Mathematics

Mathematics 2

Intermediate 2

4728

Spring 1999

HIGHER STILL

Mathematics

Mathematics 2

Intermediate 2

Support Materials

First published 1999

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STAFF NOTES

INTRODUCTION

These support materials for Mathematics were developed as part of the Higher Still Development Programme in response to needs identified at needs analysis meetings and national seminars.

Advice on learning and teaching may be found in *Achievement for All* (SOEID 1996), *Effective Learning and Teaching in Mathematics* (SOEID 1993) and in the Mathematics Subject Guide.

This support package provides student material to cover the content of Mathematics 2 within the Intermediate 2 course. The depth of treatment is therefore more than is required to demonstrate competence in the unit assessment; that is, it goes beyond minimum grade C.
The content of Mathematics 2 (Int 2) is set out in the landscape pages of content in the Arrangements document where the requirements of the unit Mathematics 2 (Int 2) are also stated. Students are unlikely to have met much of the materials of this unit before, i.e. Simultaneous Equations, Trigonometry beyond the right angled triangle or the Statistical content for Intermediate 2.

The material is designed to be directed by the teacher/lecturer, who will decide on the ways of introducing topics and on the use of exercises for consolidation and for formative assessment. The use of a scientific calculator will be necessary for Part B but students should be encouraged to set down all working and, where appropriate, use mental calculations. The use of computers is obviously highly desirable for some of the statistical content of the course.

An attempt has been made to have the ‘easy’ questions at the start of each exercise, leading to more testing questions towards the end of the exercise. While students may tackle most of the questions individually, there are opportunities for collaborative working. Staff may wish to discuss points raised with individuals, groups and the whole class.

The specimen assessment questions at the end of the package are **not** intended to be only at minimum grade C. The National Assessment Bank packages for Mathematics 2 (Int 2) contain questions that meet the requirements of this unit.

This package gives opportunities to practise core skills, particularly the components of the Numeracy core skill, Using Number and Using Graphical Information, and Problem Solving. Information on the core skills embedded in the unit, Mathematics 2 (Int 2) and in the Intermediate 2 course is given in the final version of the Arrangements document. General advice and details of the Core Skills Framework can be found in the Core Skills Manual (HSDU June 1998).

Brief notes of advice on the teaching of each topic are given.

Format of Student Material

- Exercises on Trigonometry
  Checkup for Trigonometry
- Exercises on Simultaneous Linear Equations
  Checkup for Simultaneous Linear Equations
- Specimen Assessment Questions
- Answers for all exercises

TRIGONOMETRY

Introduction

Knowledge of the sines and cosines of angles other than acute angles is needed within this unit to allow students to use the Sine and Cosine Rules in obtuse angled triangles.
The tangent is included for completeness.

An investigative approach, using the sine, cosine and tangent graphs, could be taken, though it should be noted that formal knowledge of the graphs is not required until Mathematics 3.

Exercise 1A takes the students through the drawing of $y = \sin x°$, $y = \cos x°$ and $y = \tan x°$.

Exercise 1A may now be attempted.

The class/group should then discuss when graph is positive and when negative. A summary table of results could then be drawn up, for example:

$y = \sin x°$	positive when	$0 < x < 180$
	negative when	$180 < x < 360$
$y = \cos x°$	positive when	$0 < x < 90$ and $270 < x < 360$
	negative when	$90 < x < 270$
$y = \tan x°$	positive when	$0 < x < 90$ and $180 < x < 270$
	negative when	$90 < x < 180$ and $270 < x < 360$

This should lead to

SIN	ALL
TAN	COS

A class set of graphic calculators will enable students to complete this introduction quickly.

Have students use their trigonometric calculators to look up various trig ratios such as sin 20°, cos 150°, tan 210° etc, each time checking with their graphs as to the validity of their results. (i.e the sign each time).

Exercise 1B may now be attempted.

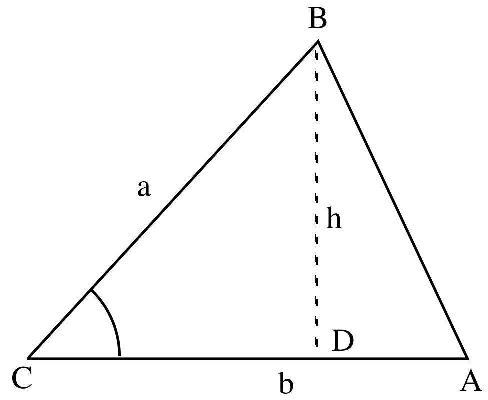
A. Area of a triangle using trigonometry

Students should be taken through the development of the formula:

$$\boxed{\text{Area} = \tfrac{1}{2}ab\sin C}$$

by starting with:

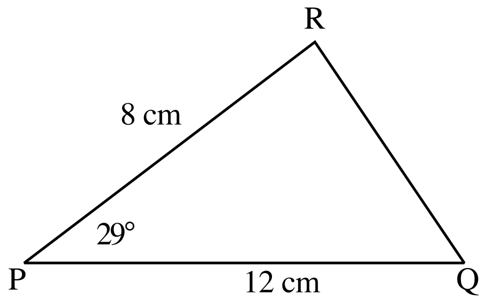
$$\text{Area} = \tfrac{1}{2}b \times h$$



and showing that in triangle BCD, $\Rightarrow \quad \sin C = h/a \quad \Rightarrow \quad h = a\sin C$

$\Rightarrow \quad \text{Area} = \tfrac{1}{2}b \times (a\sin C) \quad \Rightarrow \quad \text{Area} = \tfrac{1}{2}ab\sin C$

Go over an example with different named vertices.



$\Rightarrow \quad \text{Area} = \tfrac{1}{2}qr\sin P$

$\Rightarrow \quad \text{Area} = \tfrac{1}{2}\,8 \times 12 \times \sin 29°$

$\Rightarrow \quad \text{Area} = 23{\cdot}3\text{ cm}^2$

Exercise 2 may now be attempted

Throughout this unit, students should be encouraged to make effective use of their calculators and avoid premature rounding.

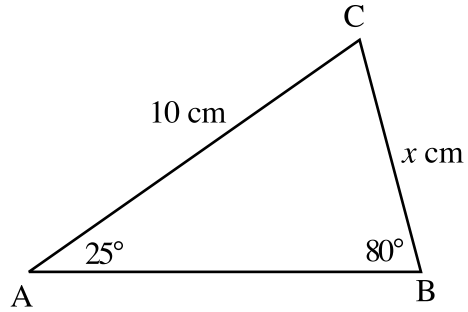
B. Sine Rule

Students should be encouraged to discuss the restrictions on their present trigonometry. A brief revision of right angled triangle trigonometry should follow with a few examples ending with the following:

$$\sin 25° = \frac{x}{10}$$

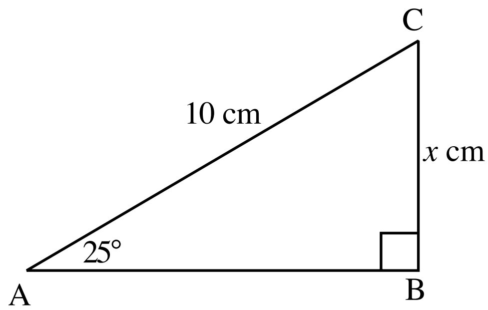
=> $x = 10 \text{ x } \sin 25°$

=> $x = 4{\cdot}23 \text{ cm}$



A slight variation can now be introduced to the above example.

Students should realise that the above method will not work here because it is no longer a right angled triangle.



Encourage students to develop the idea of dropping a perpendicular from C to AB, meeting it at point D and studying the two right angled triangles instead.

Go through this process with the above example and, once accepted, let students attempt Qu 2(a) from Exercise 3 using this method.

Develop the **Sine Rule** with the students.

in triangle ADC

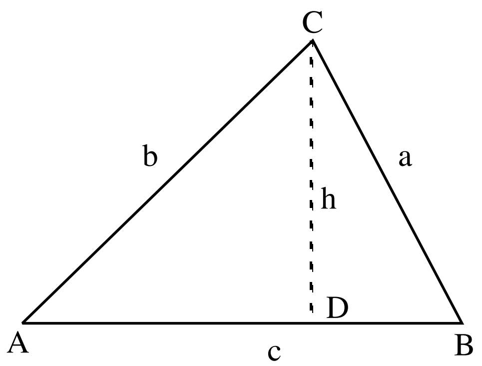
=> $h = b \sin A$

and in triangle BDC

=> $h = a \sin B$

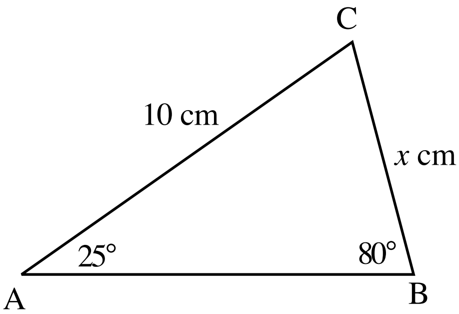
combining these 2 results

=> $a \sin B = b \sin A$



=> $\frac{a}{\sin A} = \frac{b}{\sin B}$ => by symmetry to $\boxed{\frac{a}{\sin A} \quad \frac{b}{\sin B} \quad \frac{c}{\sin C}}$

The initial example can now be solved using the Sine rule to calculate a missing side.



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \quad \frac{x}{\sin 25°} = \frac{10}{\sin 80°}$$

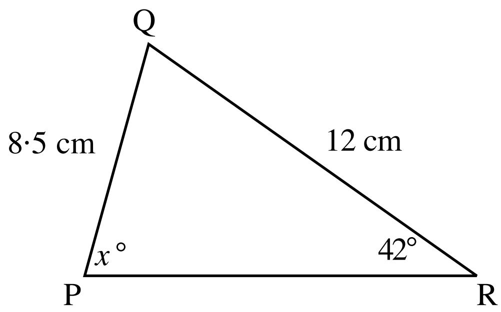
$$\Rightarrow \quad x \sin 80° = 10 \times \sin 25°$$

$$\Rightarrow \quad x = \frac{10 \sin 25°}{\sin 80°} \qquad \Rightarrow \quad x = 4{\cdot}29 \text{ cm}$$

Exercise 3, questions 1 to 4, may now be attempted.

Calculating the missing angle using the Sine rule:

Example:



$$\frac{p}{\sin P} = \frac{q}{\sin Q} = \frac{r}{\sin R}$$

$$\Rightarrow \quad \frac{12}{\sin x°} = \frac{8{\cdot}5}{\sin 42°}$$

$$\Rightarrow \quad 8{\cdot}5 \sin x° = 12 \times \sin 42°$$

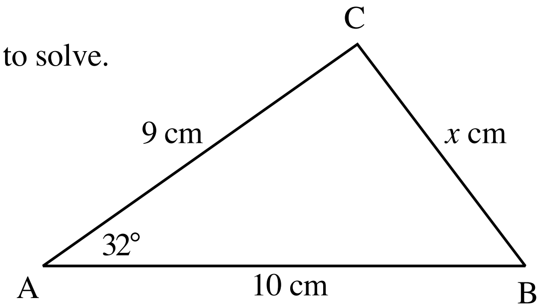
$$\Rightarrow \quad \sin x = \frac{12 \sin 42°}{8{\cdot}5} = 0{\cdot}945$$

$\Rightarrow \quad x = 70{\cdot}8°$ (remind about the use of $\sin^{-1}$)

Exercise 3, questions 5 to 10, may now be attempted.

C. Cosine rule.

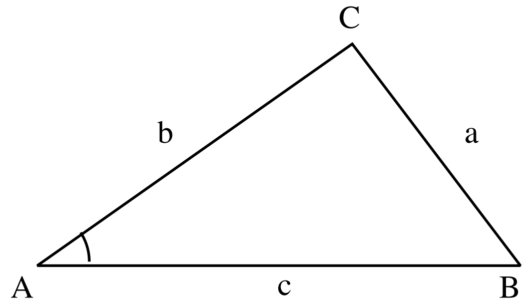
Students should be given the following example to solve. They should soon realise it cannot be solved by use of the Sine rule.



The Cosine rule should now be introduced

Define the Cosine rule:

$$\boxed{a^2 = b^2 + c^2 - 2bc \cos A}$$



Go over the 'format' and try to get students to come up with a similar formula for

Note: 'brackets' could be introduced to prevent mishandling of the values of a, b and c. i.e.

$$a^2 = b^2 + c^2 - (2bc \cos A)$$

Some other named triangles could be presented/displayed and students asked to state the corresponding formula for any required side.

The initial question can now be solved.

$\Rightarrow \quad a^2 = b^2 + c^2 - (2bc \cos A)$

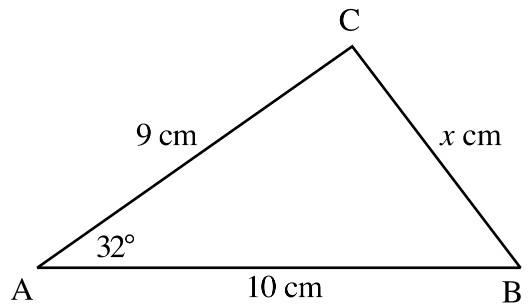
$\Rightarrow \quad a^2 = 9^2 + 10^2 - (2 \times 9 \times 10 \times \cos 32°)$

$\Rightarrow \quad a^2 = 81 + 100 - (180 \times 0{\cdot}848..)$

$\Rightarrow \quad a^2 = 181 - 152{\cdot}64..$

$\Rightarrow \quad a^2 = 28{\cdot}35...$

$\Rightarrow \quad a = 5{\cdot}32$ cm



Exercise 4A, questions 1 and 2, may now be attempted.

An example involving an obtuse angled triangle should now be introduced

$\Rightarrow \quad p^2 = q^2 + r^2 - (2qr \cos P)$

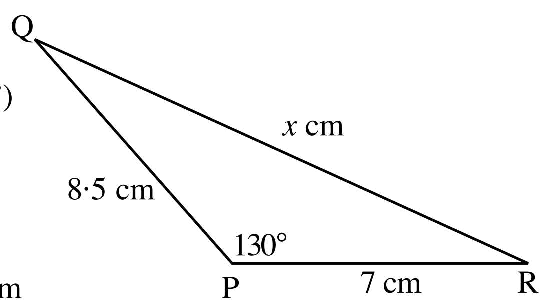
$\Rightarrow \quad p^2 = 7^2 + 8{\cdot}5^2 - (2 \times 7 \times 8{\cdot}5 \times \cos 130°)$

$\Rightarrow \quad p^2 = 49 + 72{\cdot}25 - (119 \times (-0{\cdot}642...))$

$\Rightarrow \quad p^2 = 121{\cdot}25 - (-76{\cdot}49...)$

$\Rightarrow \quad p^2 = 121{\cdot}25 + 76{\cdot}49...$

$\Rightarrow \quad p^2 = 197{\cdot}7... \qquad \Rightarrow \quad p = 14{\cdot}1$ cm

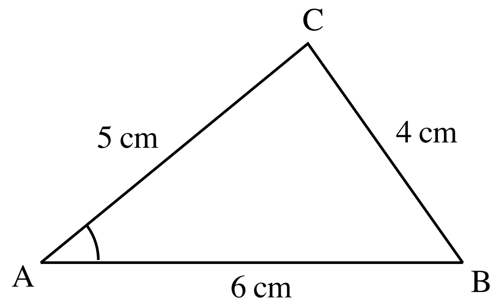


Exercise 4A, questions 3 to 6, may now be attempted.

Calculating the missing angle using the Cosine rule.

Exercise 4B covers material listed in Helvetica font and is therefore beyond Grade C.

