2007 Mathematics

Higher – Paper 1

Finalised Marking Instructions

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These Marking Instructions have been prepared by Examination Teams for use by SQA Appointed Markers when marking External Course Assessments. This publication must not be reproduced for commercial or trade purposes.
1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.

2. Award one mark for each ‘bullet’ point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.

3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of mark(s) should be made.
   This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (✓). This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (✘ or X✓). In appropriate cases attention may be directed to work which is not quite correct (e.g. bad form) but which has not been penalised, by underlining with a dotted or wavy line.
   Work which is correct but inadequate to score any marks should be corrected with a double cross tick (✘✘).

5. • The total mark for each section of a question should be entered in red in the outer right hand margin, opposite the end of the working concerned.
   • Only the mark should be written, not a fraction of the possible marks.
   • These marks should correspond to those on the question paper and these instructions.

6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked. Where a candidate has scored zero marks for any question attempted, “0” should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

8. Do not penalise:
   • working subsequent to a correct answer
   • legitimate variations in numerical answers
   • correct working in the “wrong” part of a question
   • omission of units
   • bad form
9. No piece of work should be scored through without careful checking - even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme - answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).

10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referral to the P.A. Please see the general instructions for P.A. referrals.

12. No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.

13. Transcription errors: In general, as a consequence of a transcription error, candidates lose the opportunity of gaining either the first ic mark or the first pd mark.

14. Casual errors: In general, as a consequence of a casual error, candidates lose the opportunity of gaining the appropriate ic mark or pd mark.

15. Do not write any comments on the scripts. A revised summary of acceptable notation is given on page 4.

16. Working that has been crossed out by the candidate cannot receive any credit. If you feel that a candidate has been disadvantaged by this action, make a P.A. Referral.

17. Throughout this paper, unless specifically mentioned, a correct answer with no working receives no credit.

Summary

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1. Tick correct working.
2. Put a mark in the outer right-hand margin to match the marks allocations on the question paper.
3. Do not write marks as fractions.
4. Put each mark at the end of the candidate’s response to the question.
5. Follow through errors to see if candidates can score marks subsequent to the error.
6. Do not write any comments on the scripts.
Higher Mathematics : A Guide to Standard Signs and Abbreviations

Remember - No comments on the scripts. Please use the following and nothing else.

**Signs**

- The tick. You are not expected to tick every line but of course you must check through the whole of a response.

- The cross and underline. Underline an error and place a cross at the end of the line.

- The tick-cross. Use this to show correct work where you are following through subsequent to an error.

- The roof. Use this to show something is missing such as a crucial step in a proof or a 'condition' etc.

- The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).

- The double cross-tick. Use this to show correct work but which is inadequate to score any marks. This may happen when working has been eased.

---

Bullets showing where marks are being allotted may be shown on scripts

<table>
<thead>
<tr>
<th>$rac{dy}{dx} = 4x - 7$</th>
<th>✓ •</th>
<th>margins</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4x - 7 = 0$</td>
<td>✗</td>
<td></td>
</tr>
<tr>
<td>$x = \frac{7}{4}$</td>
<td>✗ •</td>
<td>2</td>
</tr>
<tr>
<td>$y = 3\frac{2}{5}$</td>
<td>✗ •</td>
<td></td>
</tr>
</tbody>
</table>

| $C = (1,-1)$ | ✗ |
| $m = \frac{3-(-1)}{4-1}$ | |
| $m_{\text{rad}} = \frac{4}{3}$ | ✗ • |
| $m_{\text{deg}} = -\frac{1}{3}$ | |
| $m_{\text{gtr}} = -\frac{2}{4}$ | ✗ • |
| $y - 3 = -\frac{2}{4}(x - 2)$ | ✗ • |

| $x^2 - 3x = 28$ | ✓ • |
| $x = 7$ | ✓ ✗ |

| $\sin(x) = 0.75 = \text{invsin}(0.75) = 48.6^\circ$ | ✗ • |

Remember - No comments on the scripts. No abreviations. No new signs. Please use the above and nothing else.

All of these are to help us be more consistent and **accurate**.

Note: There is no such thing as a transcription error, a trivial error, a casual error or an insignificant error. These are all mistakes and as a consequence a mark is lost.

