Indices - Lesson 3

Indices - Fractional Powers

LI

- Know how to work out fractional powers.
- Simplify expressions using fractional powers.

SC

- Notation.

Fractional Powers

$a^{1/n}$ means $\sqrt[n]{a}$

(the n^{th} root of a)

Example 1

$$3^2 = 9$$

$$3 = \sqrt[2]{9}$$

$$3 = \sqrt{9}$$

$$3 = 9^{1/2}$$

Example 2

$$4^3 = 64$$

$$4 = \sqrt[3]{64}$$

$$4 = 64^{1/3}$$

Some Common Roots

$a^{\frac{1}{2}}$	$\sqrt{a}$	square root
$a^{\frac{1}{3}}$	$\sqrt[3]{a}$	cube root
$a^{\frac{1}{4}}$	$\sqrt[4]{a}$	fourth root
$a^{\frac{1}{5}}$	$\sqrt[5]{a}$	fifth root

First Form :

The 5th Rule of Indices :

$$a^{m/n} = \sqrt[n]{a^m}$$

Index Form

Root Form

Second Form :

The 5th Rule of Indices :

$$a^{m/n} = (\sqrt[n]{a})^m$$

Example 3

Write these in root form :

(a) $x^{3/5}$

$$= \sqrt[5]{x^3}$$

(b) $p^{2/7}$

$$= \sqrt[7]{p^2}$$

(c) $N^{13/11}$

$$= \sqrt[11]{N^{13}}$$

(d) $f^{-7/9}$

$$= \frac{1}{f^{7/9}}$$

$$= \frac{1}{\sqrt[9]{f^7}}$$

Example 4

Write these in index form :

(a) $\sqrt[3]{b^4}$

$= \boxed{b^{4/3}}$

(b) $\sqrt[8]{M^6}$

$= M^{6/8}$

$= \boxed{M^{3/4}}$

(c) $\sqrt[6]{v^6}$

$= v^{6/6}$

$= v^1$

$= \boxed{v}$

(d) $\dfrac{1}{\sqrt[17]{x^{15}}}$

$= \dfrac{1}{x^{15/17}}$

$= \boxed{x^{-15/17}}$

Example 5

Evaluate :

(a) $49^{1/2}$

$= \sqrt{49}$

$= \boxed{7}$

(b) $125^{-1/3}$

$= \frac{1}{125^{1/3}}$

$= \frac{1}{\sqrt[3]{125}}$

$= \boxed{\frac{1}{5}}$

(c) $125^{2/3}$

$= \left(\sqrt[3]{125}\right)^2$

$= 5^2$

$= \boxed{25}$

(d) $64^{-3/2}$

$= \frac{1}{64^{3/2}}$

$= \frac{1}{\left(\sqrt{64}\right)^3}$

$= \frac{1}{8^3}$

$= \boxed{\frac{1}{512}}$

Questions

1 Use the rules to express the following with root signs of the form $\sqrt[n]{a^m}$.

a $a^{\frac{1}{3}}$ **b** $a^{\frac{1}{5}}$ **c** $t^{\frac{1}{2}}$ **d** $a^{\frac{2}{3}}$ **e** $a^{\frac{3}{5}}$

f $t^{\frac{5}{2}}$ **g** $x^{\frac{4}{3}}$ **h** $y^{\frac{2}{5}}$ **i** $p^{\frac{1}{4}}$ **j** $m^{\frac{3}{4}}$

2 Write in index form.

a $\sqrt{t^5}$ **b** $\sqrt[4]{a^3}$ **c** $\sqrt[5]{x^3}$ **d** $\sqrt[7]{m^4}$ **e** $\sqrt[3]{a^{12}}$

3 Evaluate.

a $9^{\frac{1}{2}}$ **b** $16^{\frac{1}{4}}$ **c** $8^{\frac{2}{3}}$ **d** $49^{\frac{3}{2}}$ **e** $25^{-\frac{1}{2}}$

f $81^{-\frac{3}{4}}$ **g** $100^{-\frac{3}{2}}$ **h** $\left(\frac{1}{27}\right)^{\frac{2}{3}}$ **i** $\left(\frac{49}{81}\right)^{\frac{1}{2}}$ **j** $\left(\frac{16}{25}\right)^{\frac{3}{2}}$

Answers

1 a $\sqrt[3]{a}$	2 a $t^{\frac{5}{2}}$	3 a 3
b $\sqrt[5]{a}$	b $a^{\frac{3}{4}}$	b 2
c $\sqrt{t}$	c $x^{\frac{3}{5}}$	c 4
d $\sqrt[3]{a^2}$	d $m^{\frac{4}{7}}$	d 343
e $\sqrt[5]{a^3}$	e $a^{\frac{12}{3}} = a^4$	e $\frac{1}{5}$
f $\sqrt{t^5}$		f $\frac{1}{27}$
g $\sqrt[3]{x^4}$		g $\frac{1}{1{,}000}$
h $\sqrt[5]{y^2}$		h $\frac{1}{9}$
i $\sqrt[4]{p}$		i $\frac{7}{9}$
j $\sqrt[4]{m^3}$		j $\frac{64}{125}$