2013 Mathematics

Higher

Finalised Marking Instructions

General Comments

These marking instructions are for use with the 2013 Higher Mathematics Examination.

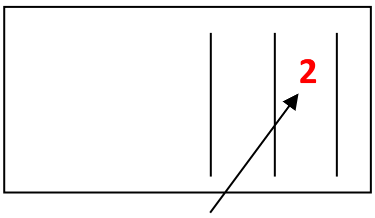
For each question the marking instructions are in two sections, namely **Illustrative Scheme** and **Generic Scheme**. The **Illustrative Scheme** covers methods which are commonly seen throughout the marking. The **Generic Scheme** indicates the rationale for which each mark is awarded. In general markers should use the **Illustrative Scheme** and only use the **Generic Scheme** where a candidate has used a method not covered in the **Illustrative Scheme**.

All markers should apply the following general marking principles throughout their marking:

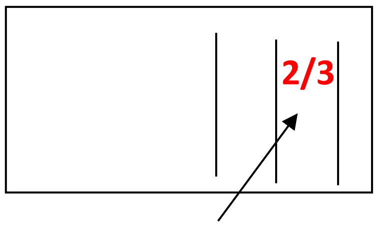
1 Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than deducted for what is wrong.

2 Award one mark for each $\bullet$. There are **no** half marks.

3 The mark awarded for **each part** of a question should be entered in the **outer** right hand margin, opposite the end of the working concerned. The marks should correspond to those on the question paper and these marking instructions. Only the mark, **as a whole number**, should be written.



Marks in this column whole numbers only



Do not record marks on scripts in this manner.

4 Where a candidate has not been awarded any marks for an attempt at a question, or part of a question, 0 should be written in the right hand margin against their answer. It should not be left blank. If absolutely no attempt at a question, or part of a question, has been made, ie a completely empty space, then NR should be written in the outer margin.

5 Every page of a candidate's script should be checked for working. Unless blank, every page which is devoid of a marking symbol should have a tick placed in the bottom right hand margin.

6 Where the solution to part of a question is fragmented and continues later in the script, the marks should be recorded at the end of the solution. This should be indicated with a down arrow (↓), in the margin, at the earlier stages.

7 Working subsequent to an error must be **followed through**, with possible full marks for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working for a follow through mark has been eased, the follow through mark cannot be awarded.

8 As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Throughout this paper, unless specifically mentioned in the marking instructions, a correct answer with no working receives no credit.

9 Marking Symbols

No comments or words should be written on scripts. Please use the following symbols and those indicated on the welcome letter and from comment 6 on the previous page.

✓ A tick should be used where a piece of working is correct and gains a mark. Markers must check through the whole of a response, ticking the work only where a mark is awarded.

___ ✗ At the point where an error occurs, the error should be underlined and a cross used to indicate where a mark has not been awarded. If no mark is lost the error should only be underlined, i.e. a cross is only used where a mark is not awarded.

A cross-tick should be used to indicate "correct" working where a mark is awarded as a result of **follow through** from an error.

A double cross-tick should be used to indicate correct working which is irrelevant or insufficient to score any marks. This should also be used for working which has been **eased**.

A tilde should be used to indicate a minor error which is not being penalised, e.g. **bad form**.

✓ This should be used where a candidate is given the **benefit of the doubt**.

^ A roof should be used to show that something is missing, such as part of a solution or a crucial step in the working.

These will help markers to maintain consistency in their marking and essential for the later stages of SQA procedures.

The examples below illustrate the use of the marking symbols .

Example 1

$y = x^3 - 6x^2$

$\frac{dy}{dx} = 3x^2 - \underline{12}$ ✓ ✗

$3x^2 - 12 = 0$

$x = 2$ ^

$y = -16$

$\bullet^1$ ✓

$\bullet^2$ ✗

$\bullet^3$

$\bullet^4$ ^

$\bullet^5$

Example 2

$\text{A}(4,4,0),\ \text{B}(2,2,6),\ \text{C}(2,2,0)$

$\overrightarrow{\text{AB}} = \underline{\mathbf{b}+\mathbf{a}} = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}$ ✗ $\bullet^1$

$\overrightarrow{\text{AC}} = \begin{pmatrix} 6 \\ 6 \\ 0 \end{pmatrix}$ $\bullet^2$ (repeated error)

Example 3

$3\sin x - 5\cos x$

$k\sin x\cos a - \cos x\sin a$ ✓ $\bullet^1$

$k\cos a = 3,\ k\sin a = 5$ ✓ $\bullet^2$

Example 4

$$\begin{array}{r|rrrr} 4 & 1 & -5 & 2 & 8 \\ & & 4 & -4 & -8 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

✓ $\bullet^1$ ✓ $\bullet^2$

Since the remainder is 0, $x-4$ must be a factor. ✓ $\bullet^3$

$(x^2 - x - 2)$ ✓ $\bullet^4$

$(x-4)(x+1)(x-2)$ ✓ $\bullet^5$

$x = 4$ or $x = -1$ or $x = 2$ ✓ $\bullet^6$

10 In general, as a consequence of an error perceived to be trivial, casual or insignificant, e.g. $6\times 6=12$, candidates lose the opportunity of gaining a mark. But note example 4 in comment 9 and the second example in comment 11.

11 Where a transcription error (paper to script or within script) occurs, the candidate should be penalised, e.g.

This is a transcription error and so the mark is not awarded.

Eased as no longer a solution of a quadratic equation.

$x^2+5x+7=9x+4$ ✓

$\underline{x}-4x+3=0$ ✗

$x=1$ ✓

Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt.

$x^2+5x+7=9x+4$ ✓

$x-4x+3=0$ ✓

$(x-3)(x-1)=0$

$x=1$ or 3 ✓

12 **Cross marking**

Where a question results in two pairs of solutions, this technique should be applied, but only if indicated in the detailed marking instructions for the question.

Example: Point of intersection of line with curve

Illustrative Scheme:		Cross marked:	
$\bullet^5$	$x=2,\ x=-4$	$\bullet^5$	$x=2,\ y=5$
$\bullet^6$	$y=5,\ y=-7$	$\bullet^6$	$x=-4,\ y=-7$

Markers should choose whichever method benefits the candidate, but **not** a combination of both.

13 In final answers, numerical values should be simplified as far as possible.

Examples: $\frac{15}{12}$ should be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\qquad$ $\frac{43}{1}$ should be simplified to 43

$\frac{15}{0{\cdot}3}$ should be simplified to 50 $\qquad$ $\frac{4/5}{3}$ should be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8

The square root of perfect squares up to and including 100 must be known.

14 Regularly occurring responses (ROR) are shown in the marking instructions to help mark common and/or non-routine solutions. RORs may also be used as a guide in marking similar non-routine candidate responses.

15 Unless specifically mentioned in the marking instructions, the following should not be penalised:

- Working subsequent to a **correct** answer;
- Correct working in the wrong part of a question;
- Legitimate variations in numerical answers, e.g. angles in degrees rounded to nearest degree;
- Omission of units;
- Bad form;
- Repeated error within a question, but not between questions or papers.

16 In any 'Show that . . .' question, where the candidate has to arrive at a formula, the last mark of that part is not available as a follow through from a previous error.

17 All working should be carefully checked, even where a fundamental misunderstanding is apparent early in the candidate's response. Marks may still be available later in the question so reference must be made continually to the marking instructions. All working must be checked: the appearance of the correct answer does not necessarily indicate that the candidate has gained all the available marks.

18 In the **exceptional** circumstance where you are in doubt whether a mark should or should not be awarded, consult your Team Leader (TL).

19 Scored out working which **has not been replaced** should be marked where still legible. However, if the scored out working **has been replaced**, only the work which has not been scored out should be marked.

20 Where a candidate has made multiple attempts using the same strategy, mark all attempts and award the lowest mark.
Where a candidate has tried different strategies, apply the above ruling to attempts within each strategy and then award the highest resultant mark. For example:

Strategy 1 attempt 1 is worth 3 marks	Strategy 2 attempt 1 is worth 1 mark
Strategy 1 attempt 2 is worth 4 marks	Strategy 2 attempt 2 is worth 5 marks
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

21 It is of great importance that the utmost care should be exercised in totalling the marks.
A tried and tested procedure is as follows:

Step 1 Manually calculate the total from the candidate's script.

