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Unit 2 : Properties of Functions - Lesson 3

Inverse Functions

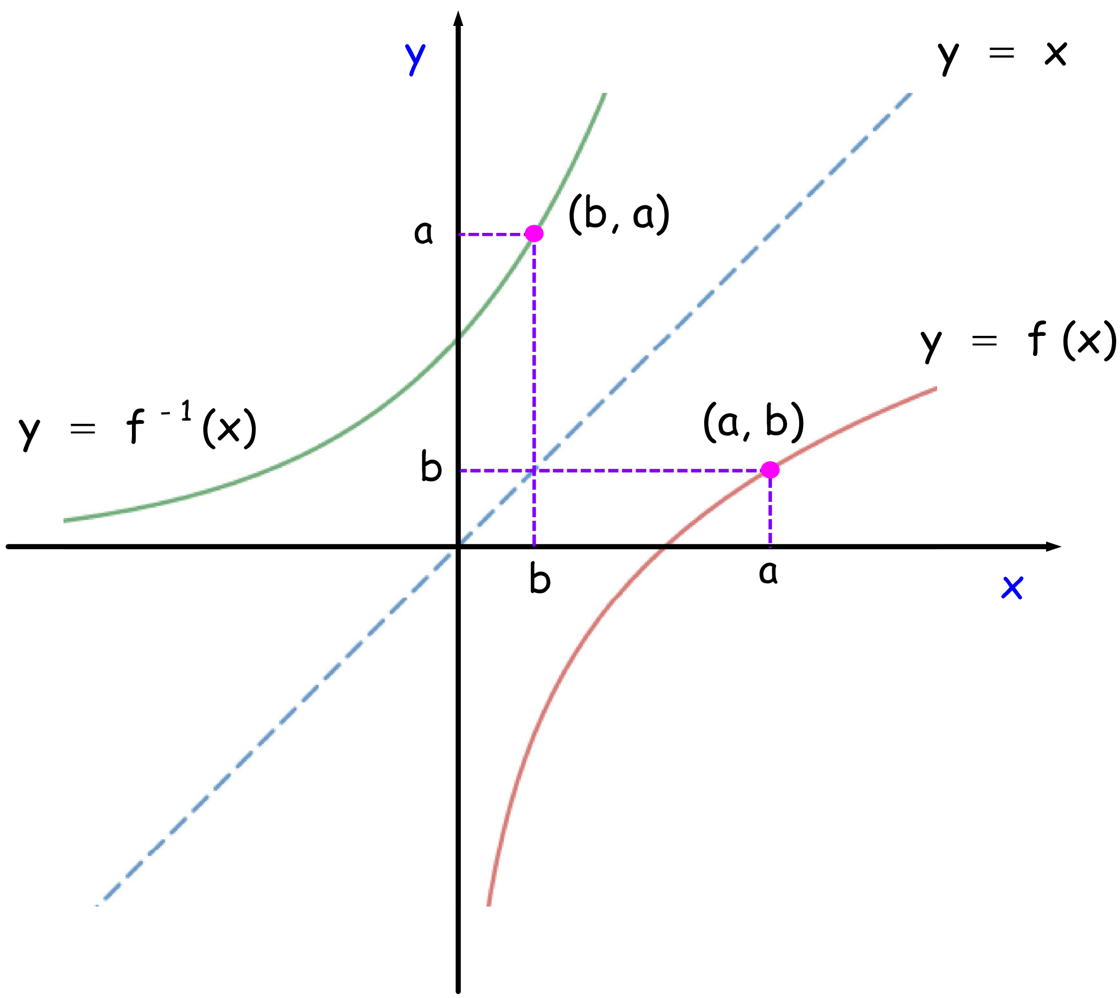
LI

- Know the graphs of $y = \sin^{-1}x, y = \cos^{-1}x$ and $y = \tan^{-1}x.$
- Find domains and ranges of functions and their inverses.
- Sketch the inverse graph of a function.

SC

- Sketch graphs.
- Find inverse functions.

