 - 3a + 2$$

$$3) \ x^2 + 6x + 8$$

$$4) \ p^2 - 6p + 8$$

$$5) \ x^2 + 9x + 18$$

$$6) \ q^2 - 9q + 18$$

$$7) \ k^2 - 8k + 12$$

$$8) \ x^2 + 7x + 10$$

$$9) \ x^2 + 7x + 12$$

$$10) \ x^2 + 8x + 15$$

$$11) \ x^2 + 10x + 21$$

$$12) \ x^2 + 8x + 12$$

$$13) \ y^2 + 12y + 35$$

$$14) \ y^2 + 11y + 24$$

$$15) \ y^2 + 10y + 25$$

$16) y^2 + 15y + 36$

$17) x^2 - 26x + 105$

$18) m^2 + 11m + 30$

$19) y^2 + 24y + 80$

$20) d^2 + 18d + 45$

$21) p^2 + 2p - 3$

$22) q^2 + 3q - 4$

$23) r^2 + r - 6$

$24) s^2 + 2s - 8$

$25) 2x^2 + 5x + 3$

$26) 2x^2 + 7x + 3$

$27) 3x^2 + 7x + 2$

$28) 2x^2 + 11x + 12$

$29) 3x^2 + 8x + 4$

$30) 2x^2 + 7x + 5$

$31) 3x^2 - 5x - 2$

$32) 2x^2 - x - 15$

$33) 2x^2 + x - 21$

$34) 3x^2 - 17x - 28$

$35) 6x^2 + 7x + 2$

$36) 12x^2 + 23x + 10$

$37) 3x^2 - 11x + 6$

$38) 3y^2 - 11y + 10$

$39) 4y^2 - 23y + 15$

$40) 6y^2 + 7y - 3$

$41) 6x^2 - 27x + 30$

$42) 10x^2 + 9x + 2$

$43) 6x^2 - 19x + 3$

$44) 8x^2 - 10x - 3$

$45) 12x^2 + 4x - 5$

$46) 16x^2 + 19x + 3$

$47) 4a^2 - 4a + 1$

$48) 12x^2 + 17x - 14$

$49) t^2 + t - 12$

$50) w^2 + 5w - 14$

$51) x^2 - 2x - 3$

$52) 15x^2 + 44x - 3$

$53) 48x^2 + 46x + 5$

$54) 64y^2 + 4y - 3$

$55) 120x^2 + 67x - 5$

$56) 1 + 3x - 18x^2$

$57) 15 - 7p - 2p^2$

$58) a^2 + 8ab + 12b^2$

$59) x^2 - 10xy + 24y^2$

$60) p^2 - pq - 2q^2$

$61) r^2 + rs - 6s^2$

$62) u^2 - 2uv + v^2$

$63) a^2 - 5ab - 14b^2$

$64) 15 - 8x + x^2$

$65) 50 + 15x + x^2$

$66) 36 - 20x + x^2$

$67) 16 + 8x + x^2$

$68) 25 - 10a + a^2$

$69) 1 - 2b + b^2$

$70) 24 + 11x + x^2$

$71) 16 - 8k + k^2$

$72) 72 + y - y^2$

$73) y^2 - 3y - 4$

$74) z^2 - z - 6$

$75) a^2 - 2a - 8$

$76) 4x^2 - 17xy + 4y^2$

$77) 12a^2 + 13ab - 4b^2$

$78) 9x^2 + xy - 10y^2$

- 79)** $8a^2 + 7ab - 15b^2$ **80)** $a^2 - 2ab - 48b^2$ **81)** $y^2 - 15yz - 54z^2$
- 82)** $h^2 - 23hk - 50k^2$ **83)** $b^2 + 14bc + 48c^2$ **84)** $k^2 - 15kl + 56l^2$
- 85)** $p^2 - 17pq + 70q^2$ **86)** $10x^2 - x - 3$ **87)** $12y^2 - 16y + 5$
- 88)** $36x^2 + x - 2$ **89)** $20x^2 - 7x - 3$ **90)** $4a^2 - 3a - 10$