Students should be shown how the formula for the Cosine rule can be rearranged:



$$a^2 = b^2 + c^2 - (2bc \cos A)$$

$$\Rightarrow \quad 2bc \cos A = b^2 + c^2 - a^2$$

$$\Rightarrow \quad \boxed{\cos A = \frac{b^2 + c^2 - a^2}{2bc}}$$

Example: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

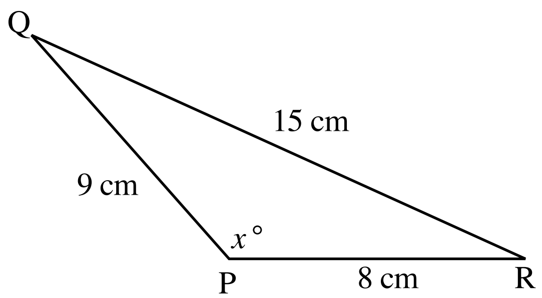
$$\Rightarrow \quad \cos A = \frac{5^2 + 6^2 - 4^2}{2 \times 5 \times 6} = \frac{45}{60} = 0{\cdot}75$$

$$\Rightarrow \quad A = 41{\cdot}4^\circ$$

Students can be asked to define similar formulae for cos B and cos C and for other triangles.

Exercise 4B, questions 1 and 2 may now be attempted.

Now introduce a problem where the cosine is negative.



$$\cos P = \frac{q^2 + r^2 - p^2}{2qr}$$

$$\Rightarrow \quad \cos P = \frac{8^2 + 9^2 - 15^2}{2 \times 8 \times 9}$$

$$\Rightarrow \quad \cos P = \frac{-80}{144} = -0{\cdot}555...$$

Students should understand that when the cosine is negative, the angle must be obtuse. However, not all calculators will give the obtuse angle immediately.
At this point, ask the students to feed in –0·555... into their calculators and press ‘$\cos^{-1}$’

=> either 123·7° appears and there is no problem

=> or –56·3° appears and the student should be told to simply +180° to get the correct answer of 123·7°.

Exercise 4B, questions 3 to 6 may now be attempted.

The checkup exercise for Trigonometry may now be attempted.

SIMULTANEOUS LINEAR EQUATIONS

A. Construction of Formulae to Describe a Linear Relationship

Two examples can be given as an introduction – one showing formula of the type $y = \mathrm{m}x$, passing through the origin, the other of the type $y = \mathrm{m}x + \mathrm{c}$, crossing the the y axis at (0,c).

On board:

Example 1. Tomatoes sold in 2 kg packets. Compare no. of bags sold with their weight.

(a) Copy and complete a table.

Number of packets (N)	1	2	3	4	5	6
Weight of tomatoes (W)	2					

(b) Write a formula. ($W = 2N$)

(c) Find the weight of 20 packets (40 kg)

(d) Draw straight line graph through O. (Watch scales and naming axes)

Example 2. Hiring a chain saw. Basic charge £5, plus £3 per day

(a) Copy and complete a table.

Number of Days (D)	1	2	3	4	5	6
Charge (C)	8	11				

(b) Write a formula. ($C = 3D + 5$)

(c) Find the charge for 10 days. (£35)

(d) Plot points from table and draw straight line graph.
Extend line to pass through (0,5) (Watch scales and naming axes)

The significance of the "5" in $C = 3D + 5$ should be pointed out.

Other lines such as $C = 2D + 4$, $C = 6D - 1$ etc. should be considered and students asked where they would cross the vertical (D) axis.

Exercise 1 may now be attempted.

B. Solving Simultaneous Equations in Two Variables Graphically

Introduction : Drawing Straight Line Graphs (Revision)

- choose three points which fit the equation of the line
- plot the points on squared paper
- draw a straight line through them

Example 1. $y = 4x$

- Choose say, $x = 0$, $x = 2$ and $x = 4$ to obtain $y = 0$, $y = 8$ and $y = 16$
- Plot (0,0) (2,8) (4,16)
- Join the points and extend line into negatives (Watch scales and naming axes)

Example 2. $y = 2x + 1$

- Choose say, $x = 0$, $x = 3$ and $x = 5$ to obtain $y = 1$, $y = 7$ and $y = 11$
- Plot (0,1) (3,7) (5,11)
- Join the points and extend line (Watch scales and naming axes)

Example 3. $2x - y = 5$ Not quite as easy

- Choose say, $x = 0$, $x = 3$ and $x = 5$
  * solve 3 simple equations to obtain $y = –5$, $y = 1$ and $y = 5$
- Plot (0,–5) (3,1) (5,5)
- Join the points and extend line (Watch scales and naming axes)

Exercise 2 may now be attempted.

Finding the Point of Intersection of 2 Straight Lines

Draw graphs of these equations to solve the pairs of simultaneous equations:

Example 1. $x + y = 3$
$y = x + 1$

As in Exercise 2 –

Choose suitable points for $x + y = 3$ and draw the line on a coordinate diagram on squared paper.

Now, choose suitable points for $y = x + 1$ and draw the line on the same coordinate diagram.

Both lines are seen to cross at the point (1,2)

Example 2. $x + y = 2$
$3x - 2y = 11$

Choose suitable points for $x + y = 2$ and draw the line on a coordinate diagram on squared paper.

Now, carefully choose suitable points for $3x - 2y = 11$ and draw the line on the same coordinate diagram.

Both lines are seen to cross at the point (3,–1)

Exercise 3 may now be attempted.

C. The Significance of the Point of Intersection of 2 Graphs

A straightforward example, such as the one below, should be given as an introduction
.

Example: 2 ice creams and 1 ice lolly cost £1·30

5 ice creams and 3 ice lollies cost £3·40

Students should be encouraged to construct formulae to represent the relationships. i.e. $2x + y = 1{\cdot}30$ and $5x + 3y = 3{\cdot}40$, where x is the cost of 1 ice cream and y is the cost of 1 ice lolly. The graphs can then be drawn and the significance of the point of intersection can be stressed.

A second example. as shown on the next page, could be used.

Prices on the Dunoon Car Ferry:

3 cars and 1 motor cycle cost £35 for a single crossing

2 cars and 3 motor cycles cost £35 for a single crossing

What is the cost for my own car ?

The following example could be used to introduce exercise 4A

Example Mr. Adam and Mrs. Bryce each own a locksmith's shop.

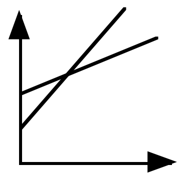
For emergency repairs, Mr. Adam charges £15 per hour plus a call–out fee of £10.

For the same repairs, Mrs. Bryce charges £10 per hour plus a call–out fee of £20.

(a) Make two tables to show the prices for up to a 5 hour call–out at Adam's and Bryce's.

Adam	0	1	2	3	4	5
	10	25	40	55	70	85
Bryce	0	1	2	3	4	5
	20	30	40	50	60	70

(b) Draw the straight line graph for both locksmith companies on the same coordinate diagram.



(c) For how many hours call–out is the cost the same at both shops ? (2 hrs)

(d) If you needed a call–out for a job which you knew would take a long time to complete, which shop would you phone to in order to save money ?

(Mrs. Bryce)

Exercise 4A may now be attempted.

The following example could be used to introduce exercise 4B

Example 2 C.D.'s and 1 cassette cost £45. 1 C.D. and 4 cassettes cost £40.
Let the cost of a C.D. be £x and the cost of a cassette be £y.

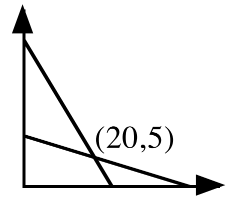
(a) Write down two equations in terms of x and y.

($2x + y = 45$; $x + 4y = 40$)

(b) Draw the two straight lines which the equations represent on the same coordinate diagram using suitable points on each line.

(c) Use your graph to find the cost of a C.D. and the cost of a cassette.

(£20) (£5)



Exercise 4B may now be attempted.

D. Solving Simultaneous Linear Equations Algebraically

Eliminating x or y by Addition, or by first Multiplying both sides of <u>one</u> Equation by –1.

Example 1. Solve: $x + y = 14$

$x - y =$

Look for the coefficient of one term being the negative of another.

Here is the ideal situation .... $+y$ and $-y$.

Simply ADD both equations to eliminate y.

$x + y = 14$ – **1**

$x - y = 8$ – **2**

1 + 2 => $2x = 22$

$\underline{\underline{x = 11}}$

Substitute $x = 11$ into either equation **1** or **2**. e.g in **1**

Giving $11 + y = 14$

$\underline{\underline{y = 3}}$

Point of intersection (11,3) CHECK by substitution!

Example 2. Solve: $x + 3y = 10$

$x - 3y = 4$

Look for the coefficient of one term being the negative of another.

Again, the ideal situation .... $+3y$ and $-3y$.

Simply ADD both equations to eliminate y.

$x + 3y = 10$ – **1**

$x - 3y = 4$ – **2**

$2x = 14$

$\underline{\underline{x = 7}}$

Substitute $x = 7$ into either equation **1** or **2**. e.g in **1**

Giving $7 + 3y = 10$

$3y = 3$

$\underline{\underline{y = 1}}$

Point of intersection (7,1) CHECK by substitution!

Example 3. Solve: $x + 2y = 4$ – **1**

$x - 3y = -1$ – **2**

NOT quite the ideal situation .... $+2y$ and $-3y$ not good.

Multiply BOTH SIDES of equation **2** by –1.

Look for the coefficient of one term being the negative of another.

$x + 2y = 4$ – **1**

2 becomes...... $-x + 3y = 1$ – **3**

Now, the ideal situation .....an x and a $-x$

ADD both equations to eliminate x.

$$x + 2y = 4 \quad - \quad \mathbf{1}$$
$$-x + 3y = 1 \quad - \quad \mathbf{3}$$
$$5y = 5$$
$$\underline{\underline{y = 1}}$$

Substitute $y = 1$ into either equation **1** or **2**. e.g in **1**

Giving $x + 2 = 4$

$$\underline{\underline{x = 2}}$$

Point of intersection (2,1) CHECK by substitution!

Exercise 5A may now be attempted.

Eliminating x or y by first Multiplying both sides of <u>one</u> Equation by a Suitable Number, then Adding.

Example 1. Solve:

$$x + 2y = 8 \quad - \quad \mathbf{1}$$
$$3x - y = 17 \quad - \quad \mathbf{2}$$

Look for the coefficient of one term being the negative of another.

NOT quite the ideal situation .... $+2y$ and $-y$ not good.

Multiply BOTH SIDES of equation **2** by 2.

$$x + 2y = 8 \quad - \quad \mathbf{1}$$

2 becomes...... $6x - 2y = 34 \quad - \quad \mathbf{3}$

Now, the ideal situation .....a $2y$ and a $-2y$

ADD both equations to eliminate y.

$$x + 2y = 8 \quad - \quad \mathbf{1}$$
$$6x - 2y = 34 \quad - \quad \mathbf{3}$$
$$7x = 42$$
$$\underline{\underline{x = 6}}$$

Substitute $x = 6$ into either equation **1, 2** or **3**. e.g in **1**

Giving $6 + 2y = 8$

$$2y = 2$$
$$\underline{\underline{y = 1}}$$

Point of intersection (6,1) CHECK by substitution!

Example 2. Solve:

$$3x + 2y = 13 \quad - \quad \mathbf{1}$$
$$x + y = 5 \quad - \quad \mathbf{2}$$

This can be best be solved by either :– (i) multiplying **2** by –3 or (ii) multiplying **2** by –2

Answer (3,2)

Exercise 5B may now be attempted.

Eliminating x or y by first Multiplying both sides of <u>one</u> or <u>both</u> Equations by a Suitable Number, then Adding.

Example . Solve: $2x - 3y = 7$ – **1**

$3x - 2y = 13$ – **2**

Point out that in a situation like this, it is not enough to change one of the equations.

BOTH equations have to be be multiplied !

In this case, suggestions should be sought – e.g. multiply **1** by 2 **and** **2** by –3.

1 becomes....... $4x - 6y = 14$ – **3**

2 becomes...... $-9x + 6y = -39$ – **4**

ADD both equations to eliminate y.

$$-5x = -25$$

$$\underline{\underline{x = 5}}$$

Substitute $x = 5$ into either equation **1, 2, 3** or **4**. e.g in **1**

Giving $10 - 3y = 7$

$$-3y = -3$$

$$\underline{\underline{y = 1}}$$

Point of intersection (5,1) CHECK by substitution !

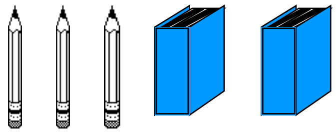
Exercise 5C may now be attempted.

E. Using Simultaneous Equations to Solve Problems

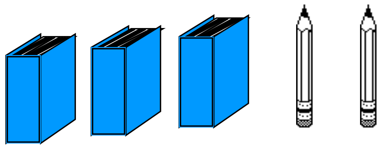
This part is a continuation of the solving of simultaneous equations in Exs. 5A, 5B and 5C.

Now though, a situation (or a picture) is given, from which both equations have to be derived before being solved.

Some problems may be simplified by drawing –



Total Cost = £10·30



Total Cost = £15·20

i.e. $3p + 2b = 10{\cdot}30$
$2p + 3b = 15{\cdot}20$

Solve as before. (1 pencil = 10 pence 1 = book £5)

From this, the cost of 20 books and 14 pencils etc. can be found

Exercise 5D may now be attempted.

The checkup exercise may now be attempted.

STUDENT MATERIALS

CONTENTS

Trigonometry

Introduction: Sine, Cosine and Tangents of non-acute angles

A. Area of a triangle

B. Sine Rule

C. Cosine Rule

Checkup

Simultaneous Equations

A. Construction of Formulae

B. Solving Simultaneous Equations (Graphically)

C. Solving Simultaneous Equations (Algebraically)

Checkup

Specimen Assessment Questions

Answers

TRIGONOMETRY

By the end of this set of exercises, you should be able to

(a) calculate the area of a triangle using trigonometry

(b) solve problems using Sine and Cosine rules.

TRIGONOMETRY

Introduction: Sine, Cosine and Tangent Graphs

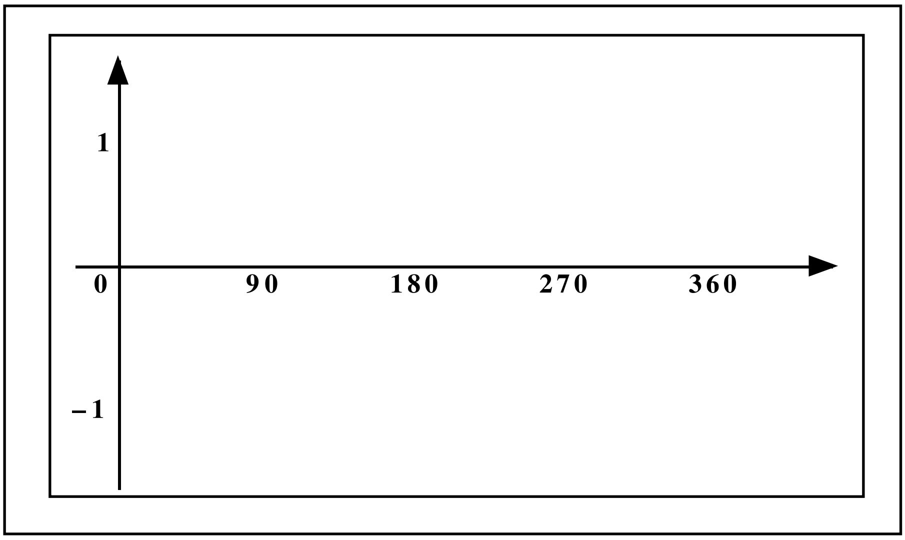
Exercise 1A

1. The Sine Graph
   (a) Make a copy of this table and use your calculator to help fill it in, giving each answer correct to 2 decimal places.

x	0°	20°	40°	60°	80°	90°	100°	120°	140°	160	180°
sin x°	0·00	0·34	0·64	0·87	0·98	1·00	...	...	...	...	...

x	200°	220°	240°	260°	270°	280°	300°	320°	340°	360
sin x°	...	...	...	...	...	...	...	...	...	...