Page 5 lists the syllabus coding for each topic. This information is given in the legend underneath the question. The calculator classification is CN(calculator neutral), CR(calculator required) and NC(non-calculator).
<table>
<thead>
<tr>
<th>UNIT 1</th>
<th>UNIT 2</th>
<th>UNIT 3</th>
</tr>
</thead>
<tbody>
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<td>A1</td>
<td>A15</td>
<td>A28</td>
</tr>
<tr>
<td>determine range/domain</td>
<td>use the general equation of a parabola</td>
<td>use the laws of logs to simplify/find equiv. expression</td>
</tr>
<tr>
<td>A2</td>
<td>A16</td>
<td>A29</td>
</tr>
<tr>
<td>recognise general features of graphs:poly,exp,log</td>
<td>solve a quadratic inequality</td>
<td>sketch associated graphs</td>
</tr>
<tr>
<td>A3</td>
<td>A17</td>
<td>A30</td>
</tr>
<tr>
<td>sketch and annotate related functions</td>
<td>find nature of roots of a quadratic</td>
<td>solve equs of the form ( A = Be^{kt} ) for ( A,B,k ) or ( t )</td>
</tr>
<tr>
<td>A4</td>
<td>A18</td>
<td>A31</td>
</tr>
<tr>
<td>obtain a formula for composite function</td>
<td>given nature of roots, find a condition on coeffs</td>
<td>solve equs of the form ( \log(a) = c ) for ( a,b,c )</td>
</tr>
<tr>
<td>A5</td>
<td>A19</td>
<td>A32</td>
</tr>
<tr>
<td>complete the square</td>
<td>form an equation with given roots</td>
<td>solve equations involving logarithms</td>
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<tr>
<td>A6</td>
<td>A20</td>
<td>A33</td>
</tr>
<tr>
<td>interpret equations and expressions</td>
<td>apply A15-A19 to solve problems</td>
<td>use relationships of the form ( y = ax^2 ) or ( y = ab^x )</td>
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<tr>
<td>A7</td>
<td>G5</td>
<td>G6</td>
</tr>
<tr>
<td>determine function(poly,exp,log) from graph &amp; vv</td>
<td>apply C1-C10 to problems</td>
<td>apply C28-A33 to problems</td>
</tr>
<tr>
<td>A8</td>
<td>C6</td>
<td></td>
</tr>
<tr>
<td>sketch/annotate graph given critical features</td>
<td>apply C11-C10 to problems</td>
<td></td>
</tr>
<tr>
<td>A9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>interpret loci such as slINES,para,poly,circle</td>
<td>apply C1-C10 to problems</td>
<td></td>
</tr>
<tr>
<td>A10</td>
<td>A21</td>
<td>G16</td>
</tr>
<tr>
<td>use the notation ( u_n ) for the nth term</td>
<td>use Rem Th. For values, factors, roots</td>
<td>calculate the length of a vector</td>
</tr>
<tr>
<td>A11</td>
<td>A22</td>
<td>G17</td>
</tr>
<tr>
<td>evaluate successive terms of a RR</td>
<td>solve cubic and quartic equations</td>
<td>calculate the 3rd given two from A,B and vector AB</td>
</tr>
<tr>
<td>A12</td>
<td>A23</td>
<td>G18</td>
</tr>
<tr>
<td>decide when RR has limit/interpret limit</td>
<td>find intersection of line and polynomial</td>
<td>use unit vectors</td>
</tr>
<tr>
<td>A13</td>
<td>A24</td>
<td>G19</td>
</tr>
<tr>
<td>evaluate limit</td>
<td>find if line is tangent to polynomial</td>
<td>use: if ( u ), ( v ) are parallel then ( v = ku )</td>
</tr>
<tr>
<td>A14</td>
<td>A25</td>
<td>G20</td>
</tr>
<tr>
<td>apply A10-A14 to problems</td>
<td>find intersection of two polynomials</td>
<td>add, subtract, find scalar mult. of vectors</td>
</tr>
<tr>
<td>G1</td>
<td>G9</td>
<td>G21</td>
</tr>
<tr>
<td>use the distance formula</td>
<td>find C/R of a circle from its equation/other data</td>
<td>simplify vector pathways</td>
</tr>
<tr>
<td>G2</td>
<td>G10</td>
<td>G22</td>
</tr>
<tr>
<td>find gradient from 2 pts./angle/equ. of line</td>
<td>find the equation of a circle</td>
<td>interpret 2D sketches of 3D situations</td>
</tr>
<tr>
<td>G3</td>
<td>G11</td>
<td>G23</td>
</tr>
<tr>
<td>find equation of a line</td>
<td>find equation of a tangent to a circle</td>
<td>find if 3 points in space are collinear</td>
</tr>
<tr>
<td>G4</td>
<td>G12</td>
<td>G24</td>
</tr>
<tr>
<td>interpret all equations of a line</td>
<td>find intersection of line &amp; circle</td>
<td>find ratio which one point divides two others</td>
</tr>
<tr>
<td>G5</td>
<td>G13</td>
<td>G25</td>
</tr>
<tr>
<td>use property of perpendicular lines</td>
<td>find if/when line is tangent to circle</td>
<td>given a ratio, find/interpret 3rd point/vector</td>
</tr>
<tr>
<td>G6</td>
<td>G14</td>
<td>G26</td>
</tr>
<tr>
<td>calculate mid-point</td>
<td>find if two circles touch</td>
<td>calculate the scalar product</td>
</tr>
<tr>
<td>G7</td>
<td>G15</td>
<td>G27</td>
</tr>
<tr>
<td>find equation of median, altitude,perp. bisector</td>
<td>apply G9-G14 to problems</td>
<td>use: if ( u ), ( v ) are perpendicular then ( v,u=0 )</td>
</tr>
<tr>
<td>G8</td>
<td></td>
<td>G28</td>
</tr>
<tr>
<td>apply G1-G7 to problems eq intersect.,concur.,collin.,</td>
<td></td>
<td>calculate the angle between two vectors</td>
</tr>
<tr>
<td></td>
<td></td>
<td>G29</td>
</tr>
<tr>
<td></td>
<td></td>
<td>use the distributive law</td>
</tr>
<tr>
<td></td>
<td></td>
<td>G30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>apply G16-G29 to problems eq geometry props.</td>
</tr>
<tr>
<td>C1</td>
<td>C12</td>
<td>C20</td>
</tr>
<tr>
<td>differentiate sums, differences</td>
<td>find integrals of ( px^n ) and sums/diffs</td>
<td>differentiate psin(ax+b), pcos(ax+b)</td>
</tr>
<tr>
<td>C2</td>
<td>C13</td>
<td>C21</td>
</tr>
<tr>
<td>differentiate negative &amp; fractional powers</td>
<td>integrate with negative &amp; fractional powers</td>
<td>differentiate using the chain rule</td>
</tr>
<tr>
<td>C3</td>
<td>C14</td>
<td>C22</td>
</tr>
<tr>
<td>express in differentiable form and differentiate</td>
<td>express in integrable form and integrate</td>
<td>integrate ( (ax + b)^n )</td>
</tr>
<tr>
<td>C4</td>
<td>C15</td>
<td>C23</td>
</tr>
<tr>
<td>find gradient at point on curve &amp; vv</td>
<td>evaluate definite integrals</td>
<td>integrate psin(ax+b), pcos(ax+b)</td>
</tr>
<tr>
<td>C5</td>
<td>C16</td>
<td>C24</td>
</tr>
<tr>
<td>find equation of tangent to a polynomial/trig curve</td>
<td>find area between curve and z-axis</td>
<td>apply C20-C23 to problems</td>
</tr>
<tr>
<td>C6</td>
<td>C17</td>
<td></td>
</tr>
<tr>
<td>find rate of change</td>
<td>find area between two curves</td>
<td></td>
</tr>
<tr>
<td>C7</td>
<td>C18</td>
<td></td>
</tr>
<tr>
<td>find when curve strictly increasing etc</td>
<td>solve differential equations(variables separable)</td>
<td></td>
</tr>
<tr>
<td>C8</td>
<td>C19</td>
<td></td>
</tr>
<tr>
<td>find stationary points/values</td>
<td>apply C12-C18 to problems</td>
<td></td>
</tr>
<tr>
<td>C9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>determine nature of stationary points</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>sketch curve given the equation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>apply C1-C10 to problems eq optimise, greatest/least</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T1</td>
<td>T17</td>
<td>T12</td>
</tr>
<tr>
<td>use gen. features of graphs of ( f(x)=\sin(ax+b) ), ( f(x)=\cos(ax+b) ), identify period/amplitude</td>
<td>solve linear &amp; quadratic equations in radians</td>
<td>solve sin,equs of form kcos(a)=p, ksin(a)=q</td>
</tr>
<tr>
<td>T2</td>
<td>T18</td>
<td>T13</td>
</tr>
<tr>
<td>use radians inc conversion from degrees &amp; vv</td>
<td>apply compound and double angle (c &amp; ( \cos ) etc) formulae</td>
<td>express ( \cos(z) + \sin(z) ) in form ( \cos(z,\pi,)etc</td>
</tr>
<tr>
<td>T3</td>
<td>T19</td>
<td>T14</td>
</tr>
<tr>
<td>know and use exact values</td>
<td>apply ( c &amp; ) ( \Delta ) the formula in geometrical cases</td>
<td>find max/min/zeros of ( \cos(z) ) ( +\sin(z) )</td>
</tr>
<tr>
<td>T4</td>
<td>T20</td>
<td>T15</td>
</tr>
<tr>
<td>recognise form of trig. function from graph</td>
<td>apply ( c ) ( \Delta ) the formula when solving equations</td>
<td>sketch graph of ( y=\cos(z) ) ( +\sin(z) )</td>
</tr>
<tr>
<td>T5</td>
<td>T21</td>
<td>T16</td>
</tr>
<tr>
<td>interpret trig. equations and expressions</td>
<td>apply T7-T10 to problems</td>
<td>solve eq of the form ( y=p\cos(rx)+\sin(rx) )</td>
</tr>
<tr>
<td>T6</td>
<td>T22</td>
<td>T17</td>
</tr>
<tr>
<td>apply T2-T5 to problems</td>
<td></td>
<td>apply T12-T16 to problems</td>
</tr>
</tbody>
</table>
Find the equation of the line through the point \((-1, 4)\) which is parallel to the line with equation \(3x - y + 2 = 0\).