Step 2 Check this total using the grid issued with these marking instructions.

Step 3 In SCORIS, enter the marks and obtain a total, which should now be compared to the manual total.

This procedure enables markers to identify and rectify any errors in data entry before submitting each candidate's marks.

22 **The candidate's script for Paper 2 should be placed inside the script for Paper 1, and the candidate's total score (i.e. Paper 1 Section B + Paper 2) written in the space provided on the front cover of the script for Paper 1.**

23 In cases of difficulty, covered neither in detail nor in principle in these instructions, markers should contact their TL in the first instance. A referral to the Principal Assessor (PA) should only be made in consultation with the TL. Further details of PA Referrals can be found in The General Marking Instructions.

Paper 1 Section A

Question	Answer
1	A
2	B
3	B
4	A
5	D
6	C
7	B
8	C
9	A
10	D
11	B
12	C
13	A
14	B
15	C
16	C
17	C
18	D
19	B
20	D

Summary	A	4
	B	6
	C	6
	D	4

Paper 1- Section B

Question	Generic Scheme	Illustrative Scheme	Max Mark
21	Express $2x^2 + 12x + 1$ in the form $a(x+b)^2 + c$.		
		Method 1	
	•1 ss identify common factor	•1 $2(x^2 + 6x\ldots$ **stated or implied by •2**	
	•2 ss complete the square	•2 $2(x+3)^2$	3
	•3 pd process for c	•3 $2(x+3)^2 - 17$	
		Method 2	
	•1 ss expands completed square form	•1 $ax^2 + 2abx + ab^2 + c$	
	•2 ss equates coefficients	•2 $a = 2 \quad 2ab = 12 \quad ab^2 + c = 1$	
	•3 pd process for b and c and write in required form	•3 $2(x+3)^2 - 17$	

Notes:

1. Correct answer without working gains full credit.

Regularly Occurring Responses:

Candidate A

$2(x^2 + 6x + \frac{1}{2})$ •1 ✓

$2(x^2 + 6x + 9 - 9 + \frac{1}{2})$ •2 ✓

$2(x+3)^2 - 8\frac{1}{2}$ •3 ✗

Candidate B

$2x^2 + 12x + 1 = (x+6)^2 - 36 + 1$ •1 ✗ •2 ✗

$= (x+6)^2 - 35$ •3 ✓✗

Candidate C

$a(x+b)^2 + c = ax^2 + 2abx + ab^2 + c$ •1 ✓

$a = 2 \quad 2ab = 12 \quad ab^2 + c = 1$ •2 ✓

$b = 3 \quad c = -17$ •3 ✓

•3 awarded as all working relates to completed square form

Candidate D

$ax^2 + 2abx + ab^2 + c$ •1 ✓

$a = 2 \quad 2ab = 12 \quad ab^2 + c = 1$ •2 ✓

$b = 3 \quad c = -17$ •3 ✗

•3 is lost as no reference is made to completed square form

Candidate E

$ax^2 + 2abx + ab^2 + c$ •1 ✓

$a = 2 \quad 2ab = 12 \quad b^2 + c = 1$ •2 ✗

$a = 2 \quad b = 3 \quad c = -8$

$2(x+3)^2 - 8$ •3 ✓✗

Candidate F

$2\left(x^2 + 12x\right) + 1$ •1 ✗

Question		Generic Scheme	Illustrative Scheme	Max Mark
22	A circle C_1 has equation $x^2 + y^2 + 2x + 4y - 27 = 0$.			
	a	Write down the centre and calculate the radius of C_1.		
		•[1] ic state centre •[2] pd find radius	•[1] $(-1, -2)$ •[2] $\sqrt{32}$	**2**

Notes:

1. Do not penalise candidates who use -1 and -2 for *g* and *f* when calculating the radius. However, candidates who use -1 and 2 or 1 and -2 lose •[2]
2. $\sqrt{32}$ need not be simplified.

Question		Generic Scheme	Illustrative Scheme	Max Mark
22	**b**	The point P(3, 2) lies on the circle C_1. Find the equation of the tangent at P.		
		•[3] ss find m_{radius} •[4] ic state m_{tangent} •[5] ic state equation of tangent	•[3] 1 •[4] -1 •[5] $y - 2 = -1(x - 3)$	**3**

Notes:

3. •[5] is only available as a result of using a perpendicular gradient.

Regularly Occurring Responses:

Candidate A		**Candidate B**	
$m_{radius} = 1$	•[3] ✓	$m_{radius} = 1$	•[3] ✓
^	•[4] ^	$m_1 m_2 = -1$ so $m_2 = 1$	•[4] ✗
equation of tangent is $y - 2 = 1(x - 3)$	•[5] ✗	$y - 2 = 1(x - 3)$	•[5] ✓

Question		Generic Scheme	Illustrative Scheme	Max Mark
22	**c**	A second circle C_2 has centre (10, –1). The radius of C_2 is half of the radius of C_1. Show that the equation of C_2 is $x^2 + y^2 - 20x + 2y + 93 = 0$.		
		$\bullet^6$ pd find radius $\bullet^7$ ic state equation of circle $\bullet^8$ pd expand and complete	$\bullet^6$ $\sqrt{8}$ **stated or implied by** $\bullet^7$ $\bullet^7$ $(x-10)^2 + (y+1)^2 = \left(\sqrt{8}\right)^2$ $\bullet^8$ $x^2 - 20x + 100 + y^2 + 2y + 1 = 8$ and complete **Accept** $2g = -20,\ 2f = 2$ ¦ Centre $(10, -1)$ $g = -10, f = 1$ ¦ $g = -10,\ f = 1$ $\bullet^6$ Centre $(10, -1)$ ¦ $2g = -20,\ 2f = 2$ $\bullet^7$ $r = \sqrt{(-10)^2 + 1^2 - 93} = \sqrt{8}$ $\bullet^8$ $\sqrt{32} = 2\sqrt{8}$ $\frac{1}{2} \times \sqrt{32} = \frac{1}{2} \times 2\sqrt{8} = \sqrt{8} = $ radius of C_2	**3**

Regularly Occurring Responses:

Candidate C C_2 centre is (10,-1) $g = -10,\ f = 1$ $2g = -20,\ 2f = 2$ $x^2 + y^2 - 20x + 2y + \ldots$ $\bullet^6$ ✓ $\bullet^7$ ✗ $\bullet^8$ ✗	**Candidate D** $x^2 + y^2 - 20x + 2y + 93 = 0$ $2g = -20,\ 2f = 2$ centre (10, -1) radius $= \sqrt{(-10)^2 + 1^2 - 93} = \sqrt{8}$ $\sqrt{32} = \sqrt{4 \times 8} = 2\sqrt{8}$ so radius of $C_2 = \frac{1}{2}$of radius of C_1 $\bullet^6$ ✓ $\bullet^7$ ✓ $\bullet^8$ ✓	**Candidate E** $x^2 + y^2 - 20x + 2y + 93 = 0$ $2g = -20,\ 2f = 2$ centre (10, -1) radius $= \sqrt{(-10)^2 + 1^2 - 93}$ $= \sqrt{8}$ $\sqrt{32} = 4\sqrt{8}\ \ldots.$ $\bullet^6$ ✓ $\bullet^7$ ✓ $\bullet^8$ ✗
Candidate F $x^2 + y^2 - 20x + 2y + 93$ $2g = -20,\ 2f = 2$ centre (10, -1) radius $= \sqrt{(-10)^2 + 1^2 - 93}$ $= \sqrt{8}$ which is half of $\sqrt{32}$ $\bullet^6$ ✓ $\bullet^7$ ✓ $\bullet^8$ ✗	**Candidate G** $x^2 + y^2 - 20x + 2y + 93 = 0$ $2g = -20,\ 2f = 2$ centre (10, -1) radius $= \sqrt{(-10)^2 + 1^2 - 93}$ $= \sqrt{8}$ $\bullet^6$ ✓ $\bullet^7$ ✓ $\bullet^8$ ✗	