   (b) Use a piece of 2 mm graph paper to draw a set of axes as illustrated below.



   (c) Plot as accurately as possible the 21 points from your table.

   (d) Join them up smoothly to create the graph of the function $y = \sin x°$.

2. Repeat question 1 (a) to (d) for the function $y = \cos x°$

3. Repeat for the graph of $y = \tan x°$ (a different scale will be required for the vertical axis). (These graphs will be studied later).

Sine, Cosine and Tangents of angles other than acute angles

Exercise 1B

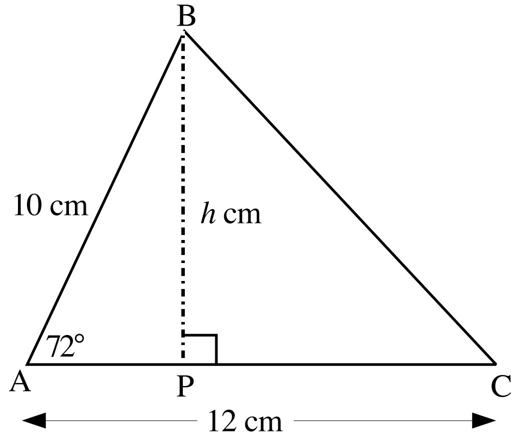
1. Use your calculator to find the following trigonometric ratios.
   Give each answer correct to 3 decimal places.

(a) sin 25°	(b) cos 95°	(c) tan 107°	(d) sin 200°
(e) cos 315°	(f) tan 181°	(g) cos 240°	(h) sin 330°
(i) tan 225°	(j) sin 300°	(k) tan 315°	(l) cos 500°
(m) tan (–75°)	(n) cos (–200°)	(o) sin 360°	(p) cos 360°

A. Area of a Triangle using Trigonometry.

Exercise 2

1. In this question you are being asked to calculate the area of triangle ABC, using two methods.



Method 1 (a) Use basic right angled trigonometry on triangle ABP to calculate the height BP (= h cm).

(b) Now use the formula Area = $^1/_2$ (base x height) to calculate the area of ΔABC.

Method 2 Use the formula:

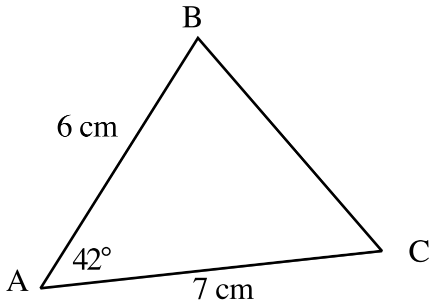
$\boxed{\text{Area} = {}^1/_2\, b\, c \sin A}$ with b = 12 cm, c = 10 cm and angle A = 72°

to calculate the area of triangle ABC.

Did you obtain the same answer? Which method was the faster?

2. Use the formula Area = $^1/_2\, a\, b \sin C$ to calculate the areas of the following six triangles: (Give all answers correct to 1 decimal place).

(a)



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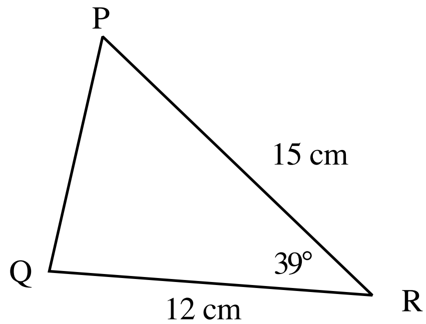

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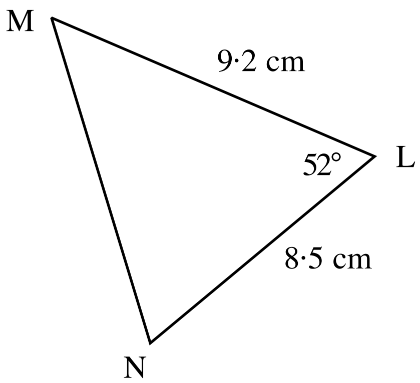

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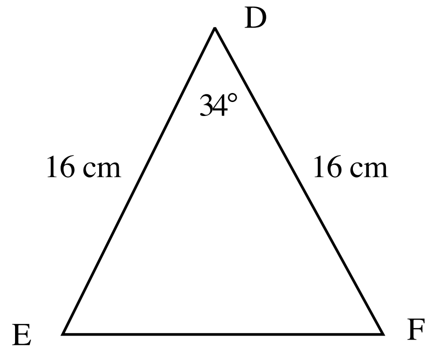
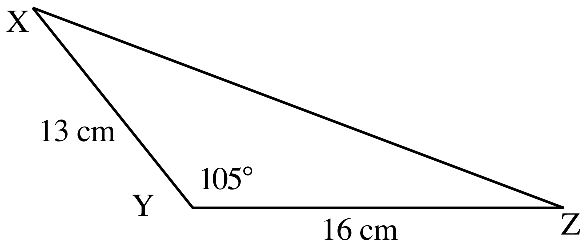


(e)



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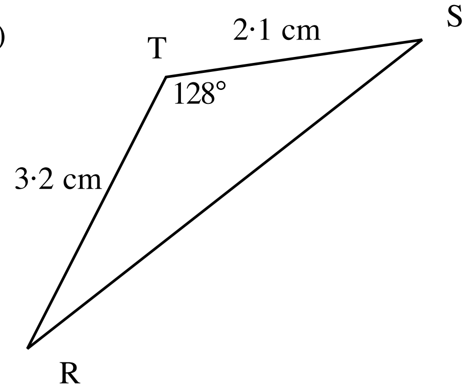
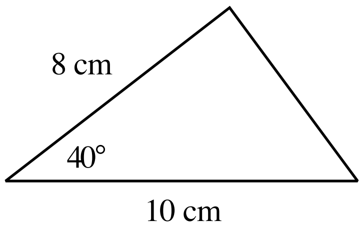
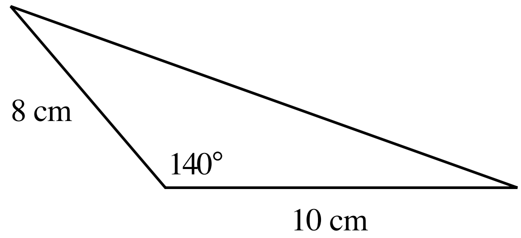


3. Calculate the areas of the following two triangles:

(a)



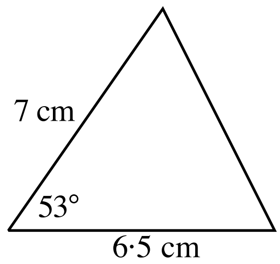
(b)



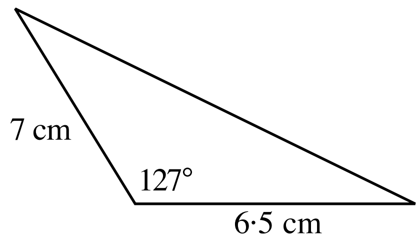
What do you notice?

4. Calculate the areas of the following two triangles:

(a)



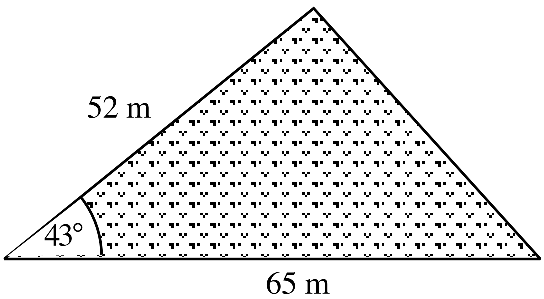
(b)



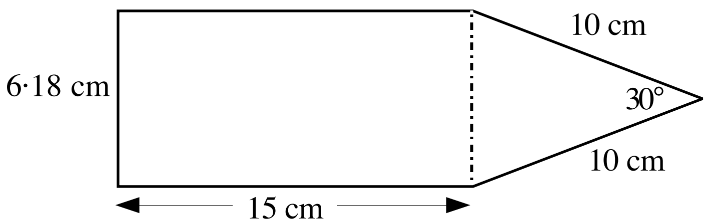
What do you notice? Can you explain your answers to questions 3 and 4?

5. Shown is a sketch of Farmer Giles' triangular field.

Calculate its area in square metres.

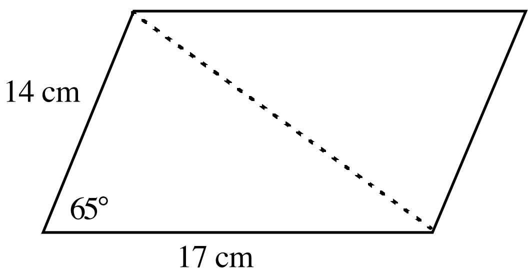


6. Calculate the area of this pentagon:

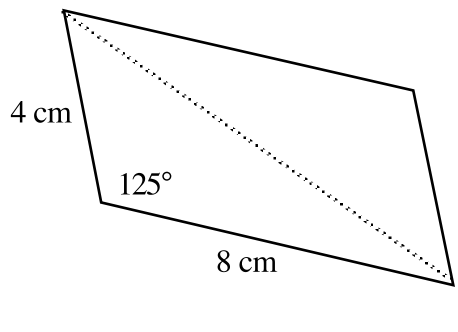


7. Calculate the areas of the following two parallelograms:

(a)



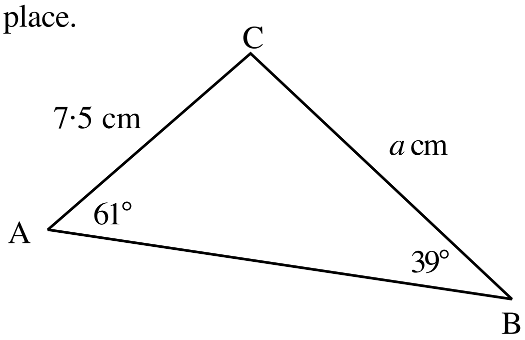
(b)



B . Sine Rule.

Exercise 3

In this exercise, give all answers correct to 1 decimal place.



1. Copy and complete the following:

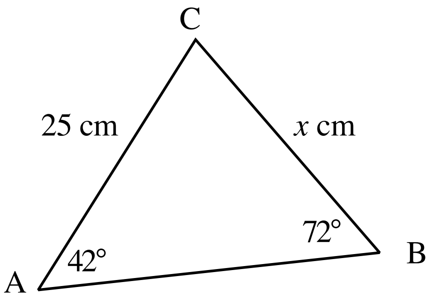
$$\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \left(\frac{c}{\text{Sin C}}\right)$$

$$\frac{a}{\text{Sin } 61°} = \frac{7{\cdot}5}{\text{Sin } 39°}$$

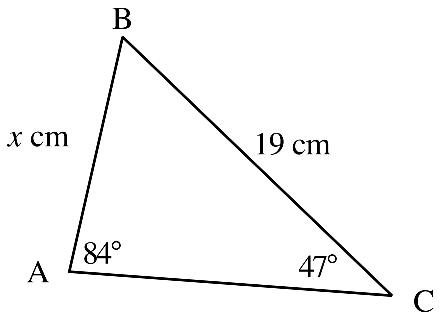
$$\Rightarrow \quad a = \frac{7{\cdot}5 \text{ x Sin } 61°}{\text{Sin } 39°} = \boxed{} \text{ cm}$$

2. Use the Sine Rule in each of the following to calculate the size of the side marked x cm.

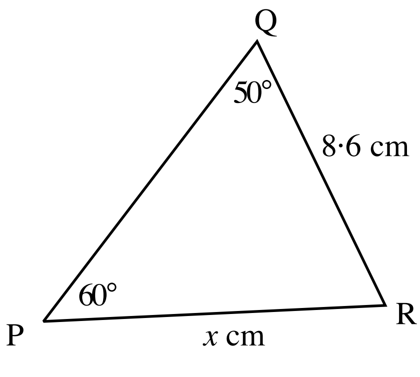
(a)



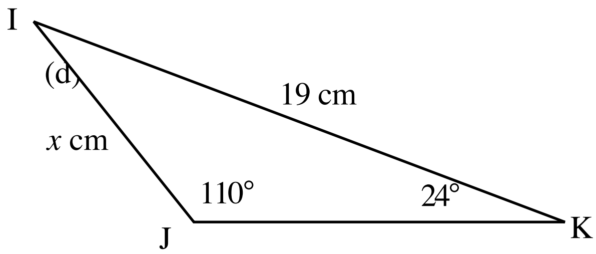
(b)



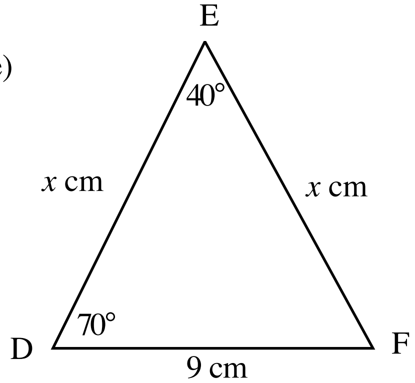
(c)



(d)



(e)



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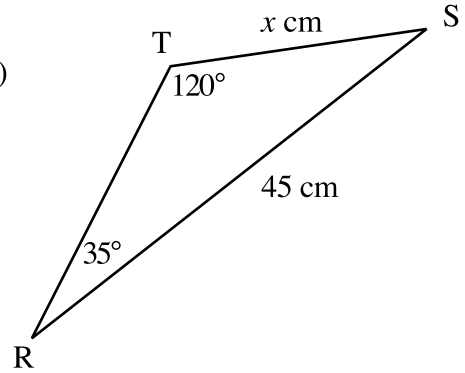
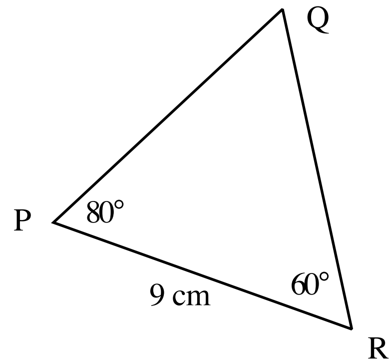


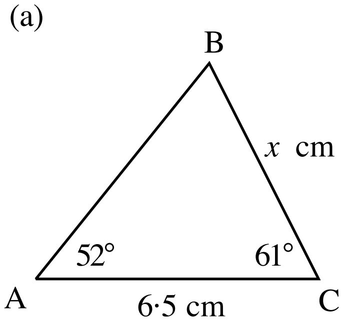
3. (a) Write down the size of $\angle$PQR.

   (b) Use the Sine rule to calculate the length of the line QR.

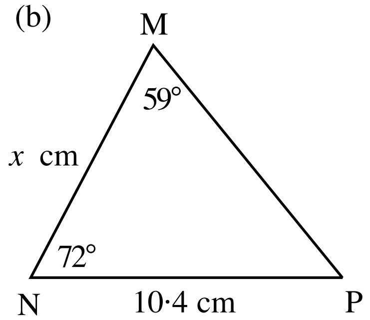


4. In each of the following, calculate the size of the third angle first before attempting to calculate the length of the side marked x cm.