The primary method is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

- 1 ss express in standard form
- 2 ic interpret gradient
- 3 ic state equation of line

**Notes**

1. Accept any form of the answer (with or without working) for 3 marks

**Primary Method:** Give 1 mark for each

1. \(y = 3x\ldots\) stated/implied by ²
2. gradient = 3 stated/implied by ³
3. \(y - 4 = 3(x - (-1))\) or
4. form is \(3x - y + c = 0\)
5. \(3 \times (-1) - 4 + c = 0\)
6. \(c = 7\)
Relative to a suitable coordinate system A and B are the points \((-2,1,-1)\) and \((1,3,2)\) respectively. A, B and C are collinear points and C is positioned such that \(BC = 2AB\). Find the coordinates of C.

The primary marking m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

- **ss** introduces vectors
- **pd** completes
- **ic** interprets positions
- **ic** finds C

**Primary Method**: Give 1 mark for each·

1. \(\overrightarrow{AB} = b - a\) stated or implied by
2. \(\overrightarrow{AB} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}\)
3. \(\overrightarrow{BC} = \begin{pmatrix} 6 \\ 6 \end{pmatrix}\)
4. \(C = (7,7,8)\)

**Alt. method 1**

1. \(c - b = 2b - 2a\)
2. \(c = 3b - 2a\)
3. \(c = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}\)
4. \(C = (7,7,8)\)

**Alt. method 2**

1. \(ic\) diagram \(\rightarrow\)
2. \(pd\) \(x = 7\)
3. \(pd\) \(y = 7\)
4. \(pd\) \(z = 8\)

Minimum response
Functions $f$ and $g$, defined on suitable domains, are given by

$$f(x) = x^2 + 1 \text{ and } g(x) = 1 - 2x.$$ 

Find

(a) \hspace{1cm} g(f(x)) \\
(b) \hspace{1cm} g(g(x)) \\

The primary method m.s. is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

1. ss know to start from the "inside"
2. ic interpret composite function
3. ss know to start from the "inside"
4. ic interpret composite function

**Notes**

1. in (a):
   - for finding $f(g(x))$:
     - $g(1 - 2x)$ \hspace{1cm} no mark
     - $(1 - 2x)^2 + 1$ \hspace{1cm} award \hspace{0.5cm} •
   - for finding $f(f(x))$:
     - no marks

2. in (b):
   - for finding $f(g(x))$:
     - no mark
   - for finding $f(f(x))$:
     - $(x^2 + 1)$ \hspace{1cm} no mark
     - $(x^2 + 1)^2 + 1$ \hspace{1cm} award \hspace{0.5cm} •

3. There are no marks available for either $g(x) \times f(x)$ or $g(x) \times g(x)$. 

**Primary Method: Give 1 mark for each**

- $g(f(x)) = g(x^2 + 1)$ s/i by •
- $g(g(x)) = g(1 - 2x)$ s/i by •
- $1 - 2(1 - 2x)$
Find the range of values of \( k \) such that the equation \( kx^2 - x - 1 = 0 \) has no real roots.

The marking scheme is based on the following generic m.s.

- **Primary Method:** Give 1 mark for each.
  - 1 \( b^2 - 4ac < 0 \)
  - 2 \( a = k, b = -1, c = -1 \) s/1 by 3
  - 3 \( 1 + 4k \)
  - 4 \( k < -\frac{1}{4} \)

**Notes**

1. The "< 0" has to appear at least once at the 1 stage or the 3 stage for 1 to be awarded.
2. If an \( x \) appears at 2 stage, none of 2, 3 or 4 are available.
3. Some candidates may start with the quadratic formula. Apply marking scheme to the part underneath the square root sign.
4. The use of any expression masquerading as the discriminant can only gain 2 at most.

**Common Error 1**

\[
\begin{align*}
1 & \quad b^2 - 4ac \\
\sqrt{2} & \quad 1 + 4k \\
\frac{1}{4} & \quad k < -\frac{1}{4}
\end{align*}
\]
The large circle has equation $x^2 + y^2 - 14x - 16y + 77 = 0$.

Three congruent circles with centres $A$, $B$ and $C$ are drawn inside the large circle with the centres lying on a line parallel to the $x$-axis.

This pattern is continued, as shown in the diagram.

Find the equation of the circle with centre $D$.

The primary method m.s. is based on the following generic m.s. This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

- $\bullet^1$ ic state centre
- $\bullet^2$ ss know to, and find, large radius
- $\bullet^3$ ss know to, and find the small radius
- $\bullet^4$ ic interpret new centre
- $\bullet^5$ ic state equation of circle

Primary Method: Give 1 mark for each $\bullet$

$\bullet^1$ $B = (7,8)$

$\bullet^2$ $r_{\text{large}} = \sqrt{7^2 + 8^2 - 77} = 6$

$\bullet^3$ $r_{\text{small}} = \frac{6}{3}$ $\text{s/i by } \bullet^5$

$\bullet^4$ $D = (15,8)$ $\text{s/i by } \bullet^5$

$\bullet^5$ $(x - 15)^2 + (y - 8)^2 = 2^2$

Note

1. If $D = (31,8)$ then $\bullet^4$ is not available; however either of

   $(x - 31)^2 + (y - 8)^2 = 2^2$

   or $(x - 31)^2 + (y - 8)^2 = 6^2$

   may be awarded $\bullet^5$

2. $\bullet^5$ is only awarded for substituting numerical values for the centre and the radius
Solve the equation \( \sin(2x°) = 6\cos(x°) \) for \( 0 \leq x \leq 360° \).

**Primary Method:** Give 1 mark for each •

1. \( 2\sin(x°)\cos(x°) \)
2. \( \cos(x°)(2\sin(x°) - 6) = 0 \)
3. \( \cos(x°) = 0 \) and \( x = 90, 270° \)
4. \( \sin(x°) = 3 \) and no solution or
5. \( \cos(x°) = 0 \) and \( \sin(x°) = 3 \)
6. \( x = 90, 270° \) and no solution

**Alt. method: Division by \( \cos(x°) \)**

1. \( 2\sin(x°)\cos(x°) \)
2. either \( \cos(x°) = 0 \) or \( \cos(x°) \neq 0 \) stated explicitly
3. \( \cos(x°) = 0 \Rightarrow x = 90 \) or 270°
4. \( 2\sin(x°) = 6 \Rightarrow \) no solution

---

**Notes**

1. •1 is NOT available for \( 2\sin A\cos A \) with no further working
2. The "\( n = 0 \)" has to appear at least once at the •1 stage or the •2 stage
3. The inclusion of extra answers which would have been correct but are outside the given interval should be treated as bad form (i.e. not penalised)
4. In following through from an error, •1 is only available for solving an equation with no solution
5. The phrase "no solution" does not always appear after \( \sin(x) = 3 \). The minimum indication that no solution exists might simply be a line drawn through or underneath the equation.
1.07

A sequence is defined by the recurrence relation \( u_{n+1} = \frac{1}{4} u_n + 16, \ u_0 = 0 \).

(a) Calculate the values of \( u_1, u_2 \) and \( u_3 \).