Question		Generic Scheme	Illustrative Scheme	Max Mark
22	d	Show that the tangent found in part (*b*) is also a tangent to circle C_2.		
			Method 1	
			Substituting for y	
		•9 ss substitute $y = 5 - x$ (or $x = 5 - y$)	•9 $x^2 + (5 - x)^2 - 20x + 2(5 - x) + 93$	
		•10 pd express in standard quadratic form	•10 $2x^2 - 32x + 128 = 0$	
		•11 ic start proof	•11 $2(x - 8)^2 = 0$ ⎮ •11 $(-32)^2 - 4 \times 2 \times 128$	
		•12 ic complete proof	•12 equal roots $\Rightarrow$ tangent ⎮ •12 $b^2 - 4ac = 0$ $\Rightarrow$ tangent	
			or	
			Substituting for x	
			•9 $(5 - y)^2 + y^2 - 20(5 - y) + 2y + 93 = 0$	
			•10 $2y^2 + 12y + 18 = 0$	
			•11 $2(y + 3)^2 = 0$ ⎮ •11 $12^2 - 4 \times 2 \times 18$	4
			•12 equal roots $\Rightarrow$ tangent ⎮ •12 $b^2 - 4ac = 0$ $\Rightarrow$ tangent	
			Method 2	
		•9 ss uses perpendicular gradients	•9 m given line $= -1$, leading to $m_{radius} = 1$	
		•10 pd find equation of radius	•10 $y + 1 = 1(x - 10)$	
		•11 ic starts proof	•11 $y = -x + 5$ $y = x - 11$ $\Rightarrow x = 8$ $y = -3$	
		•12 ic completes proof	•12 $(8)^2 + (-3)^2 - 20 \times (8) + 2(-3) + 93$ and complete	

Notes:

Method 1

4. = 0 must appear at •9 or •10 stage to gain •10.

5. Candidates who arrive at a quadratic equation which does not have equal roots cannot gain •12 as follow through. (See General Comments Note 16).

6. Where candidates do not arrive at a quadratic equation in Method 1, marks •10, •11 and •12 are not available.

7. Acceptable communication for •12, ‘only one answer so implies tangent’, ‘discriminant is 0 so tangent’, ’$x = 8$ twice so tangent’, or equivalent relating to tangency.

Question	Generic Scheme	Illustrative Scheme	Max Mark
23 a	The expression $\sqrt{3}\sin x^\circ - \cos x^\circ$ can be written in the form $k\sin(x-a)^\circ$, where $k > 0$ and $0 \le a < 360$. Calculate the values of k and a.		
	$\bullet^1$ ss use compound angle formula	$\bullet^1$ $k\sin x^o\cos a^o - k\cos x^o\sin a^o$ **stated explicitly**	4
	$\bullet^2$ ic compare coefficients	$\bullet^2$ $k\cos a^o = \sqrt{3}$ and $k\sin a^o = 1$ **stated explicitly**	
	$\bullet^3$ pd process for k	$\bullet^3$ 2 (do **not** accept $\sqrt{4}$)	
	$\bullet^4$ pd process for a	$\bullet^4$ 30	

Notes:

1. Treat $k\sin x^o\cos a^o$ - $\cos x^o\sin a^o$ as bad form only if the equations at the $\bullet^2$ stage both contain k.
2. $2\sin x^o\cos a^o - 2\cos x^o\sin a^o$ or $2(\sin x^o\cos a^o - \cos x^o\sin a^o)$ is acceptable for $\bullet^1$ and $\bullet^3$.
3. Accept $k\cos a^o = \sqrt{3}$ and $-k\sin a^o = -1$ for $\bullet^2$.
4. $\bullet^2$ is not available for $k\cos x^o = \sqrt{3}$ and $k\sin x^o = 1$, however, $\bullet^4$ is still available.
5. $\bullet^3$ is only available for a single value of k, $k > 0$.
6. $\bullet^4$ is only available for a single value of a expressed in degrees.
7. Candidates who identify and use any form of the wave equation may gain $\bullet^1$, $\bullet^2$ and $\bullet^3$, however, $\bullet^4$ is only available if the value of a is interpreted for the form $k\sin(x-a)^o$.
8. Do not penalise omission of degree sign at $\bullet^1$ or $\bullet^2$.

Regularly Occurring Responses:

Response 1: Missing information in working.

Candidate A	Candidate B	Candidate C
$\bullet^1$ ✗ $2\cos a = \sqrt{3}$ $-2\sin a = -1$ $\bullet^2$ ✓ $\tan a = \frac{1}{\sqrt{3}}$ $\bullet^3$ ✓ $a = 30$ $\bullet^4$ ✓	$\bullet^1$ ✗ $\cos a = \sqrt{3}$ $\sin a = 1$ $\bullet^2$ ✗ $\bullet^3$ ✗ $\tan a = \frac{1}{\sqrt{3}}$ $a = 30$ $\bullet^4$ ✗ Not consistent with evidence at $\bullet^2$	$k\sin x^o\cos a^o - k\cos x^o\sin a^o$ $\bullet^1$ ✓ $k\cos a = \sqrt{3}$, $k\sin a = 1$ $\bullet^2$ ✓ $k = 2$ or -2 $\bullet^3$ ✗ $\tan a = \frac{1}{\sqrt{3}}$ $\bullet^4$ ✗ $a = 30$ or 210 However candidates who then write $\sqrt{3}\sin x^o - \cos x^o = 2\sin(x-30)^o$ would gain $\bullet^3$ and $\bullet^4$

Response 2: Labelling incorrect, sin (A – B) = sin A cos B – cos A sin B from formula list.

Candidate D	Candidate E	Candidate F
$k\sin A\cos B - k\cos A\sin B$ $\bullet^1$ ✗ $k\cos a = \sqrt{3}$ $k\sin a = 1$ $\bullet^2$ ✓ $\tan a = \frac{1}{\sqrt{3}}$ $a = 30$ $\bullet^4$ ✓	$k\sin A\cos B - k\cos A\sin B$ $\bullet^1$ ✗ $k\cos x = \sqrt{3}$ $k\sin x = 1$ $\bullet^2$ ✗ $\tan x = \frac{1}{\sqrt{3}}$ $x = 30$ $\bullet^4$ ✓	$k\sin A\cos B - k\cos A\sin B$ $\bullet^1$ ✗ $k\cos B = \sqrt{3}$ $k\sin B = 1$ $\bullet^2$ ✓ $\tan B = \frac{1}{\sqrt{3}}$ $B = 30$ $\bullet^4$ ✓

Question		Generic Scheme		Illustrative Scheme	Max Mark
23	**b**	Determine the maximum value of $4 + 5\cos x^{\circ} - 5\sqrt{3}\sin x^{\circ}$, where $0 \leq x < 360$.			
•5		ic	interpret expression	•5 $4 - 5 \times 2\sin(x - 30)^{\circ}$	
•6		pd	state maximum	•6 14	**2**

Notes:

9. A solution using calculus gains no marks unless angles are converted to radian measure before differentiating.
10. 'Maximum = 14' with no working gains no marks.
11. •5 is awarded for demonstrating a clear link between the expression in (b) and the wave in part (a)
12. Candidates who start afresh, and use any form of the wave function to arrive at $4 \pm 10\cos(\ldots)$ or $4 \pm 10\sin(\ldots)$ correctly, can gain both •5 and •6 .
13. •6 is only available if, at the •5 stage, the candidate's answer in (a) is multiplied by an integer k, $k \neq \pm 1$.
14. Candidates who equate the given expression to 0 and attempt to solve gain 0 marks.

