AH Mathematics

Methods in Algebra and Calculus

Practice Assessment 3

M. Patel (July 2018)

Methods in Algebra and Calculus Assessment Standard 1.1

1 Express $\dfrac{x^2 + 24}{x^3 + 8x}$ in partial fractions. **(3)**

Methods in Algebra and Calculus Assessment Standard 1.2

2 Differentiate the following function with respect to x:

$f(x) = e^{x^2 - 6x}$. **(2)**

3 Given $y = \sqrt{\tan 5x}$, find $\dfrac{dy}{dx}$. **(2)**

4 Differentiate the following functions with respect to x:

a) $f(x) = 4x^5 \sin x$ **(2)**

b) $g(x) = \dfrac{3x + 2}{x - 4}$, $x \neq 4$. **(2)**

5 Differentiate the following function with respect to x:

$f(x) = \sin^{-1}(6x), \quad -\dfrac{1}{6} \leq x \leq \dfrac{1}{6}$. **(2)**

6 If $x^5 y + x y^2 = 11$, use implicit differentiation to find $\dfrac{dy}{dx}$. **(4)**

7 The position of a golf ball with respect to a coordinate axis system, at time t seconds, is given by:

$x = 5t, \quad y = 12t - 2t^2, \quad 0 \leq t \leq 6$.

Find the speed of the golf ball when $t = 2$. **(2)**

Methods in Algebra and Calculus Assessment Standard 1.3

8 Find :

a) $\int \frac{8}{\sqrt{1 - (3x)^2}}\, dx$ **(2)**

b) $\int \frac{7}{14x - 1}\, dx$ **(2)**

c) $\int_0^{\frac{\pi}{20}} \sec^2 5x \; dx$. **(3)**

9 Using the substitution $u = \cos x$, find $\int \frac{\sin x}{\cos^7 x}\, dx$. **(3)**

10 Using integration by parts, evaluate $\int_1^2 x^6 \ln x \; dx$. **(4)**

Methods in Algebra and Calculus Assessment Standard 1.4

11 Find the general solution of the differential equation $\frac{dy}{dx} = \frac{3y}{x - 5}$. **(4)**

12 Find the general solution, in the form $y = f(x)$, of the first-order linear differential equation

$$\frac{dy}{dx} + 3y = 7e^{6x}.$$ **(5)**

13 Find the particular solution of the second-order differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 12y = 0 \text{ when } x = 0, y = 1 \text{ and } \frac{dy}{dx} = 18.$$ **(6)**