Exercise 4

Factorise fully

- 1)** $5x + 15y$ **2)** $x^2 - 25$ **3)** $a^2 + 6a + 9$
- 4)** $x^2 - x$ **5)** $y^2 - y - 6$ **6)** $1 - p^2$
- 7)** $a^2 + 4a + 4$ **8)** $ax - ap + ad$ **9)** $3x^2 - 12$
- 10)** $p^2 - q^2$ **11)** $p^2 - 2p$ **12)** $p^2 - 2p + 1$
- 13)** $a^2 - 1$ **14)** $a^2 - a$ **15)** $a^2 - a - 2$
- 16)** $2x^2 - 18$ **17)** $2x^2 - 8x$ **18)** $x^3 - x^2$
- 19)** $2k^2 + 3k - 5$ **20)** $k^2 - 6k + 9$ **21)** $9k^2 + 6k + 1$
- 22)** $16 - x^2$ **23)** $18 - 2y^2$ **24)** $12z^2 - 6z$
- 25)** $9 - 4a^2$ **26)** $2p^2 - p - 1$ **27)** $6x^2 + 13x + 6$
- 28)** $14a^2 + 21b^2$ **29)** $14a^2 - 56b^2$ **30)** $16x^2 - 8x + 1$
- 31)** $25x^2 - 10x + 1$ **32)** $1 - 2x + x^2$ **33)** $ax + bx - 2x$
- 34)** $2x^3 - 32x$ **35)** $6y^2 + 5y - 6$ **36)** $2x^2 + 4x + 2$
- 37)** $3a^2 + 3a - 6$ **38)** $4b^2 + 14b - 8$ **39)** $5x - 20x^3$
- 40)** $6x + 24x^3$ **41)** $km^2 - kn^2$ **42)** $2c^2 - 7c - 15$