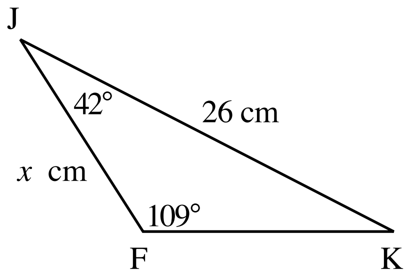
(a)



(b)



(c)



5. Copy and complete:

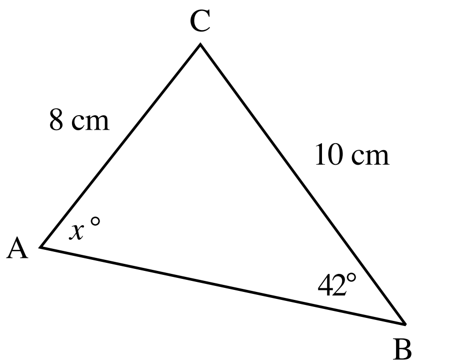
$$\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \left(\frac{c}{\text{Sin C}}\right)$$

$$\frac{10}{\text{Sin } x°} = \frac{8}{\text{Sin } 42°}$$

$$\Rightarrow \quad 8 \text{ Sin } x° = 10 \text{ Sin } 42°$$

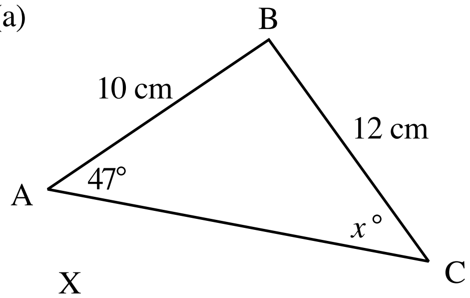
$$\Rightarrow \quad \text{Sin } x = \frac{10 \text{ Sin } 42°}{8} = \boxed{0\cdot\ .....}$$

$$\Rightarrow \quad x = \boxed{}$$

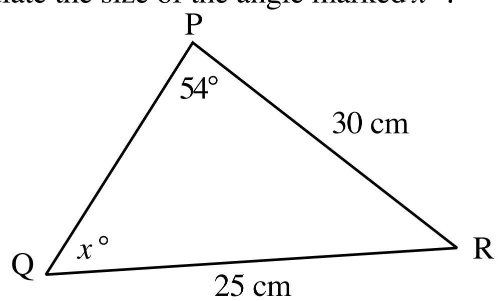


6. Use the Sine Rule in each of the following to calculate the size of the angle marked x°.

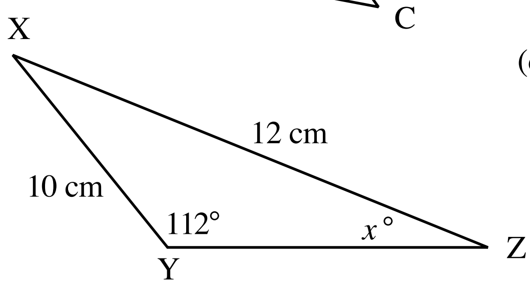
(a)



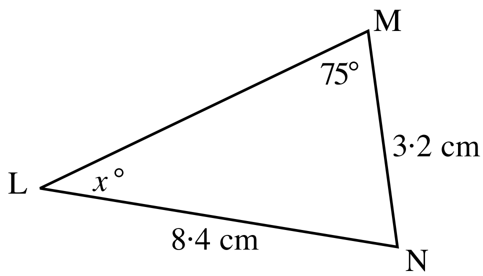
(b)



(c)

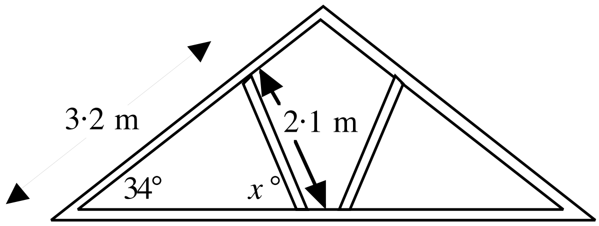


(d)



7. The diagram shows a roof truss.

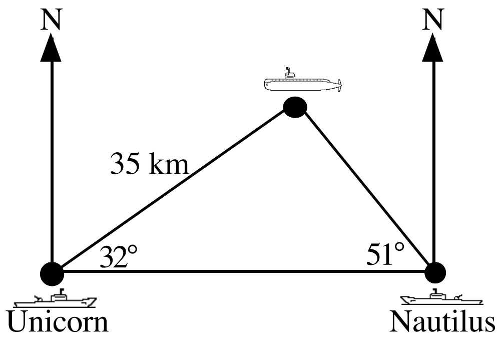
   Calculate the size of the angle marked x ° between the wooden supports.



8. H.M.S. Nautilus lies East of H.M.S. Unicorn.

   The diagram shows where an enemy submarine is in relation to the two ships.

   Calculate how far the submarine is from H.M.S. Nautilus.



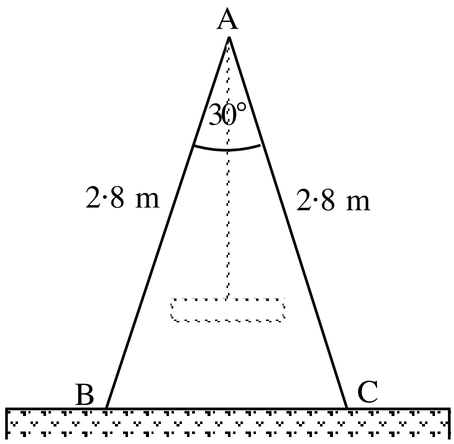
9. This is the metal frame used to support and hold a child's swing.

   It is in the shape of an isosceles triangle.

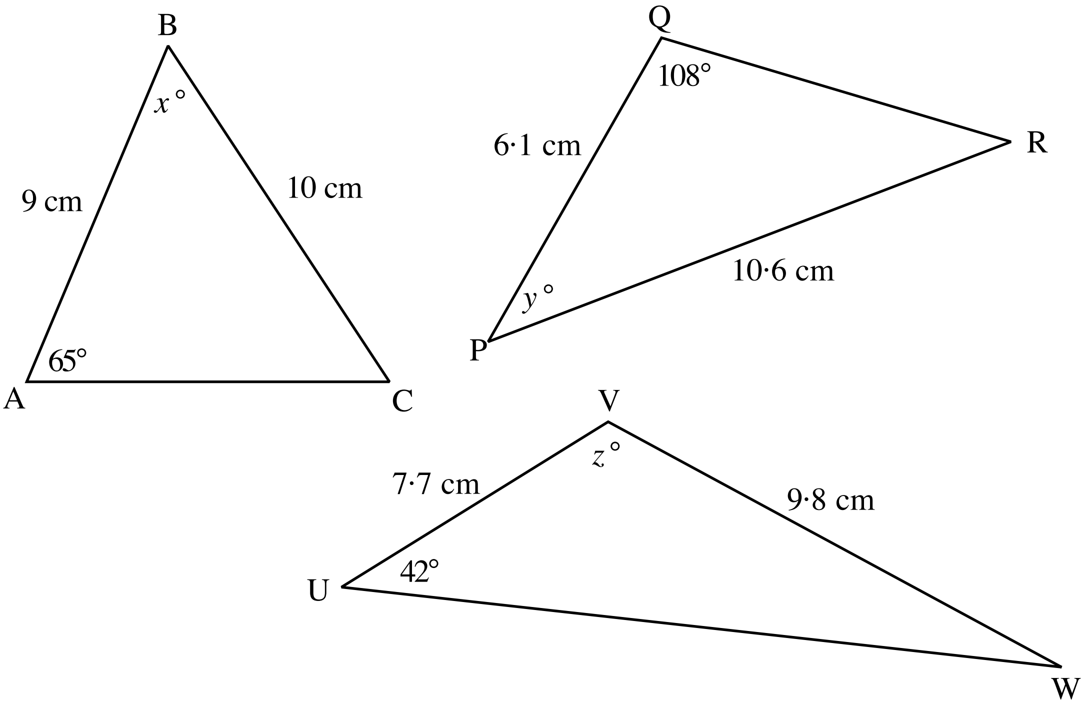
   (a) Calculate the size of ∠ABC.

   (b) Use the Sine rule to calculate how far apart points B and C are. (Answers to 2 decimal places)

   (c) Draw a vertical line through A, creating two right angled triangles and use right angled trigonometry to check your answer to part (b).



10. Calculate the size of the angles marked x °, y ° and z °. (careful!)



C. Cosine Rule

Exercise 4A

1. Copy and complete the following:

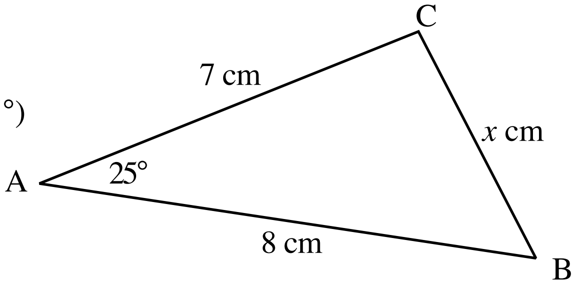
$$a^2 = b^2 + c^2 - (2bc \cos A)$$

$$\Rightarrow \ x^2 = 7^2 + 8^2 - (2 \times 7 \times 8 \times \cos 25°)$$

$$\Rightarrow \ x^2 = \ldots + \ldots - (\ldots\ldots)$$

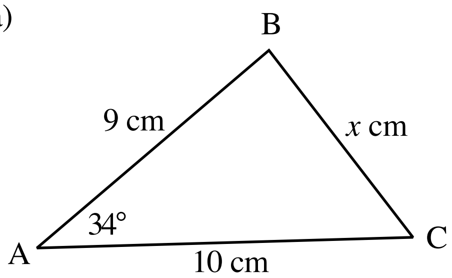
$$\Rightarrow \ x^2 = \ldots\ldots$$

$$\Rightarrow \ x = \boxed{}$$

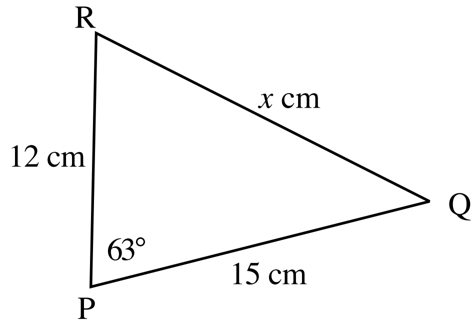


2. Use the Cosine rule to calculate the size of each side marked x cm here.

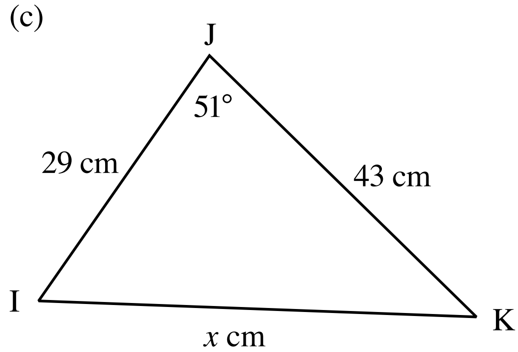
(a)



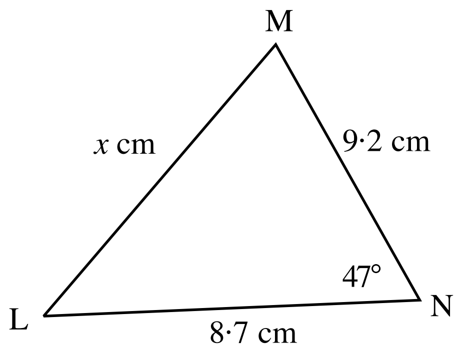
(b)



(c)



(d)



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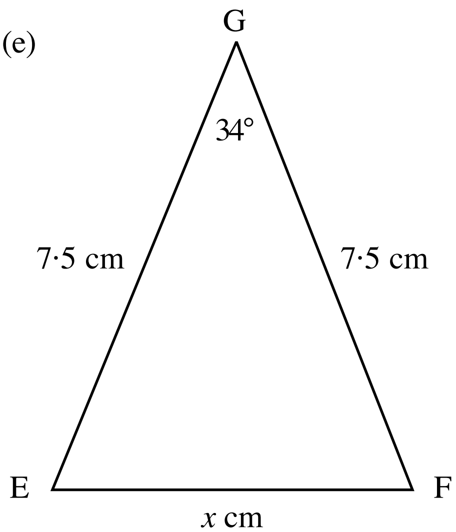

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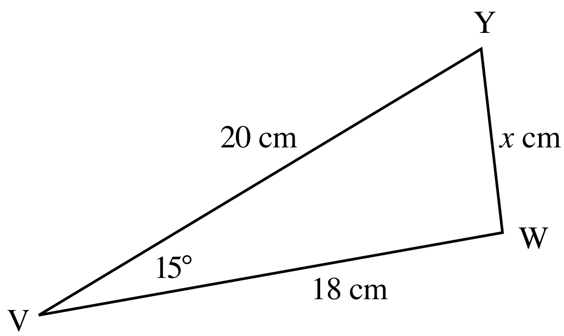


3. Copy and complete the following:

$$a^2 = b^2 + c^2 - (2bc \cos A)$$

$$=> x^2 = 8^2 + 6^2 - (2 \text{ x } 8 \text{ x } 6 \text{ x } \cos 110°)$$

$$=> x^2 = ... + ... - (96 \text{ x } (-0{\cdot}342..))$$

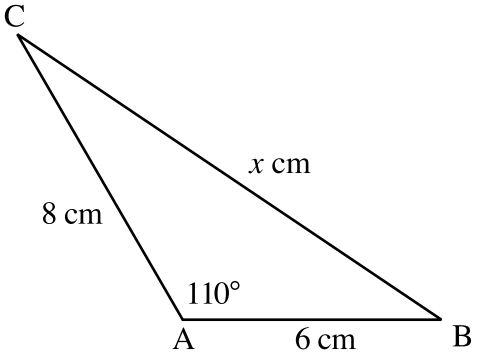
$$=> x^2 = ... - (-32{\cdot}83..)$$

$$=> x^2 = ... + 32{\cdot}83..$$

(note)

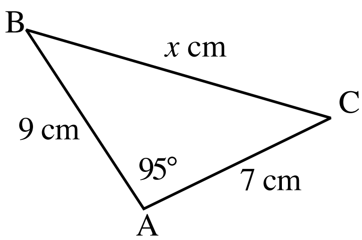
$$=> x^2 = ...$$

$$=> x = \boxed{}$$

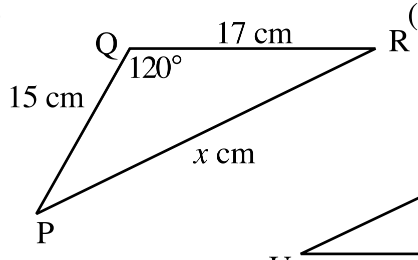


4. Calculate the lengths of the sides marked x cm.

(a)



(b)



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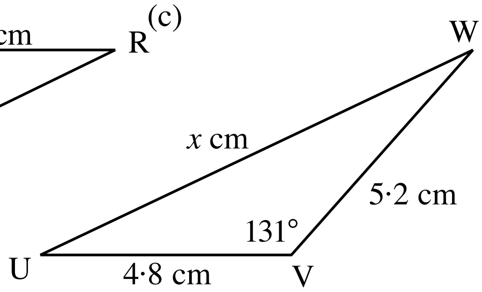
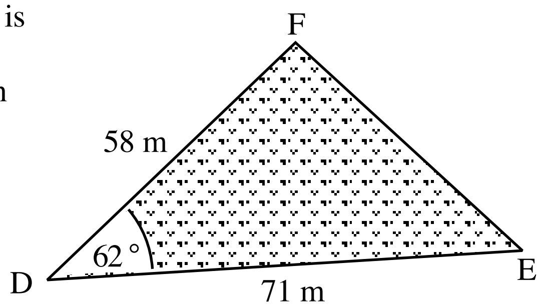


5. A farmer owns a piece of fenced land which is triangular in shape.

Calculate the length of the third side and then write down the <u>perimeter</u> of the field.