Regularly Occurring Responses:

Candidate J		Candidate K	
$4 - 5 \times 2\sin(x - 60)^{0}$	•5 ✓	$4 + 2\sin(x - 30)^{0}$	•5 ✗
Max = 14	•6 ✓	Max 2 + 4	
		Max = 6	•6 ✓̶

Question			Generic Scheme		Illustrative Scheme	Max Mark
24	**a**	**i**	Show that the points A(–7, –8, 1), T(3, 2, 5) and B(18, 17, 11) are collinear.			
24	**a**	**ii**	Find the ratio in which T divides AB.			
•[1]	ss		use vector approach	•[1]	$\overrightarrow{AT}=\begin{pmatrix}10\\10\\4\end{pmatrix}$ or $\overrightarrow{TB}=\begin{pmatrix}15\\15\\6\end{pmatrix}$	**4**
•[2]	ic		compare two vectors	•[2]	$\overrightarrow{TB}$ or $\overrightarrow{AT}$ **and** $\overrightarrow{AT}=\frac{2}{3}\overrightarrow{TB}$ or equivalent	
•[3]	ic		complete proof	•[3]	$\overrightarrow{AT}$ and $\overrightarrow{TB}$ are parallel and since there is a common point A, B and T are collinear	
•[4]	ic		state ratio	•[4]	2:3 **stated explicitly** (see Note 4)	

Notes:

1. Any appropriate combination of vectors is acceptable.
2. •[3] can only be awarded if a candidate has stated, common point, parallel (common direction) and collinear.
3. Treat $\begin{pmatrix}10\\10\\4\end{pmatrix}$ written as $(10, 10, 4)$ as bad form.
4. Accept $1:\frac{3}{2}$ or $\frac{2}{3}:1$
5. •[3] requires evidence of vectors being parallel, simply stating parallel is insufficient.

Regularly Occurring Responses:

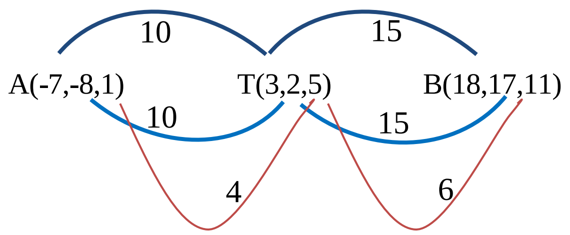
Candidate A	Candidate B	Candidate C
$\overrightarrow{AT}=2\begin{pmatrix}5\\5\\2\end{pmatrix}$ $\overrightarrow{TB}=3\begin{pmatrix}5\\5\\2\end{pmatrix}$ •[2] ✓	$\overrightarrow{AT}=\begin{pmatrix}10\\10\\4\end{pmatrix}$ or $\overrightarrow{TB}=\begin{pmatrix}15\\15\\6\end{pmatrix}$ •[1] ✓	$\overrightarrow{AT}=\begin{pmatrix}10\\10\\4\end{pmatrix}$ *or* $\overrightarrow{TB}=\begin{pmatrix}15\\15\\6\end{pmatrix}$ •[1] ✓
	$\overrightarrow{TB}=\frac{2}{3}\overrightarrow{AT}$ •[2] ✗	$\overrightarrow{TB}=\frac{2}{3}\overrightarrow{AT}$ •[2] ✗
	TB and AT are parallel, T is a common point so A, T and B are collinear. •[3] ✓	TB and AT are parallel, T is a common point so A,T and B are collinear. •[3] ✓
	AT:TB = 2:3 •[4] ✗	AT:TB = 3:2 •[4] ✓

Candidate D

$\overrightarrow{AT}=\begin{pmatrix}10\\10\\4\end{pmatrix}$ or $\overrightarrow{TB}=\begin{pmatrix}15\\15\\6\end{pmatrix}$ •[1] ✓

$\overrightarrow{TB}=\frac{2}{3}\overrightarrow{AT}$ •[2] ✗

TB and AT are parallel. T is a common point so A, T and B are collinear. •[3] ✓



•[4] ✓

10:15 = 10:15 = 4:6 = 2:3

Question		Generic Scheme		Illustrative Scheme	Max Mark
24	b	The point C lies on the x-axis. If TB and TC are perpendicular, find the coordinates of C.			
		Method 1		**Method 1**	
•5	ic	interpret C	•5	$(c, 0, 0)$	
•6	pd	use vector approach	•6	$\overrightarrow{TC}=\begin{pmatrix} c-3 \\ -2 \\ -5 \end{pmatrix}$	
•7	ss	know to use scalar product equal to 0	•7	$\overrightarrow{TB}.\overrightarrow{TC}=0$	
•8	pd	start to solve	•8	$15(c-3)+15\times(-2)+6\times(-5)\ \ldots$	
•9	pd	complete	•9	$c=7$	5
		Method 2		**Method 2**	
•5	ic	interpret C	•5	$(c, 0, 0)$	
•6	pd	use vector approach	•6	$\overrightarrow{TC}=\begin{pmatrix} c-3 \\ -2 \\ -5 \end{pmatrix}$	
•7	ss	know to use Pythagoras and calculate $\lvert\overrightarrow{TC}\rvert$ or $\lvert\overrightarrow{TB}\rvert$	•7	$\lvert\overrightarrow{TC}\rvert=\sqrt{(c-3)^2+4+25}$	
•8	pd	calculate the other two lengths	•8	$\lvert\overrightarrow{TB}\rvert=\sqrt{486}$ and $\lvert\overrightarrow{BC}\rvert=\sqrt{(c-18)^2+289+121}$	
•9	pd	complete	•9	$c=7$	

Notes:

6. In Method 1, = 0 must appear at •7 or •8 for •9 to be available.
7. In Method 1, candidates who use $\overrightarrow{TB}.\overrightarrow{TC}=-1$ can gain a maximum of 4 marks.
8. C must appear in coordinate form at •5 or •9 for •5 to be awarded.
9. If C has more than one non-zero coordinate •9 is not available.
10. •8 is only available for expressions with an unknown.

Regularly Occurring Responses:

Candidate E

$C=(c,\ \ 0,\ \ 0)$ •5 ✓

$\overrightarrow{TC}=\begin{pmatrix} c-3 \\ -2 \\ -5 \end{pmatrix}$ •6 ✓

$\overrightarrow{TB}.\overrightarrow{TC}=-1$ •7 ✗

$15(c-3)+15(-2)+6(-5)=-1$ •8 ✓

$c=\frac{104}{15}$ •9 ✓

Candidate F

$15(c-3)+15\times(-2)+6\times(-5)=0$
$c=7$
$(7, 0, 0)$ **Gains full marks**

Candidate G

$\overrightarrow{TC}=c-\begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$ •6 ^

$\overrightarrow{TB}.\overrightarrow{TC}=15(c-3)+15(c-2)+6(c-5)$ •8 ✗

It is not clear at •6 what is meant by 'c' so •8 cannot be awarded as follow through.

However

$\overrightarrow{TC}=\begin{pmatrix} x-3 \\ y-2 \\ z-5 \end{pmatrix}$ •6 ✓

Paper 2

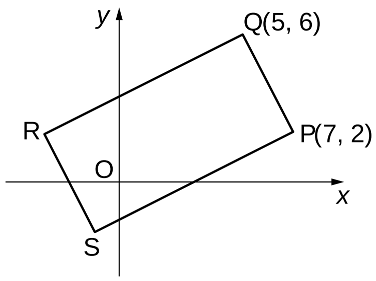
Question	Generic Scheme			Illustrative Scheme		Max Mark
1	The first three terms of a sequence are 4, 7 and 16. The sequence is generated by the recurrence relation $u_{n+1} = mu_n + c$, with $u_1 = 4$. Find the values of m and c.					
	•1	ic	interpret recurrence relation	•1	$7 = 4m + c$	
	•2	ic	interpret recurrence relation	•2	$16 = 7m + c$	**4**
	•3	ss	know to use simultaneous equation	•3	$7m + c = 16$ $4m + c = 7$ leading to	
	•4	pd	find m and c	•4	$m = 3$, $c = -5$	

Notes:

1. Treat equations like $7 = m4 + c$ or $7 = m(4) + c$ as bad form.

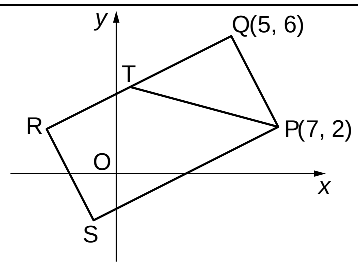
Regularly Occurring Responses:

Candidate A	Candidate B	Candidate C
No working $m = 3$ and $c = -5$ or $u_{n+1} = 3u_n - 5$ 1 mark out of 4	Only one equation $7 = 4m + c$ $m = 3$ and $c = -5$ 2 marks out of 4	Partial verification $m = 3$ and $c = -5$ $3 \times 4 - 5 = 7$ 2 marks out of 4
Candidate D	**Candidate E**	
by verification $m = 3$ and $c = -5$ $3 \times 4 - 5 = 7$ **and** $3 \times 7 - 5 = 16$ 3 marks out of 4	$7 = 4m + c$ $16 = 7m + c$ $m = 3$ and $c = -5$ 4 marks out of 4	

Question		Generic Scheme			Illustrative Scheme	Max Mark
2	a	The diagram shows rectangle PQRS with P(7, 2) and Q(5, 6). Find the equation of QR.				
		•1	ss	know to find gradient	•1 $m_{PQ} = -2$	
		•2	ic	use perpendicular gradient	•2 $m_{QR} = \frac{1}{2}$	3
		•3	ic	state equation of line	•3 $y - 6 = \frac{1}{2}(x - 5)$	

Notes:

1. •3 is only available as a consequence of using a perpendicular gradient and the point Q.
2. $m = \frac{1}{2}$ appearing ex nihilo leading to the correct equation for QR gains 0 marks.

Question		Generic Scheme			Illustrative Scheme	Max Mark
2	b	The line from P with the equation $x + 3y = 13$ intersects QR at T. Find the coordinates of T.				
		•4	ss	prepare to solve	•4 $x + 3y = 13$ **and** $x - 2y = -7$	
		•5	pd	solve for one variable	•5 $x = 1$ **or** $y = 4$	3
		•6	pd	solve for second variable	•6 $y = 4$ **or** $x = 1$	

Notes:

3. Subsequent to making an error in rearranging the equation of QR, •4 can still be awarded but •5 is lost.
4. Stepping out from P to Q and then the reverse from Q is not a valid strategy for obtaining T.
5. •4, •5 and •6 are not available to candidates who: (i) equate zeroes , (ii) give answers only without working.

Regularly Occurring Responses:

Candidate A

$y - 6 = \frac{1}{2}(x - 5)$ leading to

$2y - x = -17$

$x + 3y = 13$ •4 ✓

$5y = -4$ •5 ✗

$y = -\frac{4}{5}$

$x = 15\frac{2}{5}$ •6 ✓

Question		Generic Scheme			Illustrative Scheme		Max Mark
2	c	Given that T is the midpoint of QR, find the coordinates of R and S.					
		•7	ss	valid method eg vectors or stepping out or mid-point formula	•7	eg $\overrightarrow{QT} = \begin{pmatrix}-4\\-2\end{pmatrix}$	3
		•8	ss	know how to find R	•8	R(–3, 2)	
		•9	ss	know how to find S using $\overrightarrow{RS} = \overrightarrow{QP} = \begin{pmatrix}2\\-4\end{pmatrix}$	•9	S (–1, –2)	

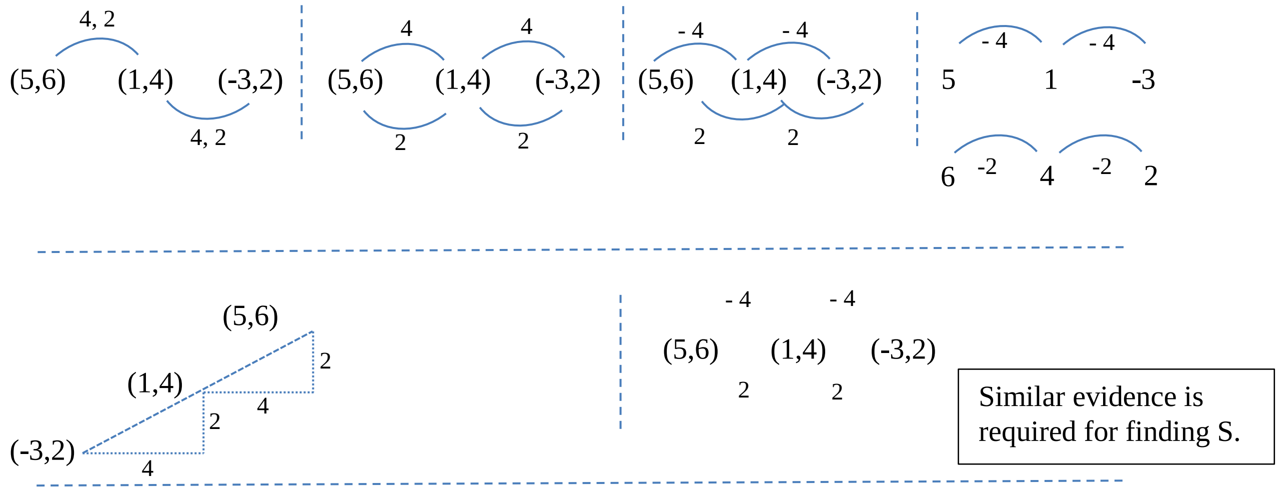
Notes:

6. Any strategy that relies upon the rectangle being composed of two congruent squares can only be given credit if this fact has been justified. Candidates who have already been penalised in 2(b) for making this assumption can gain full credit in (c).

7. If $R(-3,2)$ and $S(-1,-2)$ appear without working then •7, •8 and •9 are not available.

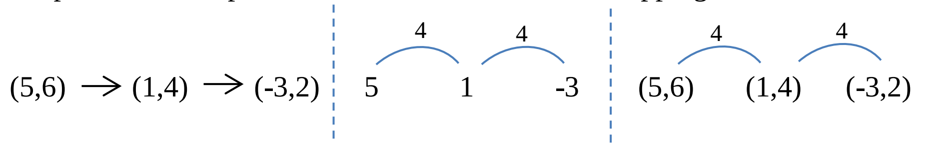
Regularly Occurring Responses:

Response 1: Examples of evidence for stepping out.

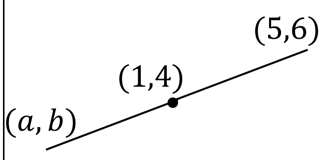


Similar evidence is required for finding S.

Response 2: Examples of insufficient evidence for stepping out.



Candidate B



•7	•8	
$\frac{a+5}{2} = 1$	$\frac{b+6}{2} = 4$	•7
$a = -3$	$b = 2$	•8

Question		Generic Scheme			Illustrative Scheme	Max Mark
3	a	Given that $(x-1)$ is a factor of x^3+3x^2+x-5, factorise this cubic fully.				4
		$\bullet^1$	ss	know to use $x=1$ in synthetic division	$\bullet^1$ $\begin{array}{r\|rrrr} 1 & 1 & 3 & 1 & -5 \\ & & 1 & 4 & 5 \\ \hline \end{array}$	
		$\bullet^2$	pd	complete evaluation	$\bullet^2$ $\begin{array}{rrrr} 1 & 4 & 5 & 0 \end{array}$	
		$\bullet^3$	ic	state quadratic factor	$\bullet^3$ x^2+4x+5	
		$\bullet^4$	ic	valid reason for irreducible quadratic	$\bullet^4$ $(x-1)(x^2+4x+5)$ with valid reason	

Notes:

1. Accept any of the following for $\bullet^4$
   a) $b^2-4ac=16-20<0$, so does not factorise.
   b) $b^2-4ac=16-4\times5<0$, so does not factorise.
   c) $16-4\times5<0$, so does not factorise.

2. Do **not** accept any of the following for $\bullet^4$
   a) $b^2-4ac<0$, so does not factorise.
   b) $(x-1)(x^2+4x+5)$ does not factorise.
   c) $(x-1)(x\ \ldots\ldots)(x\ \ldots\ldots)$ cannot factorise further.