Graphical Interpretation of the Inverse

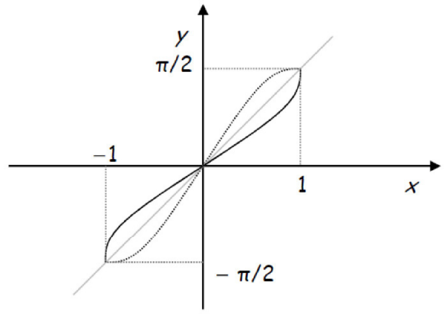


To sketch the graph of the inverse of a function, reflect the graph in the line $y = x$

If $f(a) = b$, then $f^{-1}(b) = a$ and vice versa

Inverse Trig. Functions

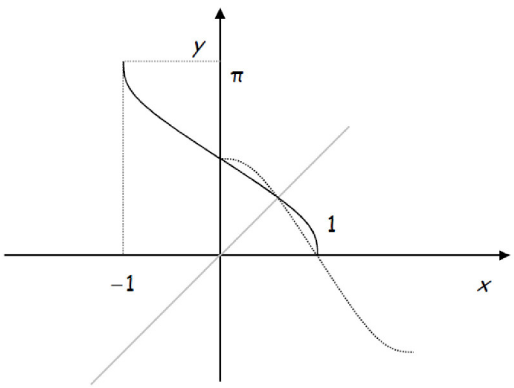
Inverse Sine (aka arcsine)



$\text{dom}(\sin^{-1} x) = [-1, 1]$

$\text{ran}(\sin^{-1} x) = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Inverse Cosine (aka arccosine)

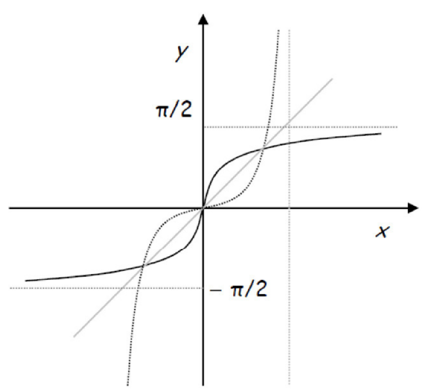


$\text{dom}(\cos^{-1} x) = [-1, 1]$

$\text{ran}(\cos^{-1} x) = [0, \pi]$

These 2 graphs do not repeat and do not extend

Inverse Tangent (aka arctangent)



$\text{dom}(\tan^{-1} x) = \mathbb{R}$

$\text{ran}(\tan^{-1} x) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

This graph does not repeat but does extend infinitely far in both x - directions

Example 1

If $f(x) = 9x^2 + 1$, find the inverse function and state the largest suitable domain and range of f for f^{-1} to exist.

$$y = 9x^2 + 1$$

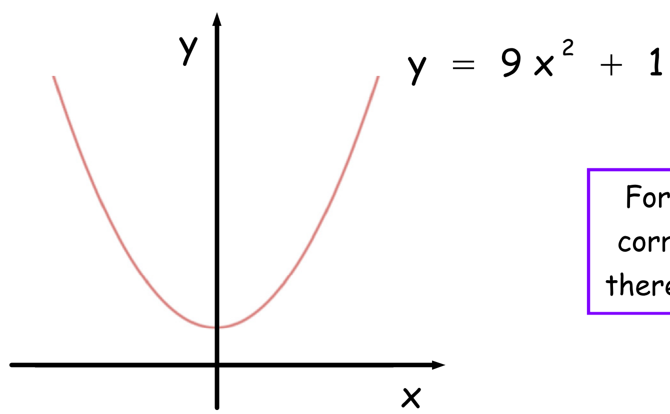
$$\therefore \quad x = 9y^2 + 1$$

$$\Rightarrow \quad 9y^2 = x - 1$$

$$\Rightarrow \quad y^2 = \frac{x-1}{9}$$

$$\Rightarrow \quad y = \pm\frac{\sqrt{x-1}}{3}$$

There are thus two options for a choice of inverse; either can be chosen. Traditionally, the positive root is chosen.