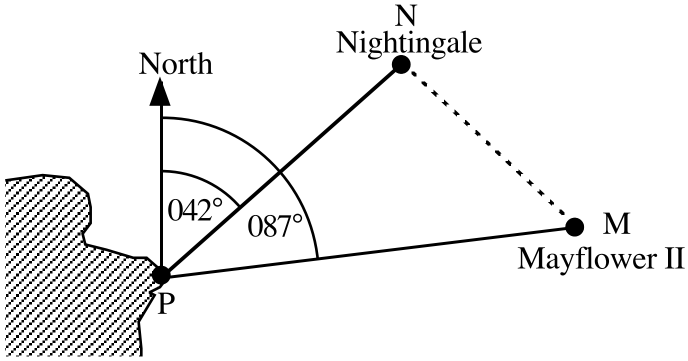


6. Two ships leave Peterborough harbour at 1300. The Nightingale sails at 20 miles per hour on a bearing 042°. The Mayflower II sails at 25 miles per hour on a bearing 087°.

(a) Calculate the size of ∠NMP.

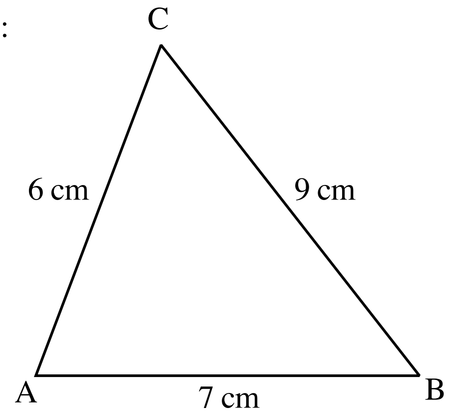
(b) How far apart will the 2 ships be after 1 hour?

(c) How far apart will they be at 1600?



Exercise 4B

1. Copy and complete the following to find ∠BAC:



$$a^2 = b^2 + c^2 - (2bc\cos A)$$

$$\Rightarrow \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\Rightarrow \quad \cos A = \frac{6^2 + 7^2 - 9^2}{2 \times 6 \times 7}$$

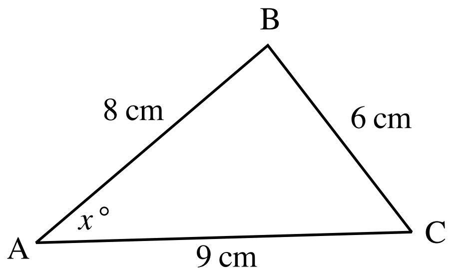
$$\Rightarrow \quad \cos A = 0\cdot \ldots\ldots$$

$$\Rightarrow \quad A = \boxed{}$$

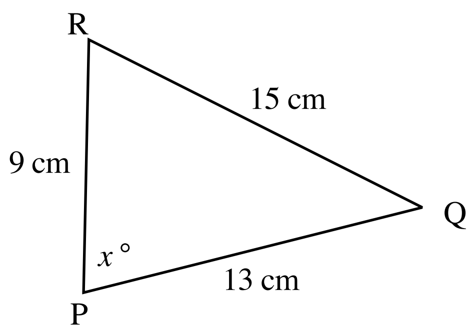
2. Use this 'reverse' form of the Cosine rule to calculate the size of each angle marked $x°$ here.

$$\boxed{\cos A = \frac{b^2 + c^2 - a^2}{2bc}}$$

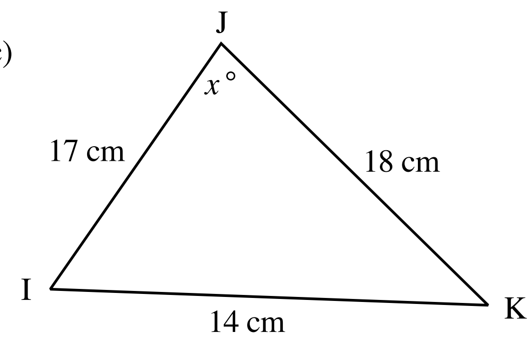
(a)



(b)



(c)



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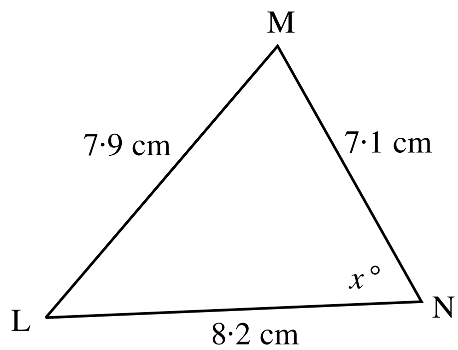
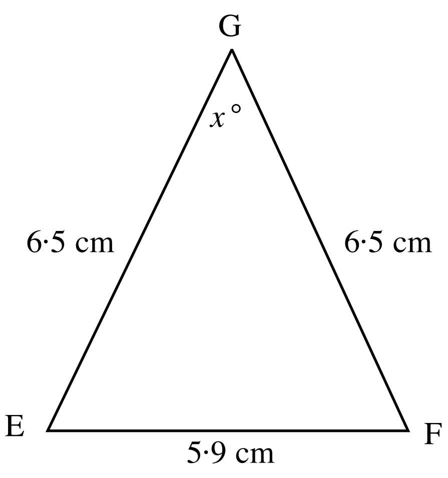
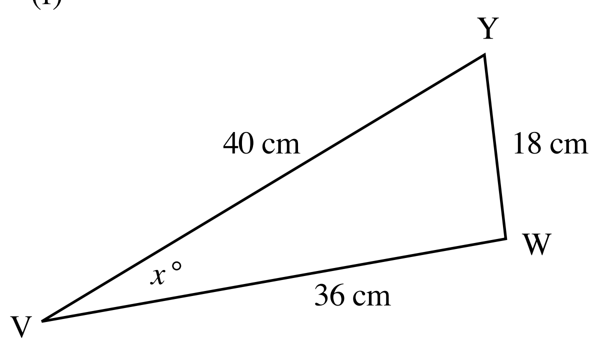


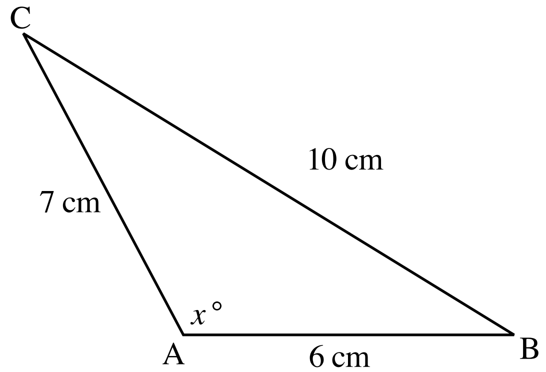
(e)



(f)



3. Copy and complete the following to find ∠BAC:



$$a^2 = b^2 + c^2 - (2bc \cos A)$$

$$\Rightarrow \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

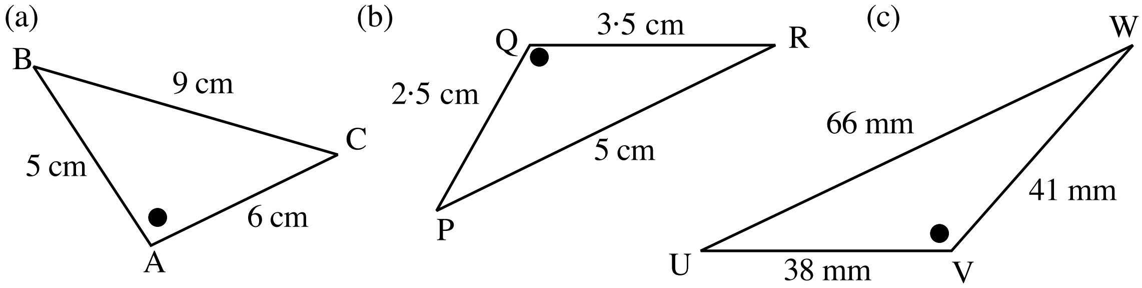
$$\Rightarrow \quad \cos A = \frac{7^2 + 6^2 - 10^2}{2 \times 7 \times 6}$$

$$\Rightarrow \quad \cos A = -0{\cdot}178..$$

$$\Rightarrow \quad A = ?????$$

Hint :- try finding SHIFT (or INV) cos (–0·178..)
if you obtain the correct answer of 100·3°, your calculator can handle negatives.
if you obtain the wrong answer of –79·7°, ask your teacher/lecturer for help.

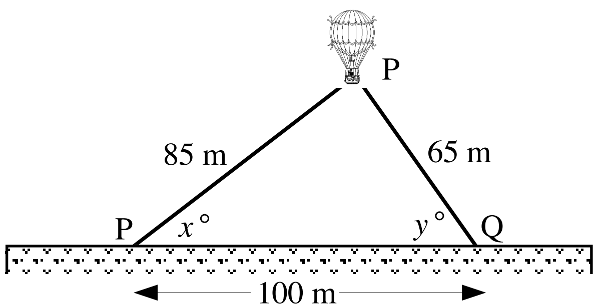
4. Calculate the size of each of the obtuse angles in the following three triangles:



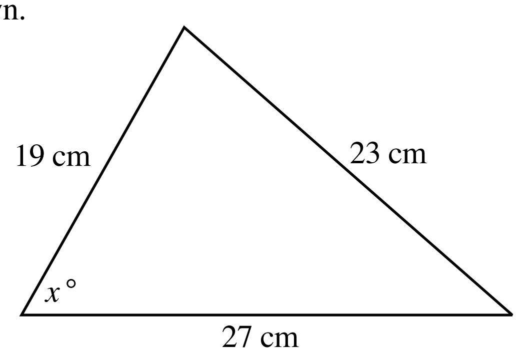
5. Two guy ropes are used to restrain a balloon.

The ropes are 85 metres and 65 metres long, and are tethered at points 100 metres apart.

Calculate the sizes of the two angles marked x ° and y °.



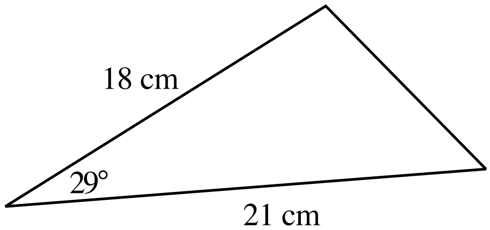
6. This triangular metal plate has its 3 sides as shown.
   (a) Calculate the size of the angle marked x °.
   (b) Calculate the <u>area</u> of the triangular plate.



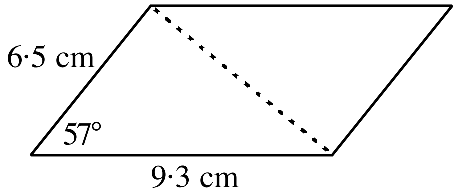
CHECKUP FOR TRIGONOMETRY

1. Write down the values of the following to 3 decimal places:
   (a) sin 200° (b) tan 320° (c) cos (–265°)

2. Calculate the area of this triangle:

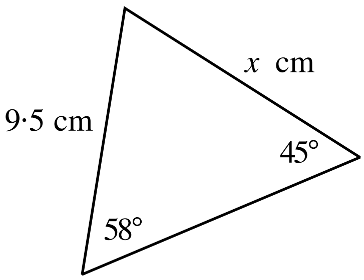


3. Calculate the area of this parallelogram:

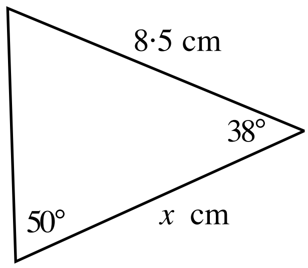


4. Use the Sine Rule or the Cosine rule (2 formats) to calculate the value of x each time here:

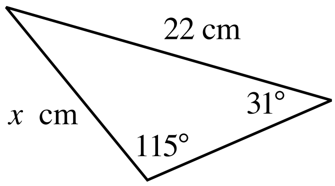
(a)



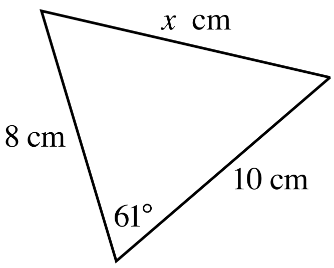
(b)



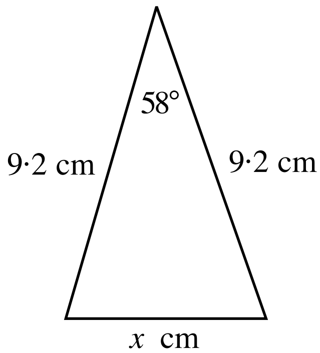
(c)



(d)



(e)



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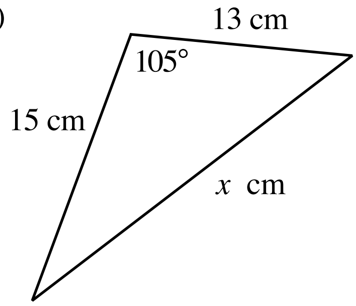
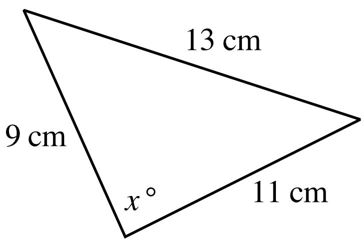
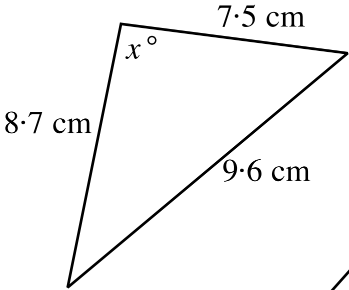


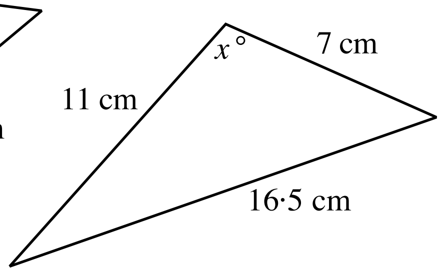
(g)



(h)

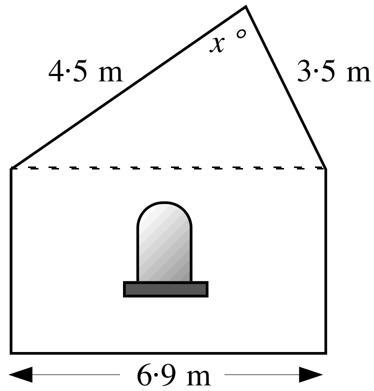


(i)



5. The diagram shows the side view of a house with a sloping roof.

Calculate the size of the angle, $x°$, between the two sloping sides of the roof.

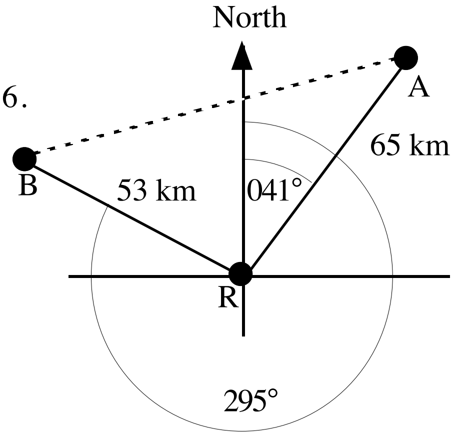


6. From a radar station at R, signals from two ships are picked up.

Ship A is on a bearing 041° from R and is 65 kilometres away.

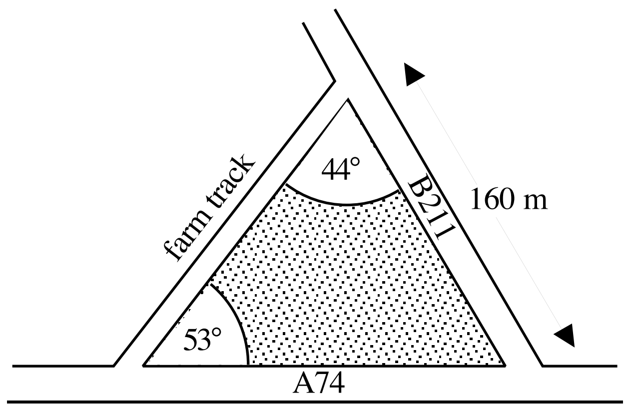
Ship B is on a bearing 295° from R and is 53 kilometres away.