Four terms of this sequence, \( u_1, u_2, u_3 \) and \( u_4 \) are plotted as shown in the graph. As \( n \to \infty \), the points on the graph approach the line \( u_n = k \), where \( k \) is the limit of this sequence.

(b) (i) Give a reason why this sequence has a limit. 
(ii) Find the exact value of \( k \).

The primary method m.s is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

1. ic interpret r.r. 
2. pd process 
3. pd interpret and process 
4. ic interpret "a"
5. ss know how to find limit 
6. pd complete

Notes 1
1. In (a) only numerical values for \( u_1, u_2 \) and \( u_3 \) are acceptable.
2. For (b)(i) accept 
   \[ |\frac{1}{4}| < 1 \]
   \[ 0 < \frac{1}{4} < 1 \]
   \[ \frac{1}{4} \text{ lies between } -1 \text{ and } 1 \]
   \[ \frac{1}{4} \text{ is a proper fraction} \]
3. For (b)(i) do NOT accept 
   \[ -1 \leq \frac{1}{4} \leq 1 \]
   \[ \frac{1}{4} < 1 \]
   \[ -1 < a < 1 \text{ unless } a \text{ is clearly identified/replace by a } \frac{1}{4} \text{ anywhere in the answer} \]

Notes 2
4. For (b)(ii) 
   \[ k = \frac{b}{1-a} \text{ and nothing else gains no marks} \]
5. For (b)(iii) 
   \[ k = \frac{16}{\frac{4}{3}} \text{ or } k = \frac{16}{0.75} \text{ may be awarded } ^6 \]
   \[ k = \frac{16}{\frac{4}{3}} \text{ or } k = \frac{16}{0.75} \text{ or } 21.3 \text{ does NOT gain } ^6 \]
6. Accept \( L \) in lieu of \( k \)
7. An answer of \( \frac{4a}{3} \) without any working cannot gain \( ^5 \) or \( ^6 \)
8. Any calculations based on formulae masquerading as a limit rule cannot gain \( ^5 \) or \( ^6 \).
The diagram shows a sketch of the graph of \( y = x^3 - 4x^2 + x + 6 \).

(a) Show that the graph cuts the \( x \)-axis at (3,0)

(b) Hence or otherwise find the coordinates of A.

(c) Find the shaded area.

### Primary Method: Give 1 mark for each

1. \( f(3) = 27 - 36 + 3 + 6 = 0 \)
2. \( (x - 3)(x^2 - \ldots) \)
3. \( (x - 3)(x^2 - x - 2) \)
4. \( (x - 3)(x - 2)(x + 1) \) so \( A = (2,0) \)
5. \( \int (x^3 - 4x^2 + x + 6) \, dx \)
6. \( \int_0^2 (x^3 - 4x^2 + 12) \, dx \)
7. \( \frac{1}{4} x^4 - \frac{4}{3} x^3 + \frac{2}{1} x^2 + 6x \)
8. \( \frac{1}{4} x^4 - x^2 - \frac{1}{2} x^3 + \frac{1}{2} x^2 + 6x \)
9. \( \frac{2}{3} \)

### Alternative Method 1 for \( \bullet^1 \) to \( \bullet^4 \)

<table>
<thead>
<tr>
<th>Mark</th>
<th>1</th>
<th>-4</th>
<th>1</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3</td>
<td>-3</td>
<td>-6</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>-2</td>
<td>0</td>
</tr>
</tbody>
</table>

\( x^2 - x - 2 \)
\( x = 2, x = -1 \) AND \( x_A = 2 \)

### Alternative Method 2 for \( \bullet^1 \) to \( \bullet^4 \)

1. \( f(3) = \ldots = 0 \)
2. \( \text{try } f(n) = \ldots \) where \( n > 0 \)
3. \( f(2) = \ldots = 0 \)
4. \( x_A = 2 \)
A function $f$ is defined by the formula $f(x) = 3x - x^3$.

(a) Find the exact values where the graph of $y = f(x)$ meets the $x$- and $y$-axes.

(b) Find the coordinates of the stationary points of the function and determine their nature.

(c) Sketch the graph of $y = f(x)$. 

**Notes 1**

1. $\bullet^2$ is only available if $\bullet^1$ has been awarded
2. The "0" shown at $\bullet^5$ must appear at least once somewhere in the working between $\bullet^4$ and $\bullet^6$
3. $\bullet^6$ is only available as a consequence of solving $f'(x) = 0$
4. An unsimplified $\sqrt{1}$ should be penalised at the first occurrence
5. The evidence for $\bullet^7$ and $\bullet^9$ may not appear until the sketch
6. The nature table must reflect previous working from $\bullet^4$ and $\bullet^6$
7. The minimum requirement for the sketch is a cubic passing through the origin and with turning points annotated

**Notes 2**

8. The use of the 2nd derivative is an acceptable strategy for $\bullet^8$
9. As shown in the Primary Method, $\bullet^6$ & $\bullet^7$, and $\bullet^8$ & $\bullet^9$ may be marked in series or in parallel [see foot of next page]
10. A " $-\sqrt{3}$ " appearing for the first time on the sketch may not be awarded $\bullet^1$ / $\bullet^2$ retrospectively
11. See foot of next page for examples of a nature table.
Given that \( y = \sqrt{3x^2 + 2} \), find \( \frac{dy}{dx} \).

**Primary Method:** Give 1 mark for each.

1. \( \left(3x^2 + 2\right)^{\frac{1}{2}} \)
2. \( \frac{1}{2}(3x^2 + 2)^{-\frac{1}{2}} \)
3. \( x = 6x \)

**Common Errors**

1. \( \sqrt{y} = (3x^2 + 2)^{-1} \)
2. \( \frac{dy}{dx} = -(3x^2 + 2)^{-2} \)
3. \( x \sqrt{...} \times 6x \)

Example of a minimum requirement nature table

<table>
<thead>
<tr>
<th>( f' )</th>
<th>( \cdot )</th>
<th>( \cdot )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>+</td>
<td>0</td>
</tr>
</tbody>
</table>

Example of a preferred nature table

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \cdot )</th>
<th>( \cdot )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( f' )</th>
<th>( \cdot )</th>
<th>( \cdot )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>+</td>
<td>0</td>
</tr>
</tbody>
</table>

| \( x = -1 \) | \( \cdot \) | \( x = 1 \) |
(a) Express \( f(x) = \sqrt{3} \cos(x) + \sin(x) \) in the form \( k \cos(x - a) \), where \( k > 0 \) and \( 0 < a < \frac{\pi}{2} \).

(b) Hence or otherwise sketch the graph of \( y = f(x) \) in the interval \( 0 \leq x \leq 2\pi \).

**Notes 1**

1. In the whole question, do not penalise more than once for not using radians.
2. Table showing marks lost for using degrees:

<table>
<thead>
<tr>
<th>Graph in degrees</th>
<th>30°</th>
<th>60°</th>
<th>π/6</th>
<th>π/3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph in radians</td>
<td>-1</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>

In (a)

2. \( k(\cos x \cos a + \sin x \sin a) \) is acceptable for \( \bullet^1 \).
3. \( k = \sqrt{3} \) does NOT earn \( \bullet^3 \).
4. \( 2(\cos x \cos a + \sin x \sin a) \) etc is acceptable for \( \bullet^2 \) & \( \bullet^3 \).
5. Candidates may use any form of the wave equation as long as their final answer is in the form \( k \cos(x - a) \). If not then \( \bullet^4 \) is NOT available.
6. Treat \( k \cos x \cos a + \sin x \sin a \) as bad form ONLY if \( \bullet^2 \) is gained.