3. Candidates who use algebraic long division to arrive at $(x-1)(x^2+4x+5)$ gain marks $\bullet^1$, $\bullet^2$ and $\bullet^3$.

4. Candidates who complete the square and make a relative comment regarding no real roots gain $\bullet^4$.

5. Treat $(x-1)x^2+4x+5$, with a valid reason, as bad form for $\bullet^4$.

Regularly Occurring Responses:

Candidate A	Candidate B
x^2+4x+5 $\bullet^3$ ✓ $(x-1)(x+5)(x-1)$ $\bullet^4$ ✗	$x^3+3x^2+x-5=(x-1)(..x^2+..x+..)$ $\bullet^1$ ✓ $\bullet^2$ ✓ $=(x-1)(x^2+4x+5)$ $\bullet^3$ ✓ $\bullet^4$ ✓ With a valid reason.

Candidate C
x^2+4x+5 $\bullet^3$ ✓ $(x-1)x^2+4x+5$ $b^2-4ac=16-20<0$ so does not factorise. $\bullet^4$ ✓

Question		Generic Scheme			Illustrative Scheme		Max Mark
3	b	Show that the curve with equation $y = x^4 + 4x^3 + 2x^2 - 20x + 3$ has only one stationary point. Find the x-coordinate and determine the nature of this point.					5
		•5	ss	start to differentiate	•5	two non-zero terms correct	
		•6	pd	complete derivative and equate to 0	•6	$4x^3 + 12x^2 + 4x - 20 = 0$	
		•7	ic	factorise	•7	$4(x-1)(x^2 + 4x + 5)$	
		•8	pd	process for x	•8	$x = 1$	
		•9	ic	justify nature and state conclusion	•9	nature table **and** minimum	

Notes:

6. $= 0$ must appear at •6 or •7 for mark •6 to be gained.
7. •9 can be gained using the second derivative to determine the nature.
8. Candidates who incorrectly obtain more than one linear factor in (a) and use this result in (b) **must** solve to get more than one solution in order to gain •8. Mark •9 is not available.
9. If the equation solved at •8 is not a cubic then •8 and •9 are not available.

Regularly Occurring Responses:

Candidate D

$(x-1)(x+5)(x-1)$ from (a)

leading to

$4x^3 + 12x^2 + 4x - 20 = 0$ •5 ✓ •6 ✓

$4(x^3 + 3x^2 + x - 5) = 0$

$4(x-1)(x+5)(x-1) = 0$ •7

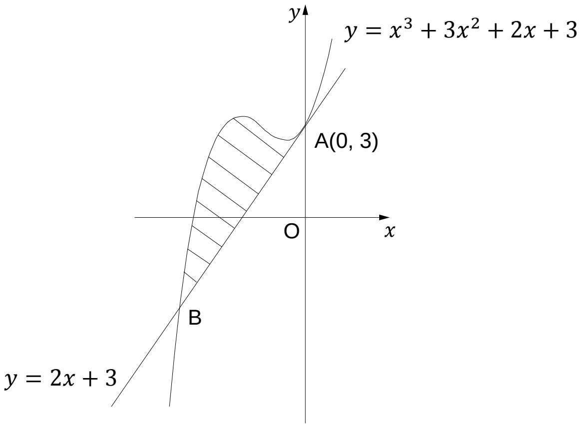
$x = 1$ or $x = -5$ •8 •9 ✗

Candidate E

x	…..	1	…….
$\frac{dy}{dx}$	$-$	0	$+$
		Min	

•9 ✓

Minimum acceptable response.

Question	Generic Scheme			Illustrative Scheme		Max Mark
4	The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at A(0, 3), as shown in the diagram. The line meets the curve again at B. Show that B is the point (–3, –3) and find the area enclosed by the line and the curve.  					
	$\bullet^1$	ss	know how to show that B is the point of intersection of the line and curve.	$\bullet^1$	$(-3)^3 + 3(-3)^2 + 2(-3) + 3 = -3$ **and** $2(-3) + 3 = -3$ **or** solving simultaneous equations	
	$\bullet^2$	ss	know to integrate and interpret limits.	$\bullet^2$	$\int_{-3}^{0} \dots\dots\dots$	**6**
	$\bullet^3$	ic	use " upper – lower"	$\bullet^3$	$\int_{-3}^{0}(x^3 + 3x^2 + 2x + 3) - (2x + 3)dx$	
	$\bullet^4$	pd	integrate	$\bullet^4$	$\frac{1}{4}x^4 + x^3$	
	$\bullet^5$	pd	substitute limits	$\bullet^5$	$0 - \left(\frac{1}{4}(-3)^4 + (-3)^3\right)$	
	$\bullet^6$	pd	evaluate area	$\bullet^6$	$\frac{27}{4}$ units^2	

Notes:

1. Where a candidate differentiates one or more terms at $\bullet^4$ then $\bullet^5$ and $\bullet^6$ are not available.
2. Candidates who substitute without integrating at $\bullet^3$ do not gain $\bullet^4$, $\bullet^5$ and $\bullet^6$.
3. Candidates must show evidence that they have considered the upper limit 0 at $\bullet^5$.
4. Where candidates show no evidence for both $\bullet^4$ and $\bullet^5$, but arrive at the correct area, then $\bullet^4$, $\bullet^5$ and $\bullet^6$ are not available.
5. The omission of dx at $\bullet^3$ should not be penalised.

Regularly Occurring Responses:

Candidate A

$\int_{0}^{-3}(lower - upper\,)dx$ $\bullet^3$ ✓

$= \frac{27}{4}$ $\bullet^6$ ✓

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Candidate B

$\int_{-3}^{0} x^3 + 3x^2 + 2x + 3 - 2x + 3$ $\bullet^3$ ✓

$= \frac{x^4}{4} + x^3$ $\bullet^4$ ✓

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Candidate C

$\int_{-3}^{0} (x^3 + 3x^2 + 2x + 3) - (2x + 3)dx$

$= -\frac{27}{4}$ cannot be negative so $= \frac{27}{4}$ $\bullet^6$ ✗

However .... $= -\frac{27}{4}$ so Area $= \frac{27}{4}$ $\bullet^6$ ✓

Reference to 'Area' **must** be made.

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Candidate D

$\int_{-3}^{0} (x^3 + 3x^2 + 2x + 3) - (2x + 3)dx$ $\bullet^2$ ✓ $\bullet^3$ ✓

$= \left[\frac{1}{4}x^4 + x^3 + x^2 + 3x - x^2 + 3x\right]_{-3}^{0}$ $\bullet^4$ ✗

$=[0] - \left[\frac{1}{4}(-3)^4 + (-3)^3 + (-3)^2 + 3(-3) - (-3)^2 + 3(-3)\right]$ $\bullet^5$ ✓

$= \frac{-45}{4}$ $\bullet^6$ ✗

See Candidate C

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Candidate E

$\int_{-3}^{3} (x^3 + 3x^2 + 2x + 3) - (2x + 3)dx$ $\bullet^2$ ✗ $\bullet^3$ ✓

$= \left[\frac{1}{4}x^4 + x^3\right]_{-3}^{3}$ $\bullet^4$ ✓

$= \left[\frac{1}{4}(3)^4 + (3)^3\right] - \left[\frac{1}{4}(-3)^4 + (-3)^3\right]$ $\bullet^5$ ✓

$= 54$ units ^{2} $\bullet^6$ ✓

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Candidate F

$\int_{-3}^{3} (x^3 + 3x^2 + 2x + 3) - (2x + 3)dx$ $\bullet^2$ ✗ $\bullet^3$ ✓

$= \left[\frac{1}{4}x^4 + x^3 + x^2 + 3x - x^2 + 3x\right]_{-3}^{3}$ $\bullet^4$ ✗

$= \left[\frac{1}{4}(3)^4 + (3)^3 + (3)^2 + 3(3) - (3)^2 + 3(3)\right] - \left[\frac{1}{4}(-3)^4 + (-3)^3 + (-3)^2 + 3(-3) - (-3)^2 + 3(-3)\right]$ $\bullet^5$ ✓