Calculate how far apart the two ships are.

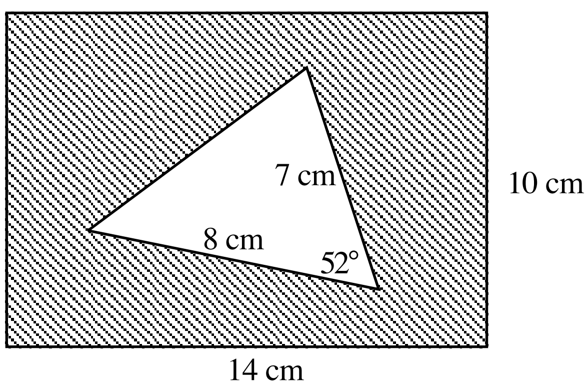


7. A farmer owns a triangular piece of land trapped between 2 main roads and the farm track.

Calculate the length of the farm track to the nearest whole metre.



8. Calculate the shaded area of this rectangular metal plate with a triangular hole cut out of it.



SIMULTANEOUS LINEAR EQUATIONS

By the end of this set of exercises, you should be able to

(a) Construct formulae to describe a linear relationship

(b) Understand the significance of the point of intersection of two graphs

(c) Solve simultaneous linear equations in two variables graphically

(d) Solve simultaneous linear equations in two variables algebraically

SIMULTANEOUS LINEAL EQUATIONS

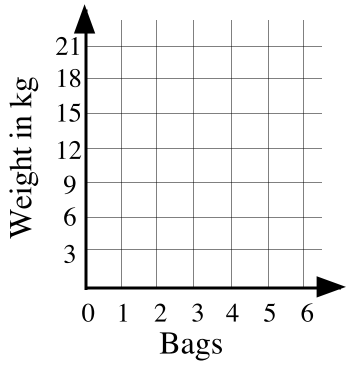
A. Construction of Formula

Exercise 1

1. A greengrocer sells Brussel Sprouts in 3 kilogram bags.
   The table compares the number of bags with the weight of sprouts sold.

Number of Bags (N)	1	2	3	4	5	6
Weight of sprouts (W)	3	6	9	12	15	18

   (a) **Copy** and complete: Weight = ... x No. Bags
   (b) Write a formula for the weight of sprouts.
   (c) Use your formula to find the weight of sprouts in 10 bags.
   (d) In your jotter, use your table to plot and join the points on a coordinate diagram like this :–
   (e) Extend your graph to show a straight line which passes through the origin.



2. A confectioner sells jelly eels in packs of ten.
   (a) Copy and complete the table:

Number of packs (P)	1	2	3	4	5	6
Number of eels (E)	10					

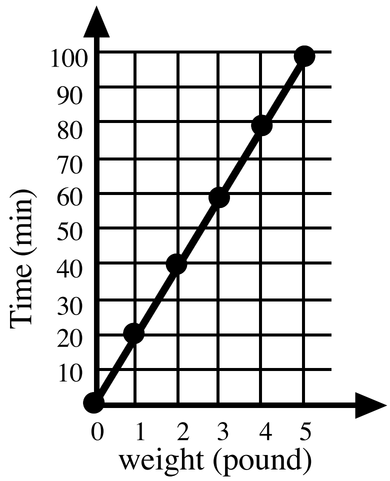
   (b) **Copy** and complete :– Number of eels = ... x No. packs
   (c) Write a formula for calculating the number of eels.
   (d) Use your formula to find the number of eels in 9 packs.
   (e) Use your table to plot and join the points on a coordinate diagram.
   (f) Extend your graph to show a straight line which passes through the origin.

3. The graph shows cooking times for roast beef.

   (a) **Copy** and complete the table:

Weight (W)	1	2	3	4	5	6
Time (T)	20					

   (b) Write a formula for the time (T) taken to cook a roast if you know its weight (W).
   (c) Use your formula to find the time taken to cook a 10 pound roast .



4. Mr. R. Highet called out Computer Fix to repair his computer. They have a ‘call out’ charge of £25 plus a charge of £8 per hour.

No. Hours (h)	1	2	3	4	5
Charge £ (C)	33	41	49		

(a) How much do Computer Fix charge for:
(i) 4 hours? (ii) 5 hours?

(b) Write a formula for the charge (C), given the number of hours worked (h).

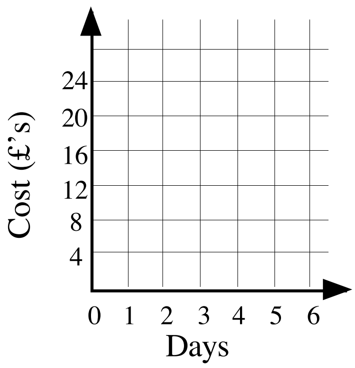
5. To hire a cement mixer it costs a basic £8 plus £4 for each day you have the machine.

(a) **Copy** and complete the table:

No. Days (D)	1	2	3	4	5
Charge £ (C)	12				

(b) Write a formula for the charge (C) given the number of days (D) for which you have the machine.

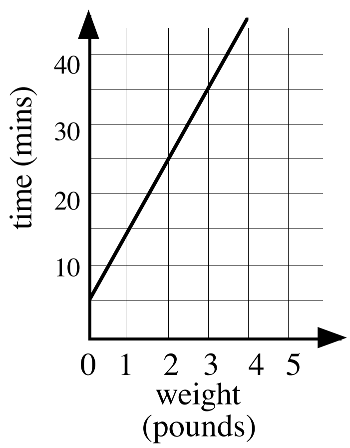
(c) In your jotter, use your table to plot and join the points on a coordinate diagram like this:



(d) Extend your graph to cut the vertical (C) axis and give the coordinates of the point where the line cuts that axis.

(e) Explain this point in relation to hiring a cement mixer.

6. The graph shows defrosting times for a chicken.



(a) Using the graph, **copy** and complete the table.

Weight (W pounds)	1	2	3	4	5	6
Time (T min)	15					

(b) Write a formula for the time (T) taken to cook a chicken if you know its weight (W).

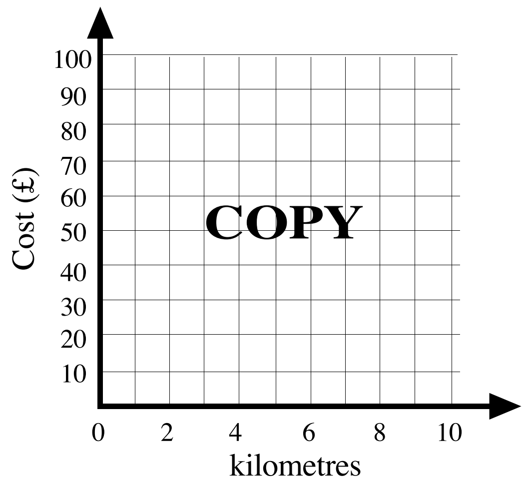
(c) Use your formula to find the time taken to cook a 10 pound chicken .

7. Fast Delivery charges £50, plus £5 per kilometre to deliver parcels.

(a) Write down a formula for the charge £C for a delivery of k kilometres.

(b) Calculate the charge for a 10 kilometre trip.

(c) Draw a graph of charges up to 10km, using these scales.



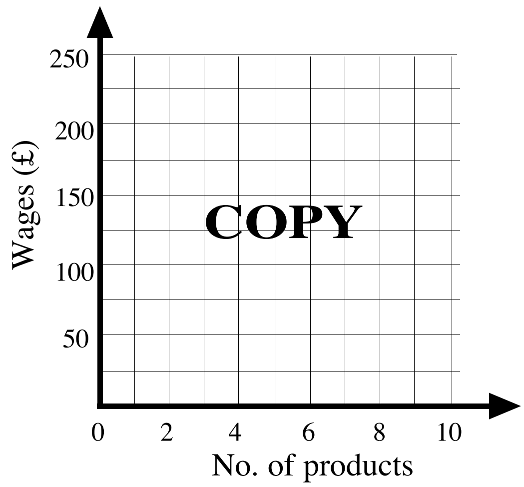
8. Mrs. Divers sells cosmetics.

She gets paid a basic £80 per week plus £10 each time she sells a product from the new Opius Perfume range.

(a) Write down a formula for her wage £W for a week in which she sells P products.

(b) Work out her wage for a for a week in which she sells 20 products.

(c) Draw a graph of her wages for up to 20 products, using these scales.



9. Mr. McGarrill, the school janitor, is ordering sweeping brushes at £10 each.

If he pays quickly he finds that he can get a discount of £5 off his <u>total</u> bill.

(a) **Copy** and complete the table:

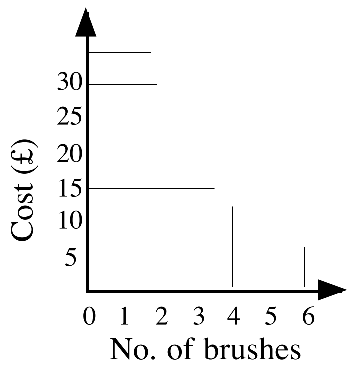
No. Brushes (B)	1	2	3	4	5
Cost £ (C)	5	15	25		

(b) What is his bill for:

(i) 4 brushes? (ii) 5 brushes?

(c) Write a formula for the cost (C) for a number of brushes (B).

(d) In your jotter, use your table to plot and join the points on a coordinate diagram like this:

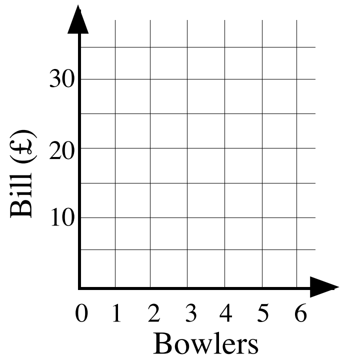


10. A group of adults are having a night out at a ten–pin bowling alley.
The cost is normally £6 each, but a midweek special is giving £4 off the total bill.

(a) Make up a table to show the total bill for 1, 2, 3, 4, 5, 6 bowlers.

(b) Write a formula for the total bill (£T) for a number of bowlers (B).

(c) In your jotter, use your table to plot and join the points on a coordinate diagram like this:



Revision:- Drawing Straight Lines

Exercise 2

For each of the following equations of a straight line:
- choose three points on the line
- plot the points on squared paper, each one on a separate diagram
- draw a straight line through them.

1. $y = x$	2. $y = 3x$	3. $y = x + 1$	4. $y = 2x + 3$
5. $y = 2x - 1$	6. $y = 2 - x$	7. $y = 5$	8. $x = 3$
9. $x + y = 6$	10. $x - y = -2$	11. $2x + y = 0$	12. $y = -x + 1$

B. Solving Simultaneous Linear Equations Graphically

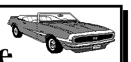
Exercise 3

By drawing the graphs represented by the following equations on squared paper, solve each pair of simultaneous equations.

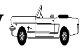
1. $x + y = 6$, $y = x$	2. $x + y = 4$, $x + 2y = 6$	3. $x - y = 4$, $x - 2y = 6$
4. $x + y = 8$, $x - y = 2$	5. $x + 2y = 5$, $x - y = -1$	6. $y = x + 2$, $y = -x - 4$
7. $x + 3y = 7$, $x - 3y = 1$	8. $y = 2x + 2$, $y = -x - 4$	9. $2x - y = 3$, $y = 5$
10. $2x + y = 4$, $3x + 2y = 9$	11. $3x - 3y = -6$, $3x - 2y = 0$	12. $x + 3y = 8$, $2x - y = -5$

Exercise 4A

1. **Goudie's Car Hire**
£40 Deposit + £10 a day



Henry's Rent a Car
£20 per day



(a) **Copy** and complete the tables showing the charges for the two car hire companies.

Goudie's

Number of days	0	1	2	3	4	5	6	7
Cost (£)	40	50	60					

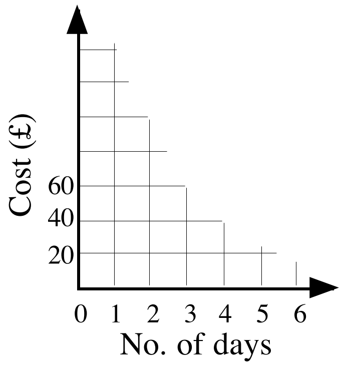
Henry's

Number of days	0	1	2	3	4	5	6	7
Cost (£)	0	20	40					

(b) Draw the straight line graph for both car hire companies on the same coordinate diagram.

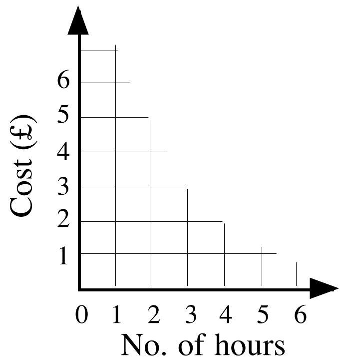
(c) The two companies charge the same amount only once. For how many days is this?

(d) Up to how many days is Henry's cheaper?

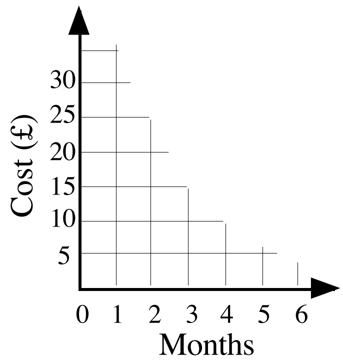


2. 'Hire a bike in Millport.'
Mr. Dawes charges **£1 deposit plus 50p per hour**.
Mr. Beckham charges **No deposit, £1 per hour**.

(a) Make two tables to show the prices for up to 6 hours hire at Dawes' and Beckham's.
(b) Draw the straight line graph for both bicycle hire companies on the same coordinate diagram.
(c) For what number of hours hire is the cost the same at both shops?
(d) If you wanted to hire a bike for 4 hours, which shop would you go to in order to save money?



3. RENT A COMPUTER are offering computers for **£20 deposit, plus £5 per month**.
COMPU HIRE are offering similar computers for **£10 per month, with no deposit**.
(a) Make two tables to show the prices for up to 5 months at each place.
(b) Draw the straight line graph for both computer rental companies on the same coordinate diagram.
(c) (i) For what number of months is the cost the same at both shops?
(ii) What price is this?



4. BLACK CAB TAXI COMPANY charge **50p** per mile.

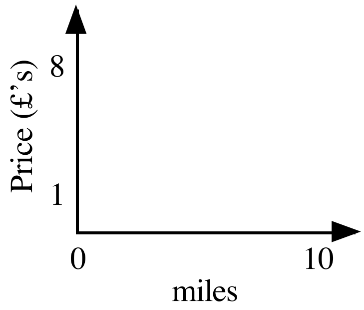
RED TAXIS charge **£2** for any journey up to **4 miles**, then **£1** per mile for each additional mile.

(a) Make two tables to show the prices for up to a 10 mile journey at both firms.

(b) Draw the straight line graph for both taxi companies on the same coordinate diagram.

(c) For how many miles is the cost the same at both firms?

(d) You are travelling only 2 or 3 miles – which taxi company would you phone to save money?



Exercise 4B

1. Third Lanark v Leith Athletic

Adult Charge £x

Child Charge £y

One adult and one child paid £8 to attend this football match.

$x + y = 8$

Two adults and one child paid £13.

$2x + y = 13$

(a) Draw the lines $x + y = 8$ and $2x + y = 13$ on the same coordinate diagram using suitable points on each line.

(b) Write down the coordinates of the point of intersection.

(c) What is significant about this point in terms of prices to get into the match?

(d) What was the charge for 10 adults and 10 children at this match?

2. The professional at Worthwent Golf Club prices her goods as follows:

Golf Balls £x Golf Gloves £y

Arnold bought 2 golf balls and 1 golf glove for £8. $2x + y = 8$

Tiger bought 4 golf balls and 1 golf glove for £12. $4x + y = 12$

(a) Draw the lines $2x + y = 8$ and $4x + y = 12$ on the same coordinate diagram using suitable points on each line.

