X100/201

NATIONAL QUALIFICATIONS 2003

WEDNESDAY, 21 MAY
1.30 PM – 2.15 PM

MATHEMATICS
INTERMEDIATE 2
Units 1, 2 and 3
Paper 1
(Non-calculator)

Read carefully

1 You may NOT use a calculator.
2 Full credit will be given only where the solution contains appropriate working.
3 Square-ruled paper is provided.
FORMULAE LIST

The roots of \( ax^2 + bx + c = 0 \) are \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

Sine rule: \( \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \)

Cosine rule: \( a^2 = b^2 + c^2 - 2bc \cos A \) or \( \cos A = \frac{b^2 + c^2 - a^2}{2bc} \)

Area of a triangle: \( \text{Area} = \frac{1}{2}ab \sin C \)

Volume of a sphere: \( \text{Volume} = \frac{4}{3} \pi r^3 \)

Volume of a cone: \( \text{Volume} = \frac{1}{3} \pi r^2 h \)

Volume of a cylinder: \( \text{Volume} = \pi r^2 h \)

Standard deviation: \( s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum x^2 - (\sum x)^2 / n}{n-1}} \), where \( n \) is the sample size.
ALL questions should be attempted.

1. Multiply out the brackets and collect like terms.

\[(2a - b)(3a + 2b)\]

2. Two spinners are used in an experiment.

The table below shows some of the possible outcomes when both spinners are spun and allowed to come to rest.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>R,1</td>
<td>R,2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yellow</td>
<td>Y,1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blue</td>
<td>B,1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Green</td>
<td>G,1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Copy and complete the table.

(b) What is the probability that one spinner comes to rest on red and the other on an even number?
3. The diagram shows a cone.

The height is 12 centimetres and the radius of the base 10 centimetres.
Calculate the volume of the cone.
Take $\pi = 3.14$. 2

4. A hotel books taxis from a company called **QUICKCARS**.
The receptionist notes the waiting time for every taxi ordered over a period
of two weeks.
The times are recorded in the stem and leaf diagram shown below.

```
<table>
<thead>
<tr>
<th>Waiting time (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>
```

$n = 14$ 1|3 represents 13 minutes

(a) For the given data, calculate:
(i) the median; 1
(ii) the lower quartile; 1
(iii) the upper quartile. 1

(b) Calculate the semi-interquartile range.
In another two week period, the hotel books taxis from a company called
**FASTCABS**.
The semi-interquartile range for **FASTCABS** is found to be 2.5 minutes.

(c) Which company provides the more consistent service?

**Give a reason for your answer.** 1
5. Part of the graph of \( y = a \sin bx^\circ \) is shown in the diagram.

State the values of \( a \) and \( b \).

6. (a) Express \( \frac{\sqrt{40}}{\sqrt{2}} \) as a surd in its simplest form.

(b) Simplify \( \frac{2x + 2}{(x + 1)^2} \).

[Turn over for Questions 7 and 8 on Page six]
C is the centre of two concentric circles.
AB is a tangent to the smaller circle and a chord of the larger circle.
The radius of the smaller circle is 6 centimetres and the chord AB has length 16 centimetres.
Calculate the radius of the larger circle.

8.  (a) Factorise $7 + 6x - x^2$.
(b) Hence write down the roots of the equation
\[ 7 + 6x - x^2 = 0. \]
(c) The graph of $y = 7 + 6x - x^2$ is shown in the diagram.

Find the coordinates of the turning point.
Read carefully

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FORMULAE LIST

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Standard deviation: $s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum x^2 - (\sum x)^2 / n}{n-1}}$, where $n$ is the sample size.
1. The tangent, MN, touches the circle, centre O, at L.
   Angle JLN = 47°.
   Angle KPL = 31°.
   Find the size of angle KLJ.

2. A sample of shoppers was asked which brand of washing powder they preferred.
   The responses are shown below.

<table>
<thead>
<tr>
<th>Washing Powder</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dazzle</td>
<td>250</td>
</tr>
<tr>
<td>Cyclo</td>
<td>375</td>
</tr>
<tr>
<td>Surfer</td>
<td>125</td>
</tr>
<tr>
<td>Cleano</td>
<td>250</td>
</tr>
</tbody>
</table>

   Construct a pie chart to illustrate this information.
   **Show all your working.**

[Turn over]
3. Seats on flights from London to Edinburgh are sold at two prices, £30 and £50.
   On one flight a total of 130 seats was sold.
   Let \( x \) be the number of seats sold at £30 and \( y \) be the number of seats sold at £50.

   (a) Write down an equation in \( x \) and \( y \) which satisfies the above condition.  
   The sale of the seats on this flight totalled £6000.

   (b) Write down a second equation in \( x \) and \( y \) which satisfies this condition.

   (c) How many seats were sold at each price?  

4. A bath contains 150 litres of water.
   Water is drained from the bath at a steady rate of 30 litres per minute.
   The graph of the volume, \( V \) litres, of water in the bath against the time, \( t \) minutes, is shown below.

   ![Graph](image)

   Write down an equation connecting \( V \) and \( t \).  

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Page four
5. A gardener grows tomatoes in his greenhouse.
The temperature of the greenhouse, in degrees Celsius, is recorded every day at noon for one week.

17 22 25 16 21 16 16

(a) For the given temperatures, calculate:
   (i) the mean;
   (ii) the standard deviation.

Show clearly all your working.

For best growth, the mean temperature should be \((20 \pm 5)^\circ\text{C}\) and the standard deviation should be less than \(5^\circ\text{C}\).

(b) Are the conditions in the greenhouse likely to result in best growth?

Explain clearly your answer.
6. A garden trough is in the shape of a prism.

The height of the trough is 25 centimetres.
The cross-section of the trough consists of a rectangle and two semi-circles with measurements as shown.

(a) Find the volume of the garden trough in cubic centimetres.
Give your answer correct to two significant figures.

A new design of garden trough is planned by the manufacturer.

The height of the trough is 20 cm.
The uniform cross-section of this trough is a quarter of a circle.
The volume of the trough is 30 000 cm$^3$.

(b) Find the radius of the cross-section.
7. Change the subject of the formula

\[ y = ax^2 + c \]

to \( x \).

\[ \text{Marks} \quad 3 \]

8. The diagram below shows a big wheel at a fairground.

The wheel has sixteen chairs equally spaced on its circumference.
The radius of the wheel is 9 metres.
As the wheel rotates in an anticlockwise direction, find the distance a chair travels in moving from position T to position P in the diagram.

\[ \text{Marks} \quad 4 \]

9. Solve the equation

\[ 2x^2 + 4x - 9 = 0, \]

giving the roots correct to one decimal place.

\[ \text{Marks} \quad 4 \]

[Turn over for Questions 10 to 12 on Page eight]
10. The sketch shows a parallelogram, PQRS.

(a) Calculate the size of angle PQR.
   Do not use a scale drawing.
   (b) Calculate the area of the parallelogram.

11. (a) Express

\[ a^{\frac{1}{2}} (a^{\frac{3}{2}} - a^{\frac{1}{2}}) \]

in its simplest form.

(b) Express

\[ \frac{a}{x} - \frac{b}{y}, \quad x \neq 0, \quad y \neq 0, \]

as a fraction in its simplest form.

12. (a) Solve the equation

\[ 2 \tan x^\circ + 7 = 0, \quad 0 \leq x < 360. \]

(b) Prove that

\[ \sin^3 x^\circ + \sin x^\circ \cos^2 x^\circ = \sin x^\circ. \]