For $x \geq 0$, there is a 1 - 1 correspondence; for $x \leq 0$, there is a 1 - 1 correspondence

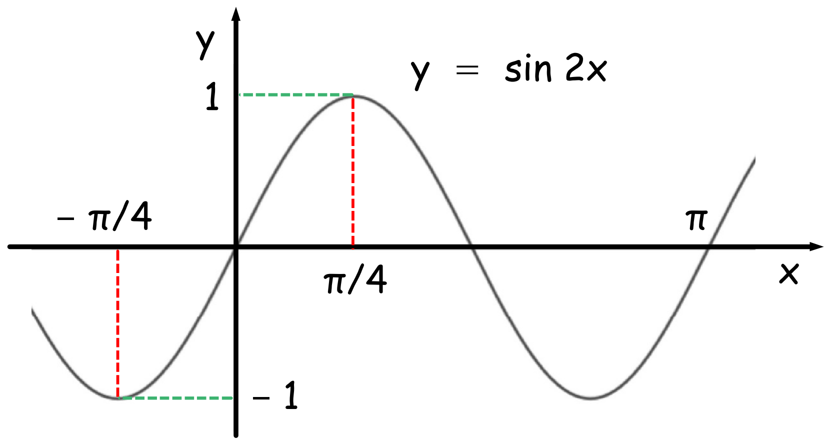
$$\therefore \quad \boxed{f^{-1}(x) = \frac{\sqrt{x-1}}{3}}$$

The domain of f^{-1} is $x \geq 1$ and the range is $y \geq 0$.
Thus,

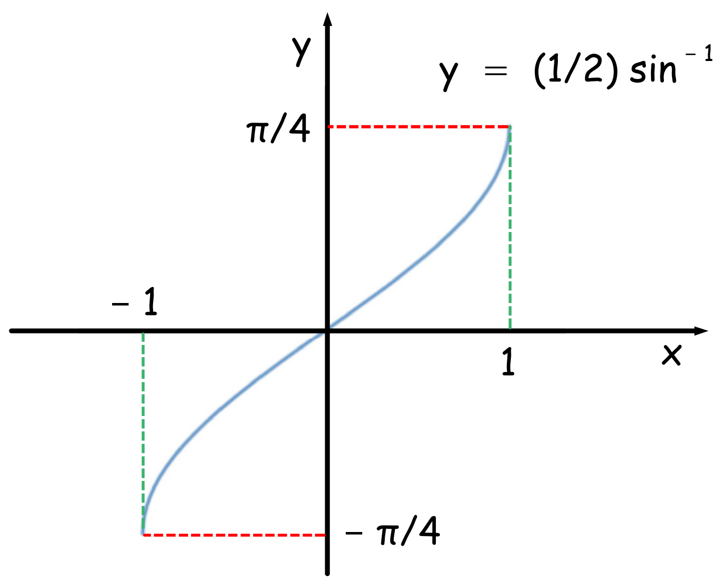
$$\boxed{\text{dom } f: x \geq 0; \text{ ran } f: y \geq 1}$$

Example 2

Sketch the graph of the inverse of $f(x) = \sin 2x$, stating a suitable domain and range for the original function for the inverse to exist.



A suitable inverse will thus exist for $-\pi/4 \le x \le \pi/4$ (with corresponding range $-1 \le y \le 1$).



$$y = \sin 2x$$
$$\therefore \quad x = \sin 2y$$
$$\Rightarrow \quad 2y = \sin^{-1} x$$
$$\Rightarrow \quad y = (1/2)\sin^{-1} x$$

$$\text{dom } f: -\pi/4 \le x \le \pi/4; \text{ ran } f: -1 \le y \le 1$$

$$\text{dom } f^{-1}: -1 \le x \le 1; \text{ ran } f^{-1}: -\pi/4 \le y \le \pi/4$$