**Notes 2**

7. Do not penalise graphs which go beyond \( 0 \leq x \leq 2\pi \).
8. A maximum of 3 marks are available for candidates who attempt to sketch graphs of \( k \cos(x + a) \), \( k \sin(x + a) \) or \( k \sin(x - a) \). No other graphs can earn any credit.
9. Alternative marking for 2 marks for candidates who do not make a sketch:

\[ \max \left( \frac{\pi}{6}, \frac{\pi}{3} \right), \min \left( \frac{\pi}{6}, \frac{\pi}{3} \right), (-2), (...) \]

(\( \frac{2\pi}{3}, 0 \) and \( \frac{5\pi}{3}, 0 \))

\( \bullet^5 \) any two from the above list

\( \bullet^6 \) another two from the above list
2007 Mathematics

Higher – Paper 2

Finalised Marking Instructions

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1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.

2. Award one mark for each ‘bullet’ point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.

3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of mark(s) should be made.

This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (✓). This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (✗ or x✓). In appropriate cases attention may be directed to work which is not quite correct (e.g. bad form) but which has not been penalised, by underlining with a dotted or wavy line.

Work which is correct but inadequate to score any marks should be corrected with a double cross tick (✗✗).

5. • The total mark for each section of a question should be entered in red in the outer right hand margin, opposite the end of the working concerned.
   • Only the mark should be written, not a fraction of the possible marks.
   • These marks should correspond to those on the question paper and these instructions.

6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked. Where a candidate has scored zero marks for any question attempted, “0” should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

8. Do not penalise:
   • working subsequent to a correct answer
   • legitimate variations in numerical answers
   • correct working in the “wrong” part of a question
   • omission of units
   • bad form
9. No piece of work should be scored through without careful checking - even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme - answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).

10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referral to the P.A. Please see the general instructions for P.A. referrals.

12. No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.

13. Transcription errors: In general, as a consequence of a transcription error, candidates lose the opportunity of gaining either the first ic mark or the first pd mark.

14. Casual errors: In general, as a consequence of a casual error, candidates lose the opportunity of gaining the appropriate ic mark or pd mark.

15. Do not write any comments on the scripts. A revised summary of acceptable notation is given on page 4.

16. Working that has been crossed out by the candidate cannot receive any credit. If you feel that a candidate has been disadvantaged by this action, make a P.A. Referral.

17. Throughout this paper, unless specifically mentioned, a correct answer with no working receives no credit.

Summary

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1. **Tick** correct working.
2. Put a mark in the **outer right-hand margin to match the marks allocations on the question paper**.
3. Do **not** write marks as fractions.
4. Put each mark at the end of the candidate’s response to the question.
5. **Follow through** errors to see if candidates can score marks subsequent to the error.
6. Do **not** write any comments on the scripts.
Higher Mathematics: A Guide to Standard Signs and Abbreviations

Remember - No comments on the scripts. Please use the following and nothing else.

**Signs**

- ✓ The tick. You are not expected to tick every line but of course you must check through the whole of a response.

- ✗ The cross and underline. Underline an error and place a cross at the end of the line.

- ✗ The tick-cross. Use this to show correct work where you are following through subsequent to an error.

- ✗ The roof. Use this to show something is missing such as a crucial step in a proof or a 'condition' etc.

- ✗ The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).

- ✗ The double cross-tick. Use this to show correct work but which is inadequate to score any marks. This may happen when working has been eased.

**Bullets showing where marks are being allotted may be shown on scripts**

<table>
<thead>
<tr>
<th>Expression</th>
<th>Mark</th>
<th>Margins</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{dy}{dx} = 4x - 7$</td>
<td>✓</td>
<td>2</td>
</tr>
<tr>
<td>$4x - 7 = 0$</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>$x = \frac{7}{4}$</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>$y = 3\frac{3}{4}$</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>$C = (1,-1)$</td>
<td>✗</td>
<td></td>
</tr>
<tr>
<td>$m = \frac{3-(-1)}{4-1}$</td>
<td>✗</td>
<td></td>
</tr>
<tr>
<td>$m_{rad} = \frac{4}{3}$</td>
<td>✗</td>
<td>3</td>
</tr>
<tr>
<td>$m_{gt} = \frac{-1}{4}$</td>
<td>✗</td>
<td></td>
</tr>
<tr>
<td>$m_{gt} = -\frac{2}{4}$</td>
<td>✗</td>
<td></td>
</tr>
<tr>
<td>$y - 3 = -\frac{2}{4}(x - 2)$</td>
<td>✗</td>
<td></td>
</tr>
<tr>
<td>$x^2 - 3x = 28$</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>$x = 7$</td>
<td>✗</td>
<td>1</td>
</tr>
<tr>
<td>$\sin(x) = 0.75 = \sin(0.75) = 48.6^\circ$</td>
<td>✓</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: There is no such thing as a transcription error, a trivial error, a casual error or an insignificant error. These are all mistakes and as a consequence a mark is lost.

