Indices - Lesson 1

Indices - Multiplying and Dividing Rules

LI

- Know what an index (pl. indices) is.
- Know and use the Rules of Indices for x and ÷.

SC

- + and − numbers.

An index is a power (aka exponent)

$$2^3 = 2 \times 2 \times 2$$

$$3^4 = 3 \times 3 \times 3 \times 3$$

Rules of Indices

$$10^3 \times 10^2 = (10 \times 10 \times 10) \times (10 \times 10)$$

$$= 1\,000 \times 100$$

$$= 100\,000$$

$$\therefore \quad 10^3 \times 10^2 = 10^5$$

We thus have the 1st Rule of Indices :

$$a^m \times a^n = a^{m+n}$$

(m, n are any numbers)

Example 1

Simplify fully :

(a) $3^{4} \times 3^{7}$

$= 3^{4+7}$

$= \boxed{3^{11}}$

(b) $w^{17} \times w^{-2}$

$= w^{17+(-2)}$

$= \boxed{w^{15}}$

Example 2

Simplify fully :

(a) $5m^{3} \times 3m^{6}$

$= 15m^{3+6}$

$= \boxed{15m^{9}}$

(b) $6w^{-3} \times 8w^{-9}$

$= 48w^{-3+(-9)}$

$= \boxed{48w^{-12}}$

$$10^6 \div 10^2 = (10 \times 10 \times 10 \times 10 \times 10 \times 10) \div (10 \times 10)$$

$$= 1\,000\,000 \div 100$$

$$= 10\,000$$

$$\therefore\ 10^6 \div 10^2 = 10^4$$

We thus have the 2nd Rule of Indices :

$$a^m \div a^n = a^{m-n}$$

(m, n are any numbers)

Example 3

Simplify fully :

(a) $5^{9} \div 5^{7}$

$= 5^{9-7}$

$= \boxed{5^{2}}$

(b) $f^{11} \div f^{-3}$

$= f^{11-(-3)}$

$= \boxed{f^{14}}$

Example 4

Simplify fully :

(a) $20\,s^{30} \div 10\,s^{20}$

$= 2\,s^{30-20}$

$= \boxed{2\,s^{10}}$

(b) $98\,x^{14} \div 4\,x^{7}$

$= (98/4)\,x^{14-7}$

$= \boxed{(49/2)\,x^{7}}$

$$10^3 \div 10^3 = (10 \times 10 \times 10) \div (10 \times 10 \times 10)$$

$$= 1\,000 \div 1\,000$$

$$= 1$$

But $10^3 \div 10^3 = 10^{3-3} = 10^0$ (using Rule 2)

We thus have the 3rd Rule of Indices :

$$a^0 = 1$$

Some Notation

$$a^{-m} = \frac{1}{a^m}$$

This is shorthand for this

Example 5

Simplify fully, expressing the answers with positive indices :

(a) $3D^{3}y^{-2} \times 4D^{-7}y^{5}$

$= 12D^{-4}y^{3}$

$= 12 \times D^{-4} \times y^{3}$

$= 12 \times \dfrac{1}{D^{4}} \times y^{3}$

$= \boxed{\dfrac{12y^{3}}{D^{4}}}$

(b) $\dfrac{8m^{13}a^{6} \times 3m^{-9}a^{2}}{48m^{4}a^{87}}$

$= \dfrac{24m^{4}a^{8}}{48m^{4}a^{87}}$

$= \boxed{\dfrac{1}{2a^{79}}}$

Questions

1 Simplify these expressions. Write your answer in index form with a positive exponent.

a $4^5 \times 4^3$	**b** $7^4 \times 7$	**c** $x^{10} \times x^2$	**d** $t^2 \times t^3 \times t^4$
e $3^2 \times 3^{-7}$	**f** $c^3 \times c^{-9}$	**g** $a^8 \times a^{-8}$	**h** $4y^3 \times 5y^6$
i $c \times 4c^2 \times 2c^3$	**j** $8c^2 \times 3c^{-7}$	**k** $10a^7 \times 3a^{-20}$	**l** $4t^3 \times 3t^{-8} \times 2t^2$

2 Simplify these expressions leaving your answer in index form.

a $3^7 \div 3^2$	**b** $6 \div 6^3$	**c** $x^8 \div x^5$	**d** $t^3 \div t$
e $p^3 \div p^{-2}$	**f** $y^{-3} \div y^{-3}$	**g** $12y^{10} \div 3y^3$	**h** $24y^3 \div 12y^8$
i $15x^2 \div 3x^{-4}$	**j** $42p^6 \div (-7p)^{-2}$	**k** $\dfrac{4t^5 \times -7t^3}{14t^{-4}}$	**l** $\dfrac{5y^2 \times 4y^{-6}}{2y^3}$

3 Simplify these expressions.

a $3x^2y \times 5x^3y^2$	**b** $3a^2b^3 \times 7ab^4$	**c** $30x^3y \div 6x^2y^4$

Answers

1 **a** 4^8	**2** **a** 3^5	**3** **a** $15x^5y^3$
b 7^5	**b** 6^{-2}	**b** $21a^3b^7$
c x^{12}	**c** x^3	**c** $5xy^{-3}$
d t^9	**d** t^2	
e $\frac{1}{3^5}$	**e** p^5	
f $\frac{1}{c^6}$	**f** y^0 = 1	
g a^0 = 1	**g** $4y^7$	
h $20y^9$	**h** $2y^{-5}$	
i $8c^6$	**i** $5x^6$	
j $\frac{24}{c^5}$	**j** $2058p^8$	
k $\frac{30}{a^{13}}$	**k** $-2t^{12}$	
l $\frac{24}{t^3}$	**l** $10y^{-7}$	