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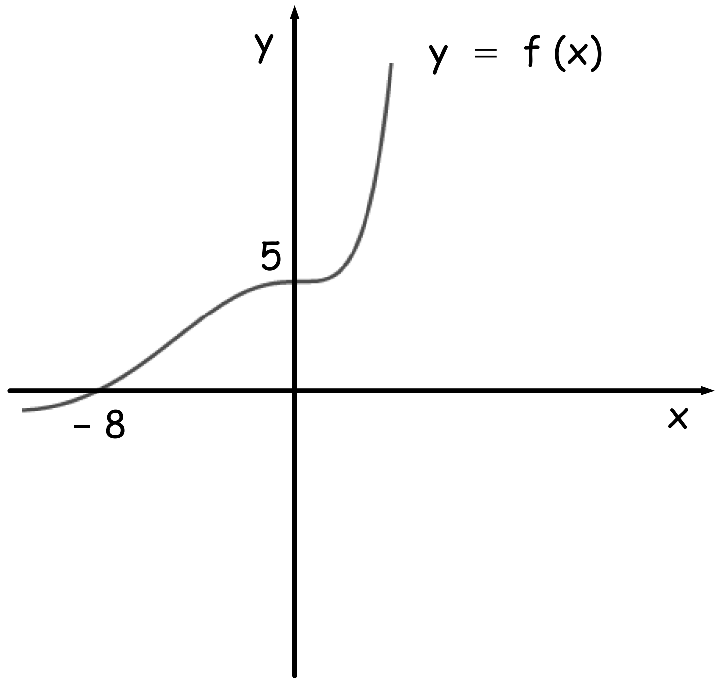
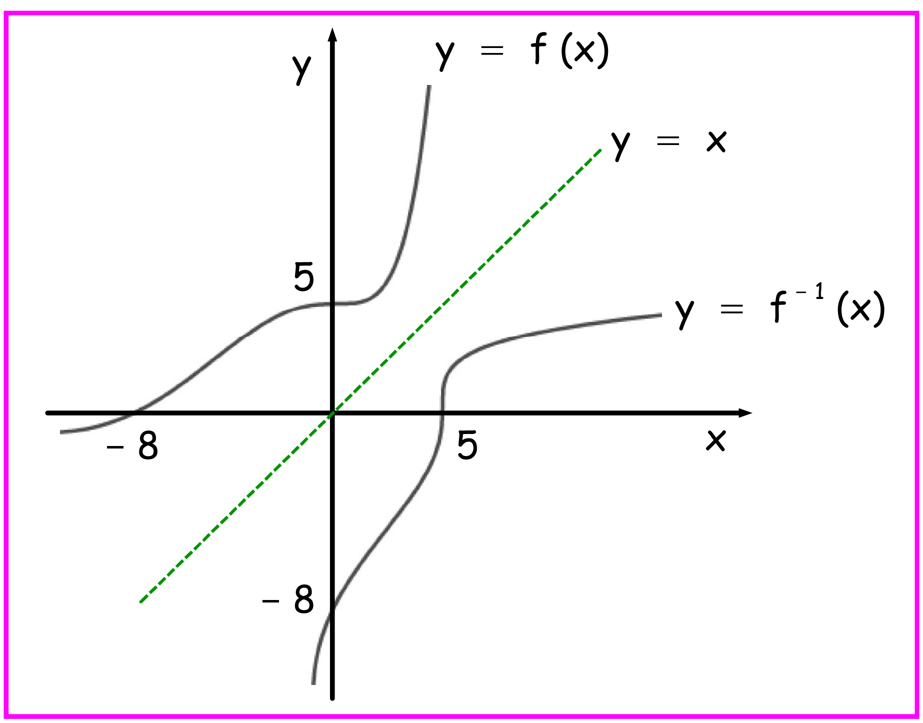
Example 3

Sketch the graph of the inverse of the function $y = f(x)$, part of whose graph is shown below :



The inverse graph is obtained by reflecting the above graph in the line $y = x$:



AH Maths - MiA (2nd Edn.)

- pg. 67 Ex. 5.3 Q 2 a - d, 3.
- pg. 68-9 Ex. 5.4 Q 4 (ii) a - c.