$= 54 + 18 + 18$

$= 90$ units^2 $\bullet^6$ ✓

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Candidate G

$\int_{-3}^{0} x^3 + 3x^2 + 2x + 3 - 2x + 3\, dx$ $\bullet^2$ ✓ $\bullet^3$ ✓

$= \int_{-3}^{0} x^3 + 3x^2 + 6\, dx$

$= \left[\frac{1}{4}x^4 + x^3 + 6x\right]_{-3}^{0}$ $\bullet^4$ ✓

$= [0] - \left[\frac{1}{4}(-3)^4 + (-3)^3 + 6(-3)\right]$ $\bullet^5$ ✓

$= \frac{99}{4}$ units^2 $\bullet^6$ ✓

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Question	Generic Scheme		Illustrative Scheme	Max Mark
5	Solve the equation $\log_5(3-2x)+\log_5(2+x)=1$, where x is a real number.			
	$\bullet^1$ ss	use correct law of logs	$\bullet^1$ $\log_5[(3-2x)(2+x)]=1$ stated or implied by $\bullet^2$	**4**
	$\bullet^2$ ic	know to and convert to exponential form	$\bullet^2$ $(3-2x)(2+x)=5^1$	
	$\bullet^3$ pd	express as an equation in standard quadratic form	$\bullet^3$ $2x^2+x-1=0$	
	$\bullet^4$ ic	solve quadratic	$\bullet^4$ $x=\frac{1}{2}$, $x=-1$	

Notes:

1. For $\bullet^2$ accept ……. = $\log_5 5$.
2. Where candidates discard an acceptable solution either by crossing out or by explicit statement, then $\bullet^4$ is not available.

Regularly Occurring Responses:

Candidate A $x=\frac{1}{2}$, $\cancel{x=-1}$ $\quad\bullet^4$ ✗	**Candidate B** $2x^2+x-1=0$ $\quad\bullet^3$ ✓ $(2x+1)(x-1)=0$ $\cancel{x=-\frac{1}{2}}$, $x=1$ $\quad\bullet^4$ ✗
Candidate C incorrect working leading to $\cancel{x=-2}$, $x=1$ $\quad\bullet^4$ ✓ Here the discard of $x=-2$ is valid in the context of the original question.	**Candidate D** $\log_5\frac{(3-2x)}{(2+x)}=1$ $\quad\bullet^1$ ✗ $\frac{(3-2x)}{(2+x)}=5^1$ $\quad\bullet^2$ ✓ $\bullet^3$ ✗ $\bullet^4$ ✗
Candidate E $\log_5[(3-2x)(2+x)]=1$ $\quad\bullet^1$ ✓ $(3-2x)(2+x)=1$ $\quad\bullet^2$ ✗ $2x^2-x-6=0$ $\quad\bullet^3$ ✗ $x=2,\ x=\frac{-3}{2}$ $\quad\bullet^4$ ✗ $\bullet^4$ is not awarded since $x=2$ is not a valid solution.	**Candidate F** $\bullet^1$ ✗ $(3-2x)(2+x)=1$ $\quad\bullet^2$ ✗ $\bullet^3$ $\bullet^4$ not available

Question	Generic Scheme			Illustrative Scheme		Max Mark
6	Given that $\int_0^a 5\sin 3x\, dx = \frac{10}{3}$, $0 \le a < \pi$, calculate the value of a.					
	$\bullet^1$	ss	integrate correctly	$\bullet^1$	$\left[\frac{-5}{3}\cos 3x\right]$	5
	$\bullet^2$	pd	process limits	$\bullet^2$	$\frac{-5}{3}\cos 3a + \frac{5}{3}\cos 0$	
	$\bullet^3$	pd	evaluate and form a correct equation	$\bullet^3$	$\frac{-5}{3}\cos 3a + \frac{5}{3} = \frac{10}{3}$	
	$\bullet^4$	pd	start to solve equation	$\bullet^4$	$\cos 3a = -1$	
	$\bullet^5$	pd	solve for a	$\bullet^5$	$a = \frac{\pi}{3}$	

Notes:

1. Candidates who include solutions outwith the range cannot gain $\bullet^5$.
2. The inclusion of $+\,c$ at $\bullet^1$ should be treated as bad form.
3. $\bullet^5$ is only available for a valid numerical answer.
4. Where the candidate differentiates $\bullet^1$ and $\bullet^2$ are not available. See Candidate A.
5. Where candidate integrate incorrectly $\bullet^2$, $\bullet^3$, $\bullet^4$ and $\bullet^5$ are still available.
6. The value of a **must** be given in radians.

Regularly Occurring Responses:

Candidate A		**Candidate B**	
$[15\cos 3x]_0^a$	$\bullet^1$ ✗	$[-5\cos 3x]_0^a$	$\bullet^1$ ✗
$15\cos 3a - 15\cos 0$	$\bullet^2$ ✗	$-5\cos 3a + 5\cos 0$	$\bullet^2$ ✓
$15\cos 3a - 15 = \frac{10}{3}$	$\bullet^3$ ✓	$-5\cos 3a + 5 = \frac{10}{3}$	$\bullet^3$ ✓
$\cos 3a = \frac{55}{45}$	$\bullet^4$ ✓	$\cos 3a = \frac{1}{3}$	$\bullet^4$ ✓
no solutions	$\bullet^5$ ✗	$a = 0 \cdot 41$ Ignore other solutions in given interval	$\bullet^5$ ✓

Candidate C		**Candidate D**	
$\frac{5}{3}\cos 3x$	$\bullet^1$ ✗	$-15\cos 3x$	$\bullet^1$ ✗
$\frac{5}{3}\cos 3a - \frac{5}{3}\cos 0$	$\bullet^2$ ✓	$-15\cos 3a + 15\cos 0$	$\bullet^2$ ✓
$\frac{5}{3}\cos 3a - \frac{5}{3} = \frac{10}{3}$	$\bullet^3$ ✓	$-15\cos 3a + 15 = \frac{10}{3}$ $-15\cos 3a = \frac{-35}{3}$	$\bullet^3$ ✓
$\cos 3a = 3$	$\bullet^4$ ✓	$\cos 3a = \frac{7}{9}$	$\bullet^4$ ✓
no solutions	$\bullet^5$ ✗	$a = 0 \cdot 23$ Ignore other solutions in given interval	$\bullet^5$ ✓

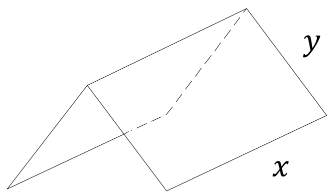
Question	Generic Scheme	Illustrative Scheme	Max Mark

7 A manufacturer is asked to design an open-ended shelter, as shown, subject to the following conditions.

Condition 1

The frame of a shelter is to be made of rods of two different lengths:

- x metres for top and bottom edges;
- y metres for each sloping edge.



Condition 2

The frame is to be covered by a rectangular sheet of material.

The total area of the sheet is 24 m^2.

a Show that the total length, L metres, of the rods used in a shelter is given by $L = 3x + \frac{48}{x}$.

	Generic Scheme		Illustrative Scheme	Max Mark
$\bullet^1$	ss	identify expression for L in x and y	$\bullet^1$ $L = 3x + 4y$	3
$\bullet^2$	ic	identify expression for y in terms of x	$\bullet^2$ $y = \frac{24}{2x}$	
$\bullet^3$	pd	complete proof	$\bullet^3$ $L = 3x + 4 \times \frac{24}{2x}$ and complete	

Notes:

1. The substitution for y at $\bullet^3$ must be clearly shown.

Question		Generic Scheme			Illustrative Scheme		Max Mark
7	b	These rods cost £8·25 per metre. To minimise production costs, the total length of rods used for a frame should be as small as possible.					
	i	Find the value of x for which L is a minimum.					
	ii	Calculate the minimum cost of a frame.					
		•4	pd	prepare to differentiate	•4	$\ldots 48x^{-1}$	7
		•5	pd	differentiate	•5	$3-48x^{-2}$	
		•6	pd	equate derivative to 0	•6	$3-48x^{-2}=0$	
		•7	pd	process for x	•7	$x=4$	
		•8	ic	verify nature	•8	nature table or 2nd derivative	
		•9	ic	evaluate L	•9	$L=24$	
		•10	pd	evaluate cost	•10	cost $24\times £8{\cdot}25=£198$	

Notes:

2. Do not penalise the non-appearance of -4 at •7. However candidates who process $x=-4$ to obtain $L=-24$ do not gain •9.
3. $y=24$ is not awarded •9.