Page 5 lists the syllabus coding for each topic. This information is given in the legend underneath the question. The calculator classification is CN(calculator neutral), CR(calculator required) and NC(non-calculator).
<table>
<thead>
<tr>
<th>UNIT 1</th>
<th>UNIT 2</th>
<th>UNIT 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A1</strong> determine range/domain</td>
<td><strong>A15</strong> use the general equation of a parabola</td>
<td><strong>A28</strong> use the laws of logs to simplify/find equiv. expression</td>
</tr>
<tr>
<td><strong>A2</strong> recognise general features of graphs:poly,exp,log</td>
<td><strong>A16</strong> solve a quadratic inequality</td>
<td><strong>A29</strong> sketch associated graphs</td>
</tr>
<tr>
<td><strong>A3</strong> sketch and annotate related functions</td>
<td><strong>A17</strong> find nature of roots of a quadratic</td>
<td><strong>A30</strong> solve eqns of the form ( A = Be^{kt} ) for ( A,B,k ) or ( t )</td>
</tr>
<tr>
<td><strong>A4</strong> obtain a formula for composite function</td>
<td><strong>A18</strong> given nature of roots, find a condition on coeffs</td>
<td><strong>A31</strong> solve eqns of the form ( \log(a) = c ) for ( a,b ) or ( c )</td>
</tr>
<tr>
<td><strong>A5</strong> complete the square</td>
<td><strong>A19</strong> form an equation with given roots</td>
<td><strong>A32</strong> solve equations involving logarithms</td>
</tr>
<tr>
<td><strong>A6</strong> interpret equations and expressions</td>
<td><strong>A20</strong> apply A15-A19 to solve problems</td>
<td><strong>A33</strong> use relationships of the form ( y = ax^2 ) or ( y = ab^t )</td>
</tr>
<tr>
<td><strong>A7</strong> determine function(poly,exp,log) from graph &amp; ( vv )</td>
<td><strong>A21</strong> use Rem Th. For values, factors, roots</td>
<td><strong>A34</strong> apply A28-A33 to problems</td>
</tr>
<tr>
<td><strong>A8</strong> sketch/annotate graph given critical features</td>
<td><strong>A22</strong> solve cubic and quartic equations</td>
<td></td>
</tr>
<tr>
<td><strong>A9</strong> interpret loci such as st.lines,parab,poly,circle</td>
<td><strong>A23</strong> find intersection of line and polynomial</td>
<td></td>
</tr>
<tr>
<td><strong>A10</strong> use the notation ( u_n ) for the ( n )th term</td>
<td><strong>A24</strong> find if line is tangent to polynomial</td>
<td></td>
</tr>
<tr>
<td><strong>A11</strong> evaluate successive terms of a RR</td>
<td><strong>A25</strong> find intersection of two polynomials</td>
<td></td>
</tr>
<tr>
<td><strong>A12</strong> decide when RR has limit/interpret limit</td>
<td><strong>A26</strong> confirm and improve on approx roots</td>
<td></td>
</tr>
<tr>
<td><strong>A13</strong> evaluate limit</td>
<td><strong>A27</strong> apply A21-A36 to problems</td>
<td></td>
</tr>
<tr>
<td><strong>A14</strong> apply A10-A14 to problems</td>
<td><strong>A28</strong> find if 2 points in space are collinear</td>
<td></td>
</tr>
<tr>
<td><strong>G1</strong> use the distance formula</td>
<td><strong>G9</strong> find C/R of a circle from its equation/other data</td>
<td></td>
</tr>
<tr>
<td><strong>G2</strong> find gradient from 2 pts./angle/equ. of line</td>
<td><strong>G10</strong> find the equation of a circle</td>
<td></td>
</tr>
<tr>
<td><strong>G3</strong> find equation of a line</td>
<td><strong>G11</strong> find equation of a tangent to a circle</td>
<td></td>
</tr>
<tr>
<td><strong>G4</strong> interpret all equations of a line</td>
<td><strong>G12</strong> find intersection of line &amp; circle</td>
<td></td>
</tr>
<tr>
<td><strong>G5</strong> use property of perpendicular lines</td>
<td><strong>G13</strong> find if/when line is tangent to circle</td>
<td></td>
</tr>
<tr>
<td><strong>G6</strong> calculate mid-point</td>
<td><strong>G14</strong> find if two circles touch</td>
<td></td>
</tr>
<tr>
<td><strong>G7</strong> find equation of median, altitude,perm. bisector</td>
<td><strong>G15</strong> apply G9-G14 to problems</td>
<td></td>
</tr>
<tr>
<td><strong>G8</strong> apply G1-G7 to problems eg intersect.,concur.,collin.</td>
<td><strong>C12</strong> find integrals of ( px^2 ) and sums/diffs</td>
<td><strong>C20</strong> differentiate ( \sin(ax+b) ), ( \cos(ax+b) )</td>
</tr>
<tr>
<td><strong>C1</strong> differentiate sums, differences</td>
<td><strong>C13</strong> integrate with negative &amp; fractional powers</td>
<td><strong>C21</strong> differentiate using the chain rule</td>
</tr>
<tr>
<td><strong>C2</strong> differentiate negative &amp; fractional powers</td>
<td><strong>C14</strong> express in integrable form and integrate</td>
<td><strong>C22</strong> integrate ( (ax+b)^n )</td>
</tr>
<tr>
<td><strong>C3</strong> express in differentiable form and differentiate</td>
<td><strong>C15</strong> evaluate definite integrals</td>
<td><strong>C23</strong> integrate ( \sin(ax+b) ), ( \cos(ax+b) )</td>
</tr>
<tr>
<td><strong>C4</strong> find gradient at point on curve &amp; ( vv )</td>
<td><strong>C16</strong> find area between curve and ( z )-axis</td>
<td><strong>C24</strong> apply C20-C23 to problems</td>
</tr>
<tr>
<td><strong>C5</strong> find equation of tangent to a polynomial/trig curve</td>
<td><strong>C17</strong> find area between two curves</td>
<td></td>
</tr>
<tr>
<td><strong>C6</strong> find rate of change</td>
<td><strong>C18</strong> solve differential equations(variables separable)</td>
<td></td>
</tr>
<tr>
<td><strong>C7</strong> find when curve strictly increasing etc</td>
<td><strong>C19</strong> apply C12-C18 to problems</td>
<td></td>
</tr>
<tr>
<td><strong>C8</strong> find stationary points/values</td>
<td><strong>C20</strong> determinenature of stationary points</td>
<td></td>
</tr>
<tr>
<td><strong>C9</strong></td>
<td></td>
<td><strong>C10</strong> sketch curve given the equation</td>
</tr>
<tr>
<td><strong>C11</strong> apply C1-C10 to problems eg optimise, greatest/least</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>T1</strong> use general features of graphs of ( f(x)=\sin(ax+b) ), ( f(x)=\cos(ax+b) ), identify period/amplitude</td>
<td><strong>T7</strong> solve linear &amp; quadratic equations in radians</td>
<td><strong>T12</strong> solve sin,equs of form ( \cos(a)=p ), ( \sin(a)=q )</td>
</tr>
<tr>
<td><strong>T2</strong> use radians inc conversion from degrees &amp; ( vv ) in numerical &amp; literal cases</td>
<td><strong>T8</strong> apply compound and double angle ( \cos ) &amp; ( \sin ) formulae</td>
<td><strong>T13</strong> express ( \cos(z)\sin(z) ) in form ( \cos(z+\angle z) ) etc</td>
</tr>
<tr>
<td><strong>T3</strong> know and use exact values</td>
<td><strong>T9</strong> apply ( \cos ) &amp; ( \sin ) formulae in geometrical cases</td>
<td><strong>T14</strong> find max/min/zero$s \ of \ \cos(z)\sin(z) )</td>
</tr>
<tr>
<td><strong>T4</strong> recognise form of trig. function from graph</td>
<td><strong>T10</strong> use ( \cos ) &amp; ( \sin ) formulae when solving equations</td>
<td><strong>T15</strong> sketch graph of ( y=\cos(z)\sin(z) )</td>
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<tr>
<td><strong>T5</strong> interpret trig. equations and expressions</td>
<td><strong>T11</strong> apply T7-T10 to problems</td>
<td><strong>T16</strong> solve eq of the form ( y=\cos(z)+\sin(z) )</td>
</tr>
<tr>
<td><strong>T6</strong> apply T2-T5 to problems</td>
<td><strong>T12</strong> apply T13-T16 to problems</td>
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</table>
OABCDEFG is a cube with side 2 units, as shown in the diagram.
B has coordinates (2, 2, 0).
P is the centre of face OCGD and Q is the centre of face CBFG.
(a) Write down the coordinates of G.
(b) Find \( p \) and \( q \), the position vectors of points P and Q.
(c) Find the size of angle POQ.

\[ \text{Primary Method: Give 1 mark for each •} \]

\[ \begin{align*}
 & & G = (0, 2, 2) \\
 & & p = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \\
 & & q = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \\
 & & \cos \angle POQ = \frac{p \cdot q}{|p||q|} \\
 & & |p| = \sqrt{2} \\
 & & |q| = \sqrt{6} \\
 & & p \cdot q = 3 \\
 & & \angle POQ = 30^\circ \\
 & & \text{[radians : } \frac{\pi}{6} \text{ (0.524); gradians : 33.3]} \\
\end{align*} \]

**Alternative Method for •\(^4\) to •\(^8\)**

\[ \begin{align*}
 & & \cos \angle POQ = \frac{OP^2 + OQ^2 - PQ^2}{2 \times OP \times OQ} \quad \text{stated or implied (s/i) by •\(^8\)} \\
 & & OP = \sqrt{2} \\
 & & OQ = \sqrt{6} \\
 & & PQ = \sqrt{2} \\
 & & \angle POQ = 30^\circ \\
 & & \text{[radians : } \frac{\pi}{6} \text{ (0.524); gradians : 33.3]} \\
\end{align*} \]
The diagram shows two right-angled triangles with angles $c$ and $d$ marked as shown.