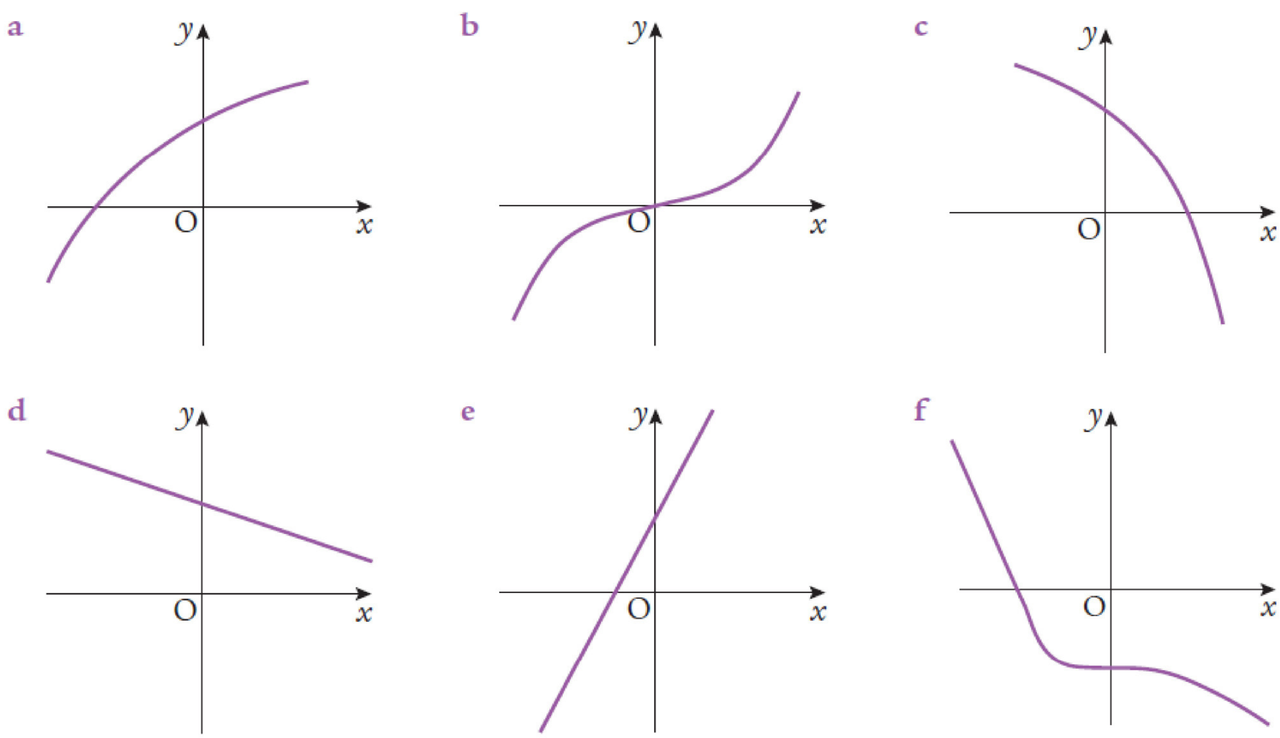
Ex. 5.3

2 With suitable restrictions on the domain and range, each of these functions has an inverse. Find the inverse and state the largest suitable domain and range for the function.

a $f(x) = x^2$ b $f(x) = x^2 - 4$ c $f(x) = (x + 1)^2$

d $f(x) = (2x - 1)^2 - 1$

3 Each of these is the graph of a function.
Sketch the graph of its inverse.



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Ex. 5.4

4 Each of these functions is defined so that an inverse exists.

ii Use it to help you sketch the inverse function

a $f(x) = \sin 3x$

b $f(x) = 3 \sin x$

c $f(x) = 3 + \sin x$

Answers to AH Maths (MiA), pg. 67, Ex. 5.3

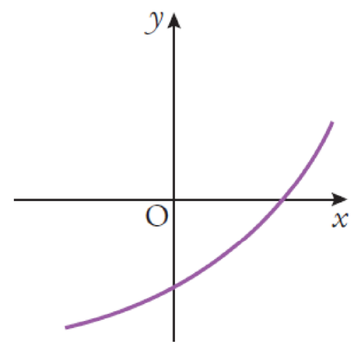
2 **a** $\sqrt{x}$; Domain $x \geqslant 0$; Range $y \geqslant 0$

