

Indices - Lesson 2

Indices - Powers of Powers

LI

- Know how to work out powers of powers.
- Simplify expressions using powers of powers.

SC

- Notation.

Reminder on previous lesson

$$a^0 \text{ equals } 1$$

Powers of Powers

$$\begin{aligned}(10^3)^2 &= (10 \times 10 \times 10)^2 \\ &= 10 \times 10 \times 10 \times 10 \times 10 \times 10 \\ &= 1\,000\,000\end{aligned}$$

$$\therefore (10^3)^2 = 10^6$$

We thus have the 4th Rule of Indices :

$$(a^m)^n = a^{m \times n}$$

(m, n are any numbers)

Example 1

Simplify :

$$(a) \quad (2^3)^4$$

$$= 2^{3 \times 4}$$

$$= 2^{12}$$

$$(b) \quad (3^0)^{-5}$$

$$= 3^{0 \times (-5)}$$

$$= 3^0$$

$$= 1$$

$$(c) \quad (4^{-1})^{-18}$$

$$= 4^{(-1) \times (-18)}$$

$$= 4^{18}$$

$$(d) \quad (6^5)^8$$

$$= 6^{5 \times 8}$$

$$= 6^{40}$$

Example 2

Simplify :

$$\begin{aligned} \text{(a)} \quad & (2x^4)^5 \\ &= 2^5 \times (x^4)^5 \\ &= 32 \times x^{20} \\ &= \boxed{32x^{20}} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & (3y^{-4})^{-3} \\ &= 3^{-3} \times (y^{-4})^{-3} \\ &= 3^{-3} \times y^{12} \\ &= \boxed{\frac{y^{12}}{27}} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & (2a^3b^4)^5 \\ &= 2^5 \times (a^3)^5 \times (b^4)^5 \\ &= 32 \times a^{15} \times b^{20} \\ &= \boxed{32a^{15}b^{20}} \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad & (3c^{-12}x^{15}w^{32})^{-4} \\ &= 3^{-4} \times (c^{-12})^{-4} \times (x^{15})^{-4} \times (w^{32})^{-4} \\ &= 3^{-4} \times c^{48} \times x^{-60} \times w^{-128} \\ &= \boxed{\frac{c^{48}x^{-60}w^{-128}}{81}} \end{aligned}$$

Questions

1 Simplify the following.

a $(3^4)^5$ **b** $(2^3)^4$ **c** $(10^5)^3$ **d** $(t^3)^{-4}$ **e** $(a^7)^3$

2 Simplify the following.

a $(3y)^2$ **b** $(x^3y^4)^5$ **c** $(ab^3)^4$
d $(3p^4q^2)^3$ **e** $(2t^3u^{-2})^4$ **f** $(10u^{-5}v^{-2})^3$

3 Simplify the following.

a $(6^4)^3$ **b** $(2^7)^4$ **c** $(a^5)^6$
d $(t^{-3})^7$ **e** $(x^{-2})^{-5}$ **f** $(6a^3b^4)^2$
g $(2x^{-3}y^5)^4$ **h** $(3a^6b^{-3})^5$ **i** $(x^4y^{-2}z^3)^3$

Answers

1	a	3^{20}	2	a	$9y^2$	3	a	6^{12}
	b	2^{12}		b	$x^{15}y^{20}$		b	2^{28}
	c	10^{15}		c	a^4b^{12}		c	a^{30}
	d	t^{-12}		d	$27p^{12}q^6$		d	$t^{-21} = \frac{1}{t^{21}}$
	e	a^{21}		e	$16t^{12}u^{-8} = \frac{16t^{12}}{u^8}$		e	x^{10}
				f	$1,000u^{-15}v^{-6} = \frac{1,000}{u^{15}v^6}$		f	$36a^6b^8$
							g	$16x^{-12}y^{20} = \frac{16y^{20}}{x^{12}}$
							h	$243a^{30}b^{-15} = \frac{243a^{30}}{b^{15}}$
							i	$x^{12}y^{-6}z^9$