$43) a^2 + a^3 + a^4$

$44) 4x^2 - 11x + 6$

$45) x^4 - x^6$

$46) x^4 - 1$

$47) (a - b)^2 - x^2$

$48) a^2 - 10ab + 25b^2$

$49) m^4 - n^4$

$50) 4a^2b^3 - 12a^3b^2$

$51) 1 - (p - q)^2$

$52) 12a^2 + 7ab - 12b^2$

$53) 16 - 8k + k^2$

$54) x^2y^6 - x^6y^2$

$55) 2x^3 + 2x^2 - 4x$

$56) 3 - 3x - 36x^2$

$57) x^2 - (y + z)^2$

Exercise 5

A Express each of the following in the form $(x + a)^2 + b$

$1) x^2 + 4x + 5$

$2) x^2 - 6x + 7$

$3) x^2 + 8x - 3$

$4) x^2 - 4x + 10$

$5) x^2 + 10x + 2$

$6) x^2 - 8x - 3$

$7) x^2 + 2x - 4$

$8) x^2 - 4x + 9$

$9) x^2 + 12x - 2$

$10) x^2 - 8x - 2$

$11) x^2 + 10x + 3$

$12) x^2 - 2x + 11$

$13) x^2 - 12x + 50$

$14) x^2 + 6x + 20$

$15) x^2 + 4x + 4$

$16) x^2 - 8x - 3$

$17) x^2 - 2x + 5$

$18) x^2 - 6x - 7$

B Express each of the following in the form $(x + a)^2 + b$

$19) x^2 - x + 4$

$20) x^2 + 3x - 1$

$21) x^2 - 5x - 2$

$22) x^2 + x - 2$

$23) x^2 - 5x - 6$

$24) x^2 - 3x + 2$

$25) x^2 + 3x + 6$

$26) x^2 - x + 8$

$27) x^2 + x - 3$

$28) x^2 - 3x - 2$

$29) x^2 + 5x + 3$

$30) x^2 + x + 3$

$31) x^2 - 3x + 5$

$32) x^2 + 5x - 2$

$33) x^2 - 3x + 7$

$34) x^2 - x + 4$

$35) x^2 + x + 4$

$36) x^2 - 3x - 2$

C Express each of the following in the form $a - (x + b)^2$

37) $2 - 4x - x^2$

38) $1 + 2x - x^2$

39) $5 + 6x - x^2$

40) $4 + 3x - x^2$

41) $3 - 5x - x^2$

42) $6 - x - x^2$

43) $-1 + 8x - x^2$

44) $4 - 6x - x^2$

45) $-8 - 4x - x^2$

46) $3 + x - x^2$

47) $-1 - 2x - x^2$

48) $2 + 5x - x^2$

49) $10 - 3x - x^2$

50) $-2 + 4x - x^2$

51) $4 - 8x - x^2$

52) $7 + 2x - x^2$

53) $3 + 3x - x^2$

54) $-1 - x - x^2$

D Express each of the following in the form $\pm(x + a)^2 + b$

55) $x^2 - 2x + 9$

56) $x^2 + 3x + 13$

57) $5 + 4x - x^2$

58) $x^2 + x - 3$

59) $x^2 + 8x + 7$

60) $10 - 3x - x^2$

61) $x^2 - 5x - 2$

62) $-5 - 6x - x^2$

63) $x^2 + 10x + 40$

64) $1 + 4x - x^2$

65) $x^2 + 8x + 3$

66) $x^2 + 3x - 5$

67) $x^2 - 6x + 7$

68) $5 - 2x - x^2$

69) $x^2 - x + 6$

70) $9 - x - x^2$

71) $x^2 + 12x - 4$

72) $x^2 - 3x + 1$

Algebraic Fraction Simplification

Exercise 1

Simplify the following:

$$1) \frac{p^2}{pq}$$

$$2) \frac{ab}{a^2}$$

$$3) \frac{xy}{xz}$$

$$4) \frac{cd}{de}$$

$$5) \frac{3x}{6x^2}$$

$$6) \frac{4ab}{12a^2}$$

$$7) \frac{2pqr}{6qrs}$$

$$8) \frac{3xy^2}{6x^2y}$$

$$9) \frac{5bc^3}{10bc^2d}$$

$$10) \frac{4a^2b^3}{6a^3b^2}$$

$$11) \frac{12x^3y^7}{18x^2y^6z^3}$$

$$12) \frac{12pqrst}{18qst^2}$$

$$13) \frac{24a^3b^2c^4d^3}{32ab^3c^2d^2}$$

$$14) \frac{15l^5m^4n^3}{25pl^2m^3n^4}$$

Now try these:

$$15) \frac{(x-1)(x+3)^2}{(x+3)}$$

$$16) \frac{(a+5)(a-3)^4}{(a-3)}$$

$$17) \frac{(x+5)^3(x-7)}{(x+5)}$$

$$18) \frac{(g-6)(g+3)^3}{(g+3)}$$

$$19) \frac{(c-1)(c-2)^7}{(c-2)}$$

$$20) \frac{(x-2)^6(x+1)}{(x-2)}$$

$$21) \frac{(y+7)(y-3)^2}{(y-3)^5}$$

$$22) \frac{(z-4)(z+3)^3}{(z+3)^4}$$

$$23) \frac{(c+5)(c-3)}{(c+5)^2}$$

$$24) \frac{(a+2)(a-3)^3}{(a-3)^6}$$

$$25) \frac{(x+2)(x-1)^4}{(x-1)^5}$$

$$26) \frac{(c+1)^2(c-1)}{(c+1)^4}$$

$$27) \frac{3(d+2)(d-1)^4}{6(d-1)^2}$$

$$28) \frac{8(x-5)^6(x+3)}{10(x-5)^4}$$

$$29) \frac{12(y-4)(y+2)^3}{18(y+2)}$$

$$30) \frac{4(e+5)(e-2)^2}{6(e-2)}$$

$$31) \frac{12(f+1)^2(f-2)}{6(f-2)^2}$$

$$32) \frac{18(f+2)(f-9)^4}{12(f+2)^3}$$

Exercise 2

Now try to simplify these:

$$1) \frac{4x+6}{2}$$

$$2) \frac{12a+8}{4}$$

$$3) \frac{14a+10b}{2}$$

$$4) \frac{10p-20q}{5}$$

$$5) \frac{8x-16y}{2}$$

$$6) \frac{ab+ac}{a}$$

$$7) \frac{a^2-3ab}{a}$$

$$8) \frac{12ab+3b^2}{3b}$$

$$9) \frac{18a^2+12a^3}{6a}$$

$$10) \frac{xy-zx}{2x}$$

$$11) \frac{3c}{6ac+9bc}$$

$$12) \frac{3x}{2xy-xz}$$

$$13) \frac{2pq}{6pr+2pq}$$

$$14) \frac{a+b}{a^2-b^2}$$

$$15) \frac{4a^2-9}{2a+3}$$

$$16) \frac{a^2-4}{a-2}$$

$$17) \frac{x^2-y^2}{x-y}$$

$$18) \frac{x+2}{x^2+3x+2}$$

$$19) \frac{3p-3}{p^2-2p+1}$$

$$20) \frac{ax+5a}{x^2+8x+15}$$

$$21) \frac{x^2-1}{x^2+2x+1}$$

$$22) \frac{2x^2+x-1}{2x^2+5x-3}$$

$$23) \frac{a^2+2a-35}{a^2+14a+49}$$

$$24) \frac{6c^2-13c+6}{3c^2+10c-8}$$

$$25) \frac{a^2-a}{a^3-a}$$

$$26) \frac{y^2-z^2}{(y-z)^2}$$

$$27) \frac{a^2+a}{a^2+3a+2}$$

$$28) \frac{p^3-p^2}{p^2+p-2}$$

$$29) \frac{b^2-4b+3}{b^2+b-2}$$

$$30) \frac{c^2+3c+2}{c^2+5c+6}$$

$$31) \frac{x^2+3x+2}{x^2+x-2}$$

$$32) \frac{a^2-2a+1}{a^2+a-2}$$

$$33) \frac{y^2-10y+25}{y^2-25}$$

$$34) \frac{p^2+pq-12q^2}{p^2-16q^2}$$

Exercise 3 - Multiplying algebraic fractions

$$1) \frac{2x}{9} \times \frac{15}{4x}$$

$$2) \frac{2}{3a} \times \frac{a}{4}$$

$$3) \frac{2x}{5} \times \frac{10}{7x}$$

$$4) \frac{4x}{5} \times \frac{10}{7x}$$

$$5) \frac{3x}{8} \times \frac{2x}{3}$$

$$6) \frac{2}{3x} \times \frac{9x}{8}$$

$$7) \frac{5a}{6} \times \frac{9}{10a}$$

$$8) \frac{2x}{3y} \times \frac{y}{5}$$

$$9) \frac{4}{5x} \times \frac{10x}{3y}$$

$$10) \frac{8x}{9y} \times \frac{3y}{4}$$

$$11) \frac{5x}{2y} \times \frac{4x}{3}$$

$$12) \frac{4x}{5y} \times \frac{8x}{15}$$

$$13) \frac{10y}{7x} \times \frac{14x^2}{5y}$$

$$14) \frac{12a^2}{5b} \times \frac{15b^2}{16a^3}$$

$$15) \frac{9d^2}{10e} \times \frac{20e^2}{d^4}$$

Exercise 4 - Dividing algebraic fractions

$$1) \frac{3x}{10} \div \frac{2x}{15}$$

$$2) \frac{5x}{6} \div \frac{7x}{8}$$

$$3) \frac{7}{12x} \div \frac{14}{15x}$$

$$4) \frac{5x}{6} \div \frac{10x}{9}$$

$$5) \frac{27}{8x} \div \frac{9}{16x}$$

$$6) \frac{3x}{5} \div \frac{2x}{y}$$

$$7) \frac{3}{4x} \div \frac{9y}{8x}$$

$$8) \frac{2x}{5y} \div \frac{8}{15y}$$

$$9) \frac{5y}{8x} \div \frac{9y}{4x}$$

$$10) \frac{30y}{7x} \div \frac{8}{21x}$$

$$11) \frac{35x}{8y} \div \frac{5x}{12y^2}$$