b $\sqrt{x+4}$; Domain $x \geqslant 0$; Range $y \geqslant -4$

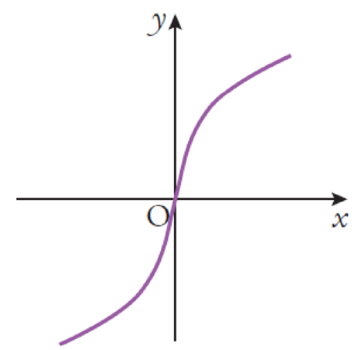
c $\sqrt{x} - 1$; Domain $x \geqslant -1$; Range $y \geqslant 0$

d $\frac{1}{2}\left(\sqrt{x+1} + 1\right)$; Domain $x \geqslant \frac{1}{2}$; Range $y \geqslant -1$

3 **a**



b



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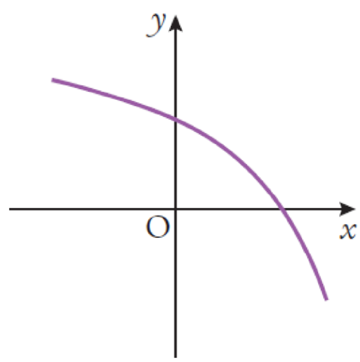
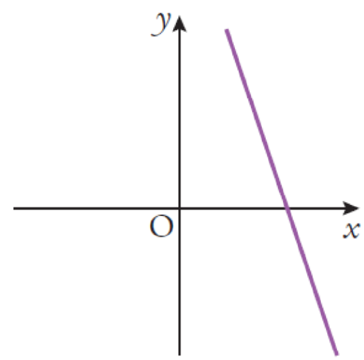
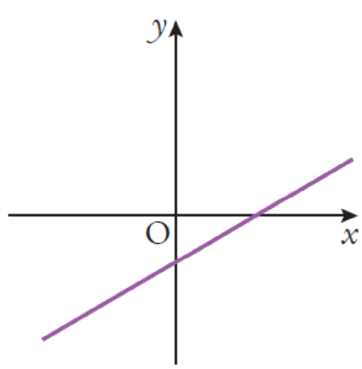


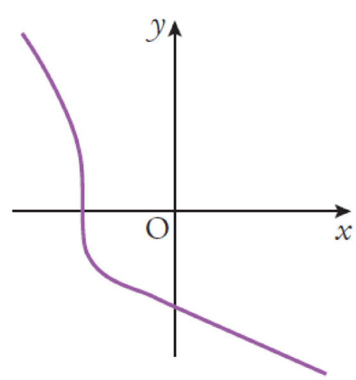
d



e

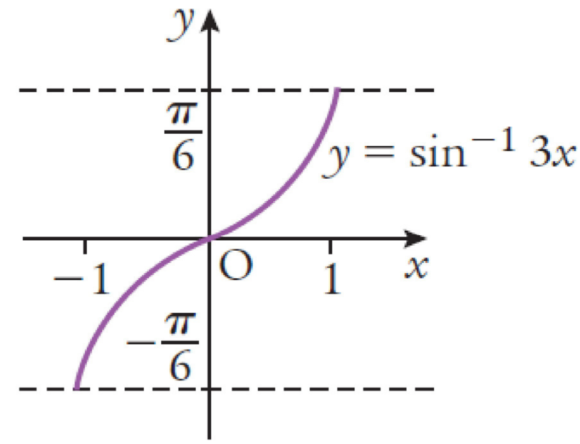


f

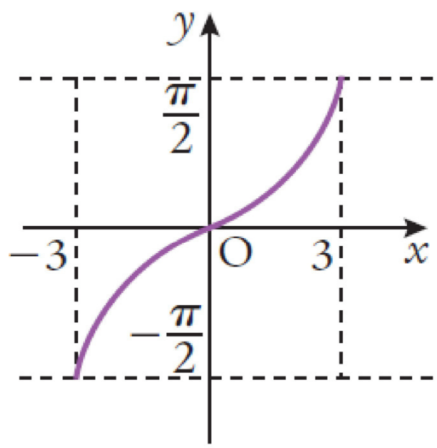


Answers to AH Maths (MiA), pg. 68-9, Ex. 5.4

4 a



b



c