Regularly Occurring Responses:

Candidate A

$L=3x+\frac{48}{x}$

$\frac{dL}{dx}=3-\frac{48}{x^2}$ •4 ✓ •5 ✓

Candidate B

x	$\longrightarrow$	4	$\longrightarrow$
$\frac{dL}{dx}$	$-$	0	$+$
		Min	

Minimum acceptable response

Candidate C

x	$\longrightarrow$	-4	$\longrightarrow$	4	$\longrightarrow$
$\frac{dL}{dx}$	$+$	0	$-$	0	$+$
				Min	

Do not penalise the inclusion of $x=-4$

Question	Generic Scheme			Illustrative Scheme	Max Mark
8	Solve algebraically the equation $\sin 2x = 2\cos^2 x$ for $0 \le x < 2\pi$				
				Method 1	
	$\bullet^1$	ss	use correct double angle formulae	$\bullet^1$ $2\sin x\cos x$	
	$\bullet^2$	ss	form correct equation	$\bullet^2$ $2\sin x\cos x - 2\cos^2 x = 0$	
	$\bullet^3$	ss	take out common factor	$\bullet^3$ $2\cos x(\sin x - \cos x) = 0$	
	$\bullet^4$	ic	proceed to solve	$\bullet^4$ $\cos x = 0$ and $\sin x = \cos x$	
	$\bullet^5$	pd	find solutions	$\bullet^5$ $\frac{\pi}{2}$ ($\bullet^5$) $\quad \frac{3\pi}{2}$ ($\bullet^6$)	
	$\bullet^6$	pd	find remaining solutions	$\bullet^6$ $\frac{\pi}{4}$ ($\bullet^5$) $\quad \frac{5\pi}{4}$ ($\bullet^6$)	6
				Method 2	
	$\bullet^1$	ss	use double angle formula	$\bullet^1$ $\cos 2x + 1$	
	$\bullet^2$	ss	form correct equation	$\bullet^2$ $\sin 2x - \cos 2x = 1$	
	$\bullet^3$	ss	express as a single trig function	$\bullet^3$ $\sqrt{2}\sin\left(2x - \frac{\pi}{4}\right) = 1$	
	$\bullet^4$	ic	proceed to solve	$\bullet^4$ $\sin\left(2x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$	
	$\bullet^5$	pd	find solutions	$\bullet^5$ $2x - \frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4}$ ($\bullet^5$) $\quad \frac{9\pi}{4}, \frac{11\pi}{4}$ ($\bullet^6$)	
	$\bullet^6$	pd	find solutions	$\bullet^6$ $x = \frac{\pi}{4}, \frac{\pi}{2}$ ($\bullet^5$) $\quad x = \frac{5\pi}{4}, \frac{3\pi}{2}$ ($\bullet^6$)	

Notes:

1. In Method 1, = 0 must appear at stage $\bullet^2$ or $\bullet^3$ for $\bullet^2$ to be available.
2. Accept the use of the wave function to solve $\sin x - \cos x = 0$ at stage $\bullet^4$ in Method 1.
3. Accept $\sin 2x - 2\cos^2 x = 0$ as evidence for $\bullet^2$.
4. For candidates who obtain all **four** solutions in degrees $\bullet^6$ can be gained but $\bullet^5$ is not available.

Regularly Occurring Responses:

Candidate A	Candidate B
Correct working leading to $x = 45^0, 90^0, 225^0, 270^0$ $\quad \bullet^5$ ✗ $\quad \bullet^6$ ✓	Correct working leading to $x = 90^0,\ 270^0$ $\quad \bullet^5$ ✗ $\quad \bullet^6$ ^

Question		Generic Scheme			Illustrative Scheme		Max Mark
9	a	The concentration of the pesticide, *Xpesto*, in soil can be modelled by the equation $P_t = P_0 e^{-kt}$ where: • P_0 is the initial concentration; • P_t is the concentration at time t; • t is the time, in days, after the application of the pesticide. Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value. If the half-life of *Xpesto* is 25 days, find the value of k to 2 significant figures.					
		•1	ic	interpret half-life	•1	$\frac{1}{2} P_0 = P_0 e^{-25k}$ **stated or implied by •2**	4
		•2	pd	process equation	•2	$e^{-25k} = \frac{1}{2}$	
		•3	ss	write in logarithmic form	•3	$\log_e \frac{1}{2} = -25k$	
		•4	pd	process for k	•4	$k \approx 0{\cdot}028$	

Notes:

1. Do not penalise candidates who substitute a numerical value for P_0 in part (a).

Regularly Occurring Responses:

Candidate A

$\frac{1}{2}P_0 = P_0 e^{-25k}$	•1 ✓
$\frac{1}{2} = e^{-25k}$	•2 ✓
$\log_{10}\left(\frac{1}{2}\right) = -25k\log_{10}e$	•3 ✓
$k = 0 \cdot 028$	•4 ✓

Question		Generic Scheme	Illustrative Scheme	Max Mark
9	b	Eighty days after the initial application, what is the percentage decrease in concentration of *Xpesto*?		3
		•5 ic interpret equation	•5 $P_t = P_0 e^{-80 \times 0{\cdot}028}$	
		•6 pd process	•6 $P_t \approx 0{\cdot}1065 P_0$	
		•7 ic state percentage decrease	•7 89%	

Notes:

2. For candidates who use a value of k which does not round to $0 \cdot 028$, •5 is not available unless already penalised in part(a).
3. For a value of k ex-nihilo then •5, •6 and •7 are not available.
4. •6 is only available for candidates who express P_t as a multiple of P_0.
5. Beware of candidates using proportion. This is not a valid strategy.

Regularly Occurring Responses:

Candidate B	**Candidate C**
$P_t = P_0 e^{-0{\cdot}03 \times 80}$ •5 ✗✓	$P_t = P_o e^{-80 \times 0{\cdot}0277\ldots\ldots}$ •5 ✓
$= 0 \cdot 0907$ •6 ^	$P_t \approx 0 \cdot 1088 \ldots . P_o$ •6 ✓
leading to $90 \cdot 9\%$ •7 ✗✓	$89 \cdot 11\ldots\%$ •7 ✓
Candidate D	**Candidate E**
•5 ✓ •6 ✓	$P_t = P_0 e^{-80 \times 0{\cdot}028}$ •5 ✓
$P_t = 89\%\ P_0$ •7 ✗	Let P_0 be 100 and $P_t = 100 \times 0{\cdot}1065$ •6 ✓
Candidate F	$P_t = 10{\cdot}65$
$P_t = 100e^{-80 \times 0{\cdot}028}$ •5 ^	⇒ Percentage decrease is 100 - 10·65 = 89·35% •7 ✓
$P_t = 10{\cdot}65$ •6 ✗	
⇒89·35% •7 ✗✓	
Candidate G	**Candidate H**
$P_t = P_0 e^{-80 \times 0{\cdot}028}$ •5 ✓	$P_t = P_0 e^{-80 \times 0{\cdot}028}$ •5 ✓
$P_t = 1 \times e^{-80 \times 0{\cdot}028}$	$P_t = \ldots.\ e^{-80 \times 0{\cdot}028}$
$P_t = 10{\cdot}65$ •6 ✗	$P_t = 0.1065\ P_0$ •6 ✓
⇒ 89·35% decrease •7 ✗✓	⇒ 89·35% decrease •7 ✓

[END OF MARKING INSTRUCTIONS]