(a) Find the exact value of $\sin(c + d)$.

(b) (i) Find the exact value of $\sin 2c$.
(ii) Show that $\cos 2d$ has the same exact value.

### Notes 1

1. Any attempt to use $\sin(c + d) = \sin c \sin d$ loses $\bullet^2$, $\bullet^3$ and $\bullet^4$
2. At $\bullet^3$ treat $\sin\left(\frac{\pi}{3}\right) \cos\left(\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{3}\right) \sin\left(\frac{\pi}{3}\right)$ as bad form if the trig functions disappear to give the answer
3. At the $\bullet^3$ stage do not penalise the use of fractions which are greater than 1
4. Neither $\bullet^4$ nor $\bullet^6$ are available for answers > 1
5. Any work based on $\sin 2c = 2\sin c \cos c$ loses $\bullet^5$ and $\bullet^6$
6. Any work based on $\cos 2d = 2\cos d - 1$ loses $\bullet^7$ and $\bullet^8$
7. In (b) candidates may calculate $\sin 2c$ and $\cos 2d$ in any order. If either $\sin 2c$ or $\cos 2d$ is correct that may be awarded 2 of the 4 marks available
8. Any working based on numerical values for $c$ and $d$ (eg 27° and 18°) earns no credit but $\bullet^1$, $\bullet^2$, $\bullet^5$ and $\bullet^7$ are still available.
9. $\bullet^8$ is only available if the answer to (b)(ii) is shown to be equivalent to the answer to (b)(i)
10. If $\sqrt{5}$ and $\sqrt{10}$ are approximated to decimal values then $\bullet^4$, $\bullet^6$ and $\bullet^8$ are not available.
Show that the line with equation \( y = 6 - 2x \) is a tangent to the circle
with equation \( x^2 + y^2 + 6x - 4y - 7 = 0 \) and find the coordinates of
the point of contact of the tangent and the circle.

<table>
<thead>
<tr>
<th>qu</th>
<th>part</th>
<th>mk</th>
<th>code</th>
<th>calc</th>
<th>source</th>
<th>ss</th>
<th>pd</th>
<th>ic</th>
<th>c</th>
<th>B</th>
<th>A</th>
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<td>2</td>
<td>6</td>
<td>g13</td>
<td>CN</td>
<td></td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Primary Method:** Give 1 mark for each

1. \( x^2 + (6 - 2x)^2 + 6x - 4(6 - 2x) - 7 = 0 \)
2. \( ...36 - 24x + 4x^2 - 24 + 8x... \)
3. \( 5x^2 - 10x + 5 = 0 \)
4. \( (x - 1)^2 = 0 \)
5. Equal roots \( \Rightarrow \) line is tangent
6. \( x = 1, \ y = 4 \)

**Alternatives for 1 and 5**

1. \( b^2 - 4ac = 0 \Rightarrow \) tangent
2. \( (-10)^2 - 4 \times 5 \times 5 = 0 \)
4. Use quad. formula to get roots
5. Equal roots \( \Rightarrow \) line is tangent

**Alternative Method:** Give 1 mark for each

1. \( m_{\text{line}} = -2 \)
2. \( (-3,2) \) and \( \frac{1}{2} \)
3. \( \text{equ. of radius} : y - 2 = \frac{1}{2}(x + 3) \)
4. \( x = 1 \)
5. \( y = 4 \)
6. Check that (1,4) lies on the circle
The diagram shows part of the graph of a function whose equation is of the form \( y = a \sin(bx) + c \).

(a) Write down the values of \( a \), \( b \) and \( c \).
(b) Determine the exact value of the \( x \)-coordinate of \( P \); the point where the graph intersects the \( x \)-axis as shown in the diagram.

Primary Method: Give 1 mark for each:

- \( a = 2 \)
- \( b = 3 \)
- \( c = -1 \)
- \( 2 \sin(3x^2) - 1 = 0 \)
- one answer from 10° or 50°
- \( x_P = 50° \)

**alternative for \( \star^4, \star^5 \) and \( \star^6 \)**

- sketch of graph with pointer to sol. point
- extraction of 50°
- confirmation of \( 2 \sin(3 \times 50°) - 1 \) does = 0

Notes 1

1. \( \star^4 \) may be awarded for \( a \sin(bx) + c = 0 \)
2. For \( \star^2 \) accept " \( b = 3x \) " as bad form
3. \( \star^6 \) may only be awarded for a value of \( x \) such that \( 30 < x < 60 \)
4. \( \star^6 \) may be awarded for (50°,0) but NOT for (0,50°)
A circle centre C is situated so that it touches the parabola with equation \( y = \frac{1}{2}x^2 - 8x + 34 \) at P and Q.

(a) The gradient of the tangent to the parabola at Q is 4. Find the coordinates of Q.

(b) Find the coordinates of P.

(c) Find the coordinates of C, the centre of the circle.

---

Notes 1

1. Treat \( y = x - 8 \) as bad form provided \( y \) is replaced by 4 at •3.

2. **Case**
   Look out for the following:
   - •3 is not available to candidates who substitute the gradient of 4 into the equation in order to find the value of \( y_Q \).

3. Alt. strategies for •5:
   - (a) substitute \( y = 10 \) into the parabola
   - (b) use the t.p. as a step to P

4. **Case**
   There are other legitimate methods for finding the coordinates of Q.

5. Candidates who solve the tangents at P and Q AND then state that \( x_C = 8 \) may be awarded •3.

---

Primary Method: Give 1 mark for each:

1. \( \frac{dy}{dx} = \ldots \) (1 term correct)
2. \( x - 8 \)
3. \( x - 8 = 4 \)
4. \( x = 12 \)
5. \( y = 10 \)
6. \( m_p = -4 \)
7. \( P = (4, 10) \)
8. \( x_C = 8 \)
9. \( y_C = 11 \)

---

Alternative Method for (c)
Solving the normals

\[ y - 10 = -\frac{1}{2}(x - 12) \]
\[ y - 10 = \frac{1}{2}(x - 4) \]

may be used. Marks are awarded as normal:

\( x = 8 \) •3 and \( y = 11 \) •9.

---

Common Errors

1. \( \frac{dy}{dx} = x - 8 \)
   \[ x - 8 = 0 \Rightarrow x = 8, y = 2 \]
   •1, •2

2. For the occasional candidate who starts with \( x - 8 = 4 \)
   award •1, •2 and •5.
A householder has a garden in the shape of a right-angled isosceles triangle. It is intended to put down a section of rectangular wooden decking at the side of the house, as shown in the diagram.