(b) Write down the coordinates of the point of intersection.

(c) What was the cost of a golf ball?

(d) What was the cost of a golf glove?

(e) What does the professional charge for 3 golf balls and 3 golf gloves?

3. 2 jotters and 2 pencils cost 80p. 1 jotter and 3 pencils cost 60p.

Let the cost of a jotter be x pence and the cost of a pencil be y pence.

One equation from the data given is $2x + 2y = 80$.

(a) Write down the other equation in terms of x and y.

(b) Draw the two straight lines which the equations represent on the same coordinate diagram using suitable points on each line.

(c) Use your graph to find the cost of a jotter and the cost of a pencil.

4. 1 packet of Weedo and 1 packet of slug pellets costs £5.

   1 packet of Weedo and 3 packets of slug pellets costs £9.

   Let the cost of a packet of Weedo be £x and the cost of a packet of slug pellets be £y.

   (a) Write down two equations in terms of x and y.

   (b) Draw the two straight lines which the equations represent on the same coordinate diagram using suitable points on each line.

   (c) Use your graph to find the cost of a packet of Weedo and the cost of a bottle of slug pellets.

5. Mary bought 3 T–shirts and 2 bottles of colour dye for £12.

   Sally bought 2 of the T–shirts and 5 bottles of colour dye for £30.

   Let the cost of a T–shirt be £x and the cost of a bottle of colour dye be £y.

   (a) Write down two equations in terms of x and y.

   (b) Draw the two straight lines which the equations represent on the same coordinate diagram using suitable points on each line.

   (c) Use your graph to find the cost of a T–shirt and the cost of a bottle of colour dye.

6. The total cost of two books is £10 and the difference in their cost is £2.

   Let the cost of a one book be £x and the cost of the other book be £y .

   (a) Write down two equations in terms of x and y.

   (b) Draw the two straight lines which the equations represent on the same coordinate diagram using suitable points on each line.

   (c) Use your graph to find the cost of each book.

C. Solving Simultaneous Linear Equations Algebraically

Exercise 5A

Solve these simultaneous equations by eliminating x or y, etc.

1. $x + y = 12$ $x - y = 8$	2 $x + y = 6$ $x - y = 4$	3. $x + y = 10$ $x - y = 8$
4. $x + 2y = 6$ $x - 2y = 2$	5. $a + 4d = 9$ $a - 4d = 1$	6. $3r + t = 10$ $3r - t = 2$
7. $5p + q = 4$ $2p + q = 1$	8. $6u + 6w = 6$ $4u + 6w = 6$	9. $7x - 3y = 1$ $4x - 3y = -2$
10. $4g - 5h = 13$ $3g - 5h = 11$	11. $5e - 2f = 8$ $-e + 2f = 0$	12. $-3x - 4y = 3$ $3x + y = 6$

Exercise 5B

Solve these simultaneous equations by first multiplying both sides of the equations by suitable numbers.

1. $x + 2y = 4$, $2x - y = 3$
2. $3a + d = 9$, $a - 2d = 3$
3. $4e - f = 11$, $e + 2f = 5$
4. $g + 2h = 7$, $2g - h = 9$
5. $m + 3n = 2$, $2m - n = 4$
6. $5p + q = 3$, $p - 2q = 5$
7. $3r + 2s = 1$, $r + s = 0$
8. $4t + 2u = 4$, $t + u = 0$
9. $3v - 4w = 13$, $v + w = 2$
10. $x - y = 4$, $3x - 2y = 8$
11. $5x - 2y = -1$, $x - 3y = 5$
12. $x - 3y = 1$, $2x - y = 7$

Exercise 5C

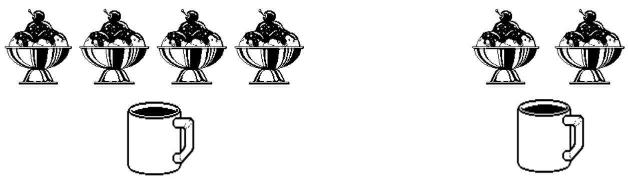
Solve these simultaneous equations by first multiplying both sides of the equations by suitable numbers.

1. $2p - 3q = 1$, $3p + 2q = 8$
2. $2x + 4y = 14$, $7x + 3y = 27$
3. $2v + 3w = 0$, $v - w = 5$
4. $7a + 4d = 1$, $5a + 2d = -1$
5. $2r - 3s = 12$, $3r - 2s = 13$
6. $5x - 8y = 0$, $4x - 3y = -17$
7. $3g + 2h - 6 = 0$, $g - h - 1 = 1$
8. $3m + 5n - 23 = 0$, $5m + 2n - 13 = 0$
9. $3f - 5g - 11 = 2$, $2f + 4g - 9 = 7$

Exercise 5D

Write down a pair of simultaneous equations for each picture, then solve them to answer the question. (Use £x and £y to represent the cost of one of each item each time).

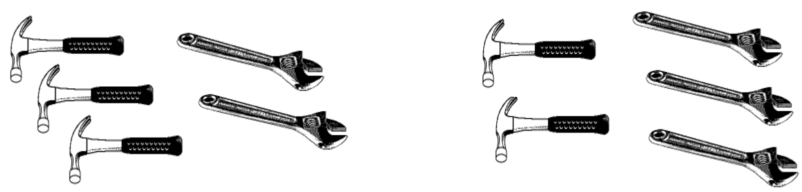
1.



Total cost £9 Total cost £5

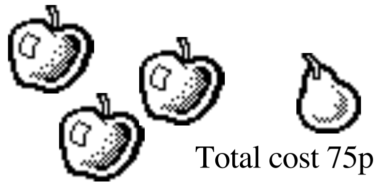
Find the cost of: (a) one ice cream sundae. (b) one mug of cocoa.

2.



Total cost £24 Total cost £21

Find the cost of: (a) one hammer. (b) one spanner.

3.  

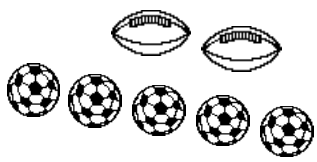
Total cost 55p | Total cost 75p

Find the cost of: (a) one apple. (b) one pear.

4.  

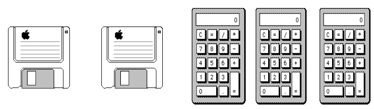
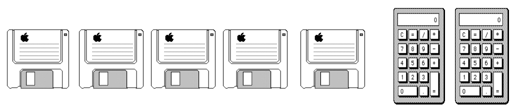
Total cost £3·50 | Total cost £2·50

Find the cost of: (a) one frothy drink. (b) one slice of cake.

5.  

Total cost £90 | Total cost £110

Find the cost of: (a) one football. (b) one rugby ball.

6.  

Total cost £7 | Total cost £6·50

Find the cost of: (a) one disk. (b) one calculator.

7.  

Total cost £6·50 | Total cost £7

Find the cost of: (a) one hot dog. (b) one hamburger.

8. At a supermarket, a lady paid £2·70 for 6 red peppers and 5 corn on the cobs.
At the same supermarket, a man paid £1·20 for 3 red peppers and 2 corn on the cobs.

Find the cost of: (a) one pepper. (b) one corn stick.

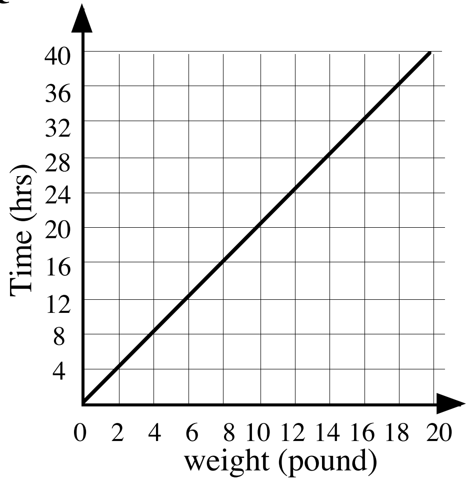
9. At a newsagent, a boy paid £1·10 for 2 memo pads and 7 pencils.
At the same shop, a girl paid £1·60 for 7 memo pads and 2 pencils.

Find the cost of: (a) one memo pad. (b) one pencil.

10. An adult's ticket for the cinema is £3 more than a child's.
The adult's ticket is also twice that of the child's.
Let the price of an adult's ticket be £x and the price of a child's ticket be £y.
Form a pair of simultaneous equations and solve them to find the price of each ticket.

CHECKUP FOR SIMULTANEOUS LINEAR EQUATIONS

1. The graph shows defrosting times at room temperature for Christmas turkey.



   (a) **Copy** and complete the table:

Weight (W)	0	2	4	6	8	10	12	14	16	18	20
Time (T)	0	4									

   (b) Write a formula for the time (T) taken to defrost a turkey if you know its weight (W).

   (c) Use your formula to find the time taken to defrost a 15 pound turkey.

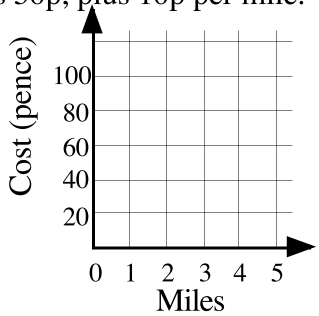
2. Pizza Point will deliver pizzas to your door. The charge is 50p, plus 10p per mile.

   (a) Write down a formula for the charge C pence for a delivery of M miles.

   (b) Work out the charge for a 5 mile delivery.

   (c) Draw a graph of charges up to 5 miles, using the scales shown.

   (d) What would be the charge for a 10 mile delivery ?



3. By drawing graphs of these equations on squared paper, solve each pair of simultaneous equations.

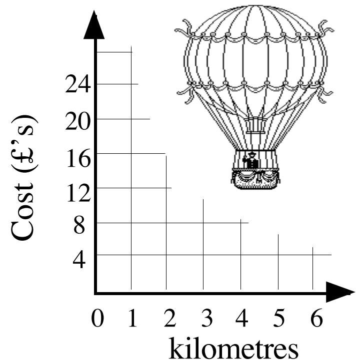
   (a) $x + y = 8$
   $y = x$

   (b) $x + 2y = 7$
   $4x - y = 10$

   (c) $x + 3y = 0$
   $x - 2y = 5$

4. HIGH FLY offer balloon trips at £10 basic, plus £2 per kilometre travelled.

   FLIGHT BALLOONS offer the same trips at £4 per kilometre, with no other charges.



   (a) Make two tables to show the prices for up to a trip of 6 km with both companies.

   (b) Draw the straight line graph for both companies on the same coordinate diagram.

   (c) (i) How many kilometres can you travel for the same price at both businesses?

   (ii) What price is this?

5. Terry bought a bottle of shampoo and a bottle of conditioner for £6.
Lesley bought 4 bottles of shampoo and a bottle of conditioner for £12.
Let the cost of a bottle of shampoo be £x and the cost of a bottle of conditioner be £y.

(a) Write down two equations in terms of x and y.
(b) Draw the two straight lines which the equations represent on the same coordinate diagram using suitable points on each line.
(c) Use your graph to find the cost of a bottle of shampoo and the cost of a bottle of conditioner.

6. Solve these simultaneous equations algebraically:

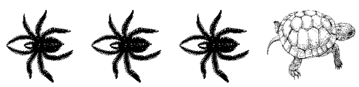
(a) $x + y = 20$, $x - y = 4$ (b) $x - 3y = -1$, $x + 3y = 11$ (c) $2x + y = 10$, $-2x + y = -10$

(d) $v + 3w = 7$, $2v - w = 0$ (e) $2p + 3q = 19$, $4p - 7q = -27$ (f) $2x - 3y = 1$, $3x + 2y = -5$

(g) $5s + 3t = 19$, $7s - 2t = 8$ (h) $4x - 3y - 1 = 4$, $3x + 4y - 10 = 0$

7. Write down a pair of simultaneous equations for each picture, then solve them to answer the question. (Use £x and £y to represent the cost of one of each item).

(a)



Total cost £36

Total cost £28

Find the cost of: (i) one spider. (ii) one turtle.

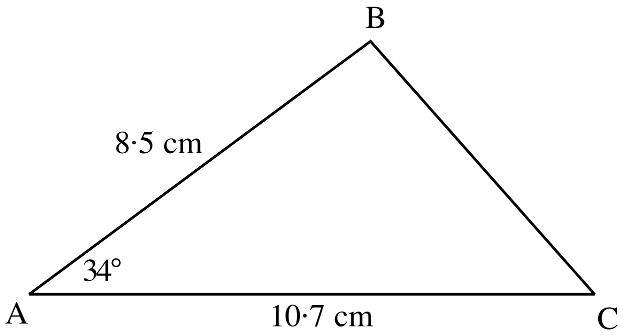
(b) 5 pairs of compasses and 2 pairs of scissors together cost £2·30.
3 pairs of compasses along with 3 pairs of scissors cost £2·10.

Find the cost of: (i) one pair of compasses. (ii) one pair of scissors .

8. The sum of two whole numbers is 112, and their difference is 36.
Form a pair of simultaneous equations and solve them to find the two numbers.

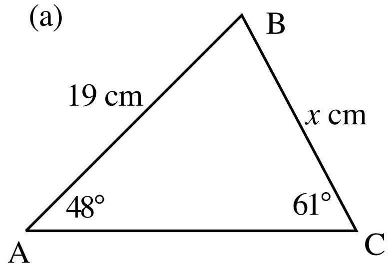
SPECIMEN ASSESSMENT QUESTIONS

1. Calculate the area of this triangle:

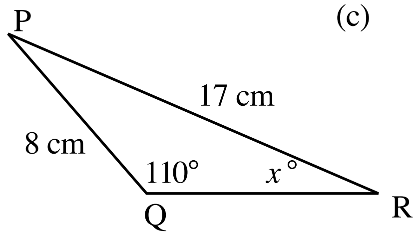


2. Use the Sine rule or Cosine rule to calculate the value of x each time.

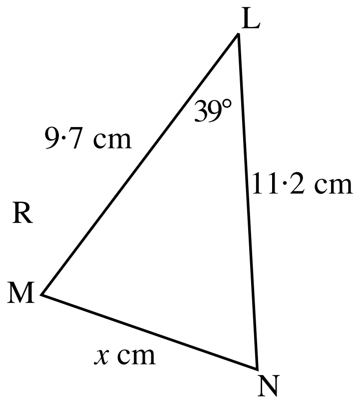
(a)



(b)



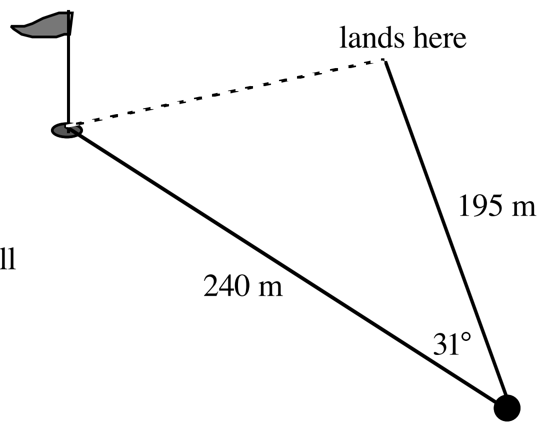
(c)



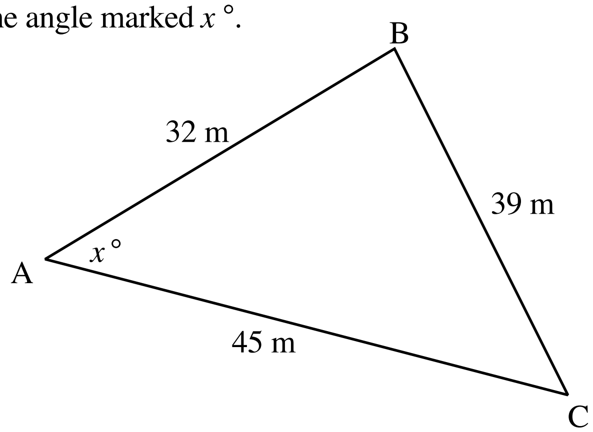
3. A golfer tees off and aims his shot for the third hole, a distance of 240 metres away.