$$12) \frac{42a^2}{15b} \div \frac{24a}{9b^2}$$

Exercise 5 - Adding algebraic fractions

$$1) \frac{x}{4} + \frac{x}{3}$$

$$2) \frac{x}{2} + \frac{x}{5}$$

$$3) \frac{3x}{8} + \frac{2x}{3}$$

$$4) \frac{7}{x} + \frac{x}{5}$$

$$5) \frac{7}{x} + \frac{5}{x}$$

$$6) \frac{2x}{7} + \frac{1}{5x}$$

$$7) \frac{x}{3} - \frac{x}{4}$$

$$8) \frac{2x}{5} - \frac{x}{6}$$

$$9) \frac{x}{2} - \frac{3x}{10}$$

$$10) \frac{4x}{5} - \frac{3x}{8}$$

$$11) \frac{6}{x} - \frac{5}{2x}$$

$$12) \frac{3a}{4} - \frac{2}{5a}$$

Exercise 6

$$1) \frac{(x+1)}{2} + \frac{(x+2)}{3}$$

$$2) \frac{(x+3)}{4} + \frac{(x-1)}{3}$$

$$3) \frac{(x-2)}{5} + \frac{(x+3)}{2}$$

$$4) \frac{(x+2)}{3} - \frac{(x+1)}{2}$$

$$5) \frac{(x+2)}{2} - \frac{(x-1)}{5}$$

$$6) \frac{(x-3)}{4} - \frac{(x-1)}{3}$$

$$7) \frac{2}{x+3} + \frac{3}{x-2}$$

$$8) \frac{3}{x+4} - \frac{2}{x-1}$$

$$9) \frac{5}{x-3} - \frac{3}{x+1}$$

$$10) \frac{1}{x-5} + \frac{3}{x+4}$$

$$11) \frac{2}{x-1} - \frac{1}{x+1}$$

$$12) \frac{3}{x+4} + \frac{4}{x-1}$$

$$13) \frac{x}{2} + \frac{x+2}{4}$$

$$14) \frac{x+2}{3} - \frac{x+3}{4}$$

$$15) \frac{x-1}{2} - \frac{x-2}{5}$$

$$16) \frac{2x-1}{3} - \frac{x+2}{4}$$

$$17) \frac{2a-5}{6} - \frac{a-7}{3}$$

$$18) \frac{2c-3}{3} + \frac{5-3c}{5}$$

$$19) \frac{1}{x^2y} + \frac{2}{xy^2}$$

$$20) \frac{5}{a^3b} - \frac{2}{ab^2}$$

$$21) \frac{1}{x+1} + \frac{1}{x-1}$$

$$22) \frac{3}{a+2} + \frac{2}{a+1}$$

$$23) \frac{4}{x+y} + \frac{5}{x-y}$$

$$24) \frac{x}{x+y} - \frac{y}{x-y}$$

$$25) \frac{2}{x^2-1} + \frac{1}{x+1}$$

$$26) \frac{1}{a^2-1} - \frac{1}{a-1}$$

$$27) \frac{3}{x^2+4x+3} + \frac{1}{x+3}$$

$$28) \frac{1}{a^2+2a+1} + \frac{1}{a+1}$$

$$29) \frac{1}{y-2} - \frac{3}{y^2+y-6}$$

$$30) \frac{1}{a^2-2a+1} + \frac{1}{a^2-1}$$

$$31) \frac{x+4}{x^2-9} - \frac{1}{x-3}$$

$$32) \frac{3}{x^2+x-2} - \frac{2}{x^2+3x+2}$$

$$33) \frac{1}{x^2} - \frac{1}{x^2+x}$$

$$35) \frac{2}{6x^2-5x-4} - \frac{3}{9x^2-16}$$

$$37) \frac{p}{p^2+3p+2} - \frac{p}{p+5p+6}$$

$$39) \frac{1}{(x-3)^2} - \frac{1}{2x^2-18}$$

$$40) \frac{2}{(2x+1)^3(3x-5)} - \frac{5}{(2x+1)(3x-5)^2}$$

$$33) \frac{x+2}{x+3} - \frac{x+3}{x+2}$$

$$34) \frac{1}{c^2-c} + \frac{1}{c^2}$$

$$36) \frac{x+3}{(x+4)^3(x-5)} + \frac{4}{(x+4)(x-5)^2}$$

$$38) \frac{a}{a^3-a^2} + \frac{1}{a^2+a-2}$$