(a) (i) Find the exact value of ST.
(ii) Given that the breadth of the decking is \(x\) metres, show that

\[
A = (10\sqrt{2})x - 2x^2
\]

(b) Find the dimensions of the decking which maximises its area.

---

**Notes 1**

In (b)

1. An "= 0 " must appear somewhere in the working between \(\bullet^4\) and \(\bullet^6\).
2. For \(\bullet^7\) accept \(\frac{dA}{dx} = -4 < 0\) at \(x = \frac{10\sqrt{2}}{4}\) ⇒ maximum

---

**Primary Method: Give 1 mark for each •**

1. \(ST = \sqrt{200}\)
2. \(length = \sqrt{200} - 2x\) s/i by their method
3. \((\sqrt{200} - 2x)\times x\) and complete proof
4. \(\frac{dA}{dx} = 0\)
5. \(\frac{dA}{dx} = 10\sqrt{2} - 4x\)
6. \(x = \frac{10\sqrt{2}}{4}\) or equivalent (3.5)
7. justification: e.g. nature table
8. \(length = 5\sqrt{2}\) (7.1)

Minimum requirement of a nature table

<table>
<thead>
<tr>
<th>...</th>
<th>3.5 ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f'(x))</td>
<td>+</td>
</tr>
</tbody>
</table>

hence maximum

**better** would be

\[
\begin{align*}
x & \rightarrow \frac{3\sqrt{2}}{2} \\
\frac{f'(x)}{2} & \rightarrow 0 \\
f(x) & \rightarrow \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\
hence maximum
\end{align*}
\]

at \(x = \frac{5\sqrt{2}}{2}\)
Find the value of \( \int_0^2 \sin(4x + 1) \, dx \).

**Primary Method:** Give 1 mark for each •

\[ \begin{align*}
\bullet^1 & \quad \text{pd} \quad \text{integrate the trig function} \\
\bullet^2 & \quad \text{pd} \quad \text{deal with the } ^{\text{4th}} \\
\bullet^3 & \quad \text{ic} \quad \text{substitute the limits} \\
\bullet^4 & \quad \text{pd} \quad \text{evaluate} \\
\end{align*} \]

\[ \begin{align*}
\bullet^1 & \quad - \cos(4x + 1) \\
\bullet^2 & \quad \times \frac{1}{4} \\
\bullet^3 & \quad - \frac{1}{4} \cos(4 \times 2 + 1) - \left( - \frac{1}{4} \cos(4 \times 0 + 1) \right) \\
\bullet^4 & \quad 0.36 \\
\end{align*} \]

**Alternative Method**

\[ \sin 4x \cos 1 + \cos 4x \sin 1 \]

\[ \begin{align*}
\bullet^1 & \quad - \frac{1}{4} \cos 4x \cos 1 \\
\bullet^2 & \quad \frac{1}{4} \sin 4x \sin 1 \\
\bullet^3 & \quad \left( - \frac{1}{4} \cos 8 \cos 1 + \frac{1}{4} \sin 8 \sin 1 \right) - \left( - \frac{1}{4} \cos 0 \cos 1 + \frac{1}{4} \sin 0 \sin 1 \right) \\
\bullet^4 & \quad 0.36 \\
\end{align*} \]
The curve with equation \( y = \log_3(x - 1) - 2.2 \), where \( x > 1 \), cuts the \( x \)-axis at the point \((a, 0)\).

Find the value of \( a \).

### Notes 1

1. Solutions given in terms of \( x \) rather than \( a \) should be treated as bad form.
The diagram shows the graph of \( y = ax \), \( a > 1 \).
On separate diagrams sketch the graphs of:

(a) \( y = a^{-x} \)
(b) \( y = a^{1-x} \)

**Notes 1**

1. For \( \bullet^1 \) and \( \bullet^3 \) the shape must be an exponential decay graph lying above the \( x \)-axis
2. There are no follow-through marks available to candidates who use an incorrect graph from (a) as a basis for their answer to (b).
2.10

The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$.

Both graphs pass through the point $(0,6)$.

The graph of $y = f'(x)$ also passes through the points $(2,0)$ and $(4,0)$.

(a) Given that $f'(x)$ is of the form $k(x-a)(x-b)$

(i) Write down the values of $a$ and $b$.

(ii) Find the value of $k$.  

(b) Find the equation of the graph of the cubic function $y = f(x)$.  

The primary method m.s is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

<table>
<thead>
<tr>
<th>ic</th>
<th>ss</th>
<th>pd</th>
<th>ic</th>
<th>Notes 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>interpret roots on diagram</td>
<td>know to use $y$-intercept</td>
<td>process</td>
<td>express as an equation</td>
<td>For candidates who fail to complete (a) but produce values for $k, a$ and $b$ ex nihilo, all 4 marks are available in (b). A deduction of 1 mark may be made if their choice eases the working.</td>
</tr>
<tr>
<td>ss</td>
<td>pd</td>
<td>ic</td>
<td>Note 1</td>
<td>In (b) For candidates who use $k = 1$, a &quot;fully correct&quot; follow-through solution may be awarded 3 out of the last 4 marks.</td>
</tr>
</tbody>
</table>
| pd | ic | Interpret constant of integration | For candidates who retain "$k$", "$a$" and "$b$", 

Primary Method: Give 1 mark for each:

1. $a = 2$ and $b = 4$ or $k(x-2)(x-4)$
2. $6 = k(0-2)(0-4)$
3. $k = \frac{3}{4}$
4. $\int \left(\frac{3}{4}(x-2)(x-4)\right) \, dx$ s/i by $\int$ any two terms integrated correctly ($\frac{3}{12} x^3$ etc)
5. $y = \frac{1}{4} x^3 - \frac{2}{4} x^2 + 6x + c$
6. $c = 6$
Two variables \( x \) and \( y \) satisfy the equation \( y = 3 \times 4^x \).

(a) Find the value of \( a \) if \((a,6)\) lies on the graph with equation \( y = 3 \times 4^x \).

(b) If \((-\frac{1}{2},b)\) also lies on the graph, find \( b \).

(c) A graph is drawn of \( \log_{10} y \) against \( x \). Show that its equation will be of the form \( \log_{10} y = Px + Q \) and state the gradient of this line.

The primary method m.s. is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

Primary Method: Give 1 mark for each.

1. \( a = \frac{1}{2} \)
2. \( b = \frac{3}{2} \)
3. \( \log_{10}(y) = \log_{10}\left(3 \times 4^x\right) \)
4. \( \log_{10}(y) = \log_{10}(3) + \log_{10}\left(4^x\right) \)
5. \( \log_{10}(y) = x \log_{10}(4) + \log_{10}(3) \)
6. gradient = \( \log_{10}(4) \) or equivalent

Alternate Method

1. \( y = 10^{Px+Q} \)
2. \( y = 10^Q \times (10^P)^x \)
3. \( 10^Q = 3 \) and \( 10^P = 4 \)
4. \( P = \log_{10} 4 \)

Cave

In (a) look out for the following:

\[
6 = 3 \times 4^a \\
2 = 4^a \\
\frac{2}{4} = a \\
\frac{a}{2} = 1
\]

This is not awarded •1