   Unfortunately, he slices his ball and it ends up at the position shown opposite.

   How far away to the nearest metre is his ball from the flag?



4. Calculate the size of the angle marked x °.



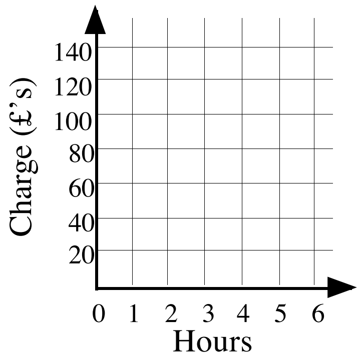
5. Mrs. Doherty called out Hoover Repair to repair her washing machine.
They have a 'call out' charge of £30 plus a charge of £20 per hour.

(a) How much do Hoover Repair charge for:

(i) 1 hour? (ii) 2 hours? (iii) 3 hours? (iv) 4 hours? (v) 5 hours?

(b) Write a formula for the charge (C), given the number of hours worked (h).

(c) Use your information to plot and join the points on a coordinate diagram like this:



6. Draw the graphs of the equations on squared paper using suitable scales and solve each pair of simultaneous equations.

(a) $x + y = 10$
$y = x - 2$

(b) $x + 2y = 80$
$3x + y = 90$

7. The price for 1 adult and 1 child to play a game of pitch and putt is £4.
2 adults and 4 children were charged £10.

Let the adult price be £x and the child price be £y.

(a) Write down two equations in terms of x and y.

(b) Draw the two straight lines which the equations represent on the same coordinate diagram using suitable points on each line.

(c) Use your graph to find the price of an adult's ticket and the price of a child's ticket.

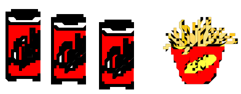
8. Solve these simultaneous equations algebraically:

(a) $5x + y = 4$
$2x + y = 1$

(b) $x + 2y = 9$
$2x - y = 8$

(c) $4x - 3y = 10$
$3x + 4y = 20$

9. Write down a pair of simultaneous equations for the picture, then solve them to answer the question. (Use £x and £y to represent the cost of one item each time).



Total cost £2·60

Total cost £2·20

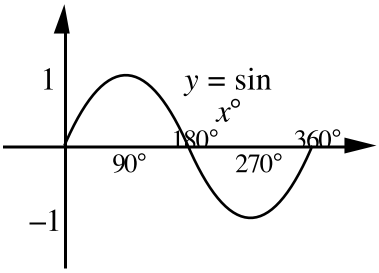
Find the cost of: (i) one can of coke. (ii) one bag of chips.

ANSWERS TO MATHEMATICS 2 (INT 2)

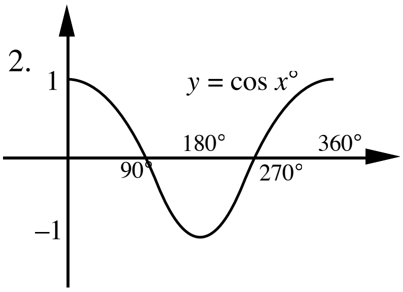
Trigonometry

Exercise 1A

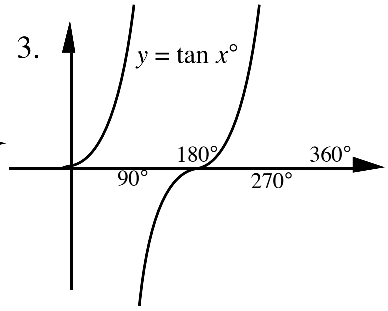
1. (a) 0·00 0·34 0·64 0·87 0·98 1·00 0·98 0·87 0·64 0·34 0·00
–0·34 –0·64 –0·87 –0·98 –1·00 –0·98 –0·87 –0·64 –0·34 0·00



2.



3.



Exercise 1B

1. (a) 0·423 (b) –0·087 (c) –3·271 (d) –0·342 (e) 0·707
(f) 0·017 (g) –0·5 (h) –0·5 (i) 1 (j) –0·866
(k) –1 (l) –0·776 (m) –3·732 (n) –0·940 (o) 0 (p) 1

Exercise 2

1. Method : (a) h = 9·51 (b) 57·06 cm^2 Method 2: —> 57·06 cm^2
2. (a) 14·1 cm^2 (b) 56·6 cm^2 (c) 30·8 cm^2
(d) 71·6 cm^2 (e) 100·5 cm^2 (f) 2·6 cm^2
3. (a) 25·7 cm^2 (b) 25·7 cm^2 same answer
4. (a) 18·2 cm^2 (b) 18·2 cm^2 same answer because sin 53° = sin 127°
5. 1152.6 cm^2 6. 117·7 cm^2 7 (a) 215·7 cm^2 (b) 26·2 cm^2

Exercise 3

1. a = 10·4 cm
2. (a) 17·6 cm (b) 14·0 cm (c) 7·6 cm (d) 8·2 cm (e) 13·2 cm (f) 29·8 cm
3. (a) 40° (b) 13·8 cm
4. (a) 67°; 5·6 cm (b) 49°; 9·2 cm (c) 29°; 13·3 cm
5. 0.836, 56·8° 6. (a) 37·6° (b) 76·1° (c) 50·6° (d) 21·6°
7. 58·4° 8. 23·9 km 9. (a) 75° (b) 1·45 m (c) 1·45 m
10. (a) x = 60·3 (b) y = 38·8 (c) z = 106·3

Exercise 4A

1. x = 3·39
2. (a) 5·6 cm (b) 14·3 cm (c) 33·5 cm (d) 7·2 cm (e) 4·4 cm (f) 5·3 cm
3. x = 11·5 cm 4. (a) 11·9 cm (b) 27·7 cm (c) 9·1 cm
5. 67·4 m; 196·4 m 6. (a) 45° (b) 17·8 km (c) 53·5 km

Exercise 4B

1. 87·3°
2. (a) 40·8° (b) 83·9° (c) 47° (d) 61·7° (e) 54·0° (f) 26·7°
3. 100·3°
4. (a) 109·5° (b) 111·8° (c) 113·3°
5. x = 40·1°, y = 57·4°
6. (a) x = 56·9° (b) Area = 214·8 cm^2

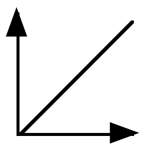
Checkup for Trigonometry

1. (a) –0·342 (b) –0·839 (c) –0·087
2. 91.6 cm^2
3. 50·7 cm^2
4. (a) 11·4 cm (b) 11·1 cm (c) 12·5 cm (d) 9·3 cm (e) 8·9 cm
   (f) 22·2 cm (g) 80·4° (h) 72·3° (i) 131·6°
5. 118·7°
6. 94·5 km
7. 199 m
8. 117·9 cm^2

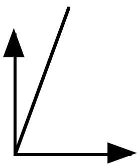
Simultaneous Linear Equations

Exercise 1

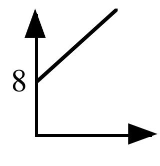
1. (a) 3 (b) W = 3N (c) 30kg (d)(e)



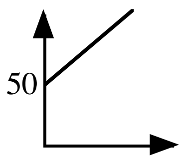
2. (a) 1/10 2/20 3/30 4/40 5/50 6/60 in table (b) 10 (c) E = 10P (d) 90
   (e) (f)



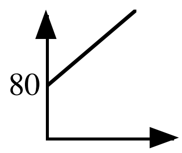
3. (a) 1/20 2/40 3/60 4/80 5/100 6/120 in table (b) T = 20W (c) 200 mins
4. (a) £57 £65 (b) C = 8h + 25
5. (a) 1/12 2/16 3/20 4/24 5/28 in table (b) C = 4D + 8 (c)
   (d) (0,8) (e) Costs £8 before even paying for any days !!



6. (a) 1/15 2/25 3/35 4/45 5/55 6/65 in table (b) T = 10W + 5
   (c) 105 mins
7. (a) C = 5k + 50 (b) £100 (c)



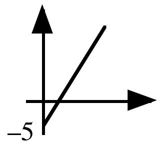
8. (a) W = 10P + 80 (b) £280 (c)



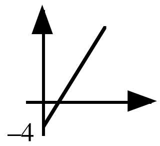
9. (a) 1/5 2/15 3/25 4/35 5/45 in table

   (b) £35; £45 (c) C = 10B – 5

   (d)



10. (a) 1/2 2/8 3/14 4/20 5/26 6/32 in table (b) T = 6B – 4 (c)



Exercise 2

1. Graph of a straight line through (0,0), (1,1) (2,2) etc.
2. Graph of a straight line through (0,0), (1,3) (2,6) etc.
3. Graph of a straight line through (0,1), (1,2) (2,3) etc.
4. Graph of a straight line through (0,3), (1,5) (2,7) etc.
5. Graph of a straight line through (0,–1), (1,1) (2,3) etc.
6. Graph of a straight line through (0,2), (1,1) (2,0) etc.
7. Graph of a straight line through (0,5), (1,5) (2,5) etc.
8. Graph of a straight line through (3,0), (3,1) (3,2) etc.
9. Graph of a straight line through (0,6), (1,5) (2,4) etc.
10. Graph of a straight line through (0,2), (1,3) (2,4) etc.
11. Graph of a straight line through (0,0), (1,–2) (2,–4) etc.
12. Graph of a straight line through (0,1), (1,0) (2,–1) etc.

Exercise 3

1. (3,3) 2. (2,2) 3. (2,–2) 4. (5,3) 5. (1,2) 6. (–3,–1) 7. (4,1)
8. (–2,–2) 9. (4,5) 10. (–1,6) 11. (4,6) 12. (–1,3)

Exercise 4A

1. (a)

Goudies	0/40	1/50	2/60	3/70	4/80	5/90	6/100	7/110
Henry's	0/0	1/20	2/40	3/60	4/80	5/100	6/120	7/140

   (b) Straight lines crossing at (4,80)

   (c) 4 days (d) 3 days

2. (a)

Dawes	0/1	1/1·50	2/2	3/2·50	4/3	5/3·50	6/4
Beckams	0/0	1/1	2/2	3/3	4/4	5/5	6/6

   (b) Straight lines crossing at (2,2) (c) 2 hours (d) Dawes

3. (a)

Rent a Computer	0/20	1/25	2/30	3/35	4/40	5/45
Compu Hire	0/0	1/10	2/20	3/30	4/40	5/50

   (b) Straight lines crossing at (4,40) (c) 4 £40

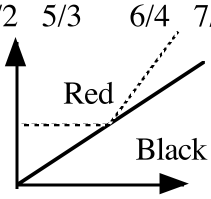
4. (a)

Black	0/0	1/0·5	2/1	3/1·5	4/2	5/2·5	6/3	7/3·5	8/4	9/4·5	10/5
Red	0/2	1/2	2/2	3/2	4/2	5/3	6/4	7/5	8/6	9/7	10/8

   (b) Lines crossing at (4,2)

   (c) 4 miles

   (d) Black Cab



Exercise 4B

1. (a)(b) Straight lines crossing at (5,3) (c) £5 adult £3 child (d) £80
2. (a)(b) Straight lines crossing at (2,4) (c) £2 (d) £4 (e) £18
3. (a) $x + 3y = 60$ (b) Straight lines crossing at (30,10) (c) jotter 30p pencil 10p
4. (a) $x + y = 5$ $x + 3y = 9$ (b) Straight lines crossing at (3,2) (c) Weedo £3 slug £2
5. (a) $3x + 2y = 12$ $2x + 5y = 30$ (b) Straight lines crossing at (0,6) (c) shirt free dye £6
6. (a) $x + y = 10$ $x - y = 2$ (b) Straight lines crossing at (6,4) (c) £6 and £4

Exercise 5A

1. (10,2)	2. (5,1)	3. (9,1)	4. (4,1)	5. (5,1)
6. (2,4)	7. (1,–1)	8. (0,1)	9. (1,2)	10. (2,–1)
11. (2,1)	12. (3,–3)			

Exercise 5B

1. (2,1)	2. (3,0)	3. (3,1)	4. (5,1)	5. (2,0)
6. (1,–2)	7. (1,–1)	8. (2,–2)	9. (3,–1)	10. (0,–4)
11. (–1,–2)	12. (4,1)			

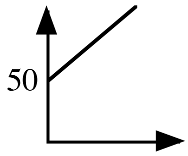
Exercise 5C

1. (2,1)	2. (3,2)	3. (3,–2)	4. (–1,2)	5. (3,–2)
6. (–8,–5)	7. (2,0)	8. (1,4)	9. (6,1)	

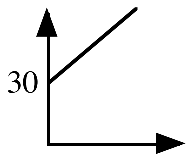
Exercise 5D

1. $4x + y = 9$	$2x + y = 5$	ice cream £2	cocoa £1
2. $3x + 2y = 24$	$2x + 3y = 21$	hammer £6	spanner £3
3. $2x + y = 55$	$3x + y = 75$	apple 20p	pear 15p
4. $2x + y = 3{\cdot}50$	$x + 2y = 2{\cdot}50$	drink £1·50	cake 50p
5. $5x + 2y = 90$	$5x + 3y = 110$	football £10	rugby ball £20
6. $2x + 3y = 7$	$5x + 2y = 6{\cdot}50$	disk 50p	calculator £2
7. $3x + 2y = 6{\cdot}50$	$2x + 4y = 7$	hot dog £1	hamburger £1·50
8. $6x + 5y = 2{\cdot}70$	$3x + 2y = 1{\cdot}20$	pepper 20p	corn 30p
9. $2x + 7y = 1{\cdot}10$	$7x + 2y = 1{\cdot}60$	pad 20p	pencil 10p
10. $x - y = 3$ $x = 2y$ or equivalent		adult £6	child £3

Checkup for Simultaneous Linear Equations

1. (a) 0/0 2/4 4/8 6/12 8/16 10/20 12/24 14/28 16/32 18/36 20/40 in table
   (b) T = 2W (c) 30 hours
2. (a) C = 10M + 50 (b) 100p (c)  (d) 150p
3. (a) (4,4) (b) (3,2) (c) (3,–1)
4. (a) High Fly 0/10 1/12 2/14 3/16 4/18 5/20 6/22 in table
   Flight Balloons 0/0 1/4 2/8 3/12 4/16 5/20 6/24 in table
   (b) Straight lines crossing at (5,20) (c) 5km £20
5. (a) $x + y = 6$ $4x + y = 12$ (b) Straight lines crossing at (2,4) (c) Sham £2 Cond £4
6. (a) (12,8) (b) (5,2) (c) (5,0) (d) (1,2) (e) (2,5)
   (f) (–1,–1) (g) (2,3) (h) (2,1)
7. (a) $3x + y = 36$ $2x + y = 28$ spider £8 turtle £12
   (b) $5x + 2y = 2{\cdot}30$ $3x + 3y = 2{\cdot}10$ compasses 30p scissors 40p
8. 74 & 38

Specimen Assessment Questions

1. 25·4 cm^2
2. (a) 16·1 cm (b) 26·2° (c) 7·1 cm
3. 124 m
4. 58·0°
5. (a) £50 £70 £90 £110 £130 (b) C = 20h + 30 (c) 
6. (a) (6,4) (b) (20,30)
7. (a) $x + y = 4$ $2x + 4y = 10$ (b) Straight lines crossing at (3,1) (c) Adult £3 Child £1
8. (a) (1,–1) (b) (5,2) (c) (4,2)
9. (a) $3x + y = 2{\cdot}60$ $x + 2y = 2{\cdot}20$ coke 60p chips 80p