

# Higher Derivatives

In the following,  $a$ ,  $b$  and  $n$  are real constants  
and  $f(x)$  a real-valued function.

Focus on the special cases.

\* indicates the only derivatives given in the exam.

## Powers of a Generic Function

$$\frac{d}{dx} (f(x))^n = n (f(x))^{n-1} f'(x)$$

## Powers of a Linear Function ( $f(x) = ax + b$ )

$$\frac{d}{dx} (ax + b)^n = na (ax + b)^{n-1}$$

## Special cases

- $b = 0$  gives  $\frac{d}{dx} Ax^n = nA x^{n-1}$  ( $A \stackrel{\text{def}}{=} a^n$ )
- $b = 0, a = 1$  gives  $\frac{d}{dx} x^n = n x^{n-1}$
- $b = 0, n = 1$  gives  $\frac{d}{dx} ax = a$
- $a = 0, n = 1$  gives  $\frac{d}{dx} b = 0$

- $b = 0, a = 1, n = \frac{1}{2}$  gives  $\frac{d}{dx} x^{\frac{1}{2}} = \frac{1}{2} x^{-\frac{1}{2}}$ , which is commonly written  $\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$

### Sine and Cosine ( $f(x) = \sin(ax + b), \cos(ax + b)$ )

$$\frac{d}{dx} (\sin(ax + b))^n = na (\sin(ax + b))^{n-1} \cos(ax + b)$$

$$\frac{d}{dx} (\cos(ax + b))^n = -na (\cos(ax + b))^{n-1} \sin(ax + b)$$

### Special cases

- $a \neq 0, n = 1$  gives

$$\frac{d}{dx} \sin(ax + b) = a \cos(ax + b)$$

$$\frac{d}{dx} \cos(ax + b) = -a \sin(ax + b)$$

- $a \neq 0, b = 0, n = 1$  gives

$$\frac{d}{dx} \sin ax = a \cos ax \quad *$$

$$\frac{d}{dx} \cos ax = -a \sin ax \quad *$$

- $a = 1, b = 0, n = 1$  gives

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

- $a \neq 0, b = 0, n = 2$  gives

$$\frac{d}{dx} \sin^2 ax = 2a \sin ax \cos ax$$

$$\frac{d}{dx} \cos^2 ax = -2a \sin ax \cos ax$$

- $a = 1, b = 0, n = 2$  gives

$$\frac{d}{dx} \sin^2 x = 2 \sin x \cos x$$

$$\frac{d}{dx} \cos^2 x = -2 \sin x \cos x$$