2006 Mathematics

Higher – Paper 1

Finalised Marking Instructions

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1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.

2. Award one mark for each ‘bullet’ point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.

3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of mark(s) should be made. This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (√). This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (× or X√). In appropriate cases attention may be directed to work which is not quite correct (e.g. bad form) but which has not been penalised, by underlining with a dotted or wavy line. Work which is correct but inadequate to score any marks should be corrected with a double cross tick (××).

5. • The total mark for each section of a question should be entered in red in the outer right hand margin, opposite the end of the working concerned. • Only the mark should be written, not a fraction of the possible marks. • These marks should correspond to those on the question paper and these instructions.

6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked. Where a candidate has scored zero marks for any question attempted, “0” should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

8. Do not penalise:
   • working subsequent to a correct answer
   • legitimate variations in numerical answers
   • correct working in the “wrong” part of a question
   • omission of units
   • bad form
9. No piece of work should be scored through without careful checking - even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme - answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).

10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referral to the P.A. Please see the general instructions for P.A. referrals.

12. No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.

13. Transcription errors: In general, as a consequence of a transcription error, candidates lose the opportunity of gaining either the first ic mark or the first pr mark.

14. Casual errors: In general, as a consequence of a casual error, candidates lose the opportunity of gaining the appropriate ic mark or pr mark.

15. **Do not write any comments on the scripts.** A revised summary of acceptable notation is given on page 4.

16. Working that has been crossed out by the candidate cannot receive any credit. If you feel that a candidate has been disadvantaged by this action, make a P.A. Referral.

17. Throughout this paper, unless specifically mentioned, a correct answer with no working receives no credit.

**Summary**

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1. **Tick** correct working.
2. Put a mark in the **outer right-hand margin** to match the marks allocations on the question paper.
3. Do **not** write marks as fractions.
4. Put each mark at the end of the candidate’s response to the question.
5. **Follow through** errors to see if candidates can score marks subsequent to the error.
6. Do **not** write any comments on the scripts.
Higher Mathematics : A Guide to Standard Signs and Abbreviations

Remember - No comments on the scripts. Please use the following and nothing else.

**Signs**

✓  The tick. You are not expected to tick every line but of course you must check through the whole of a response.

✗ The cross and underline. Underline an error and place a cross at the end of the line.

✗ The tick-cross. Use this to show correct work where you are following through subsequent to an error.

∧ The roof. Use this to show something is missing such as a crucial step in a proof or a 'condition' etc.

~ The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).

✗ The double cross-tick. Use this to show correct work but which is inadequate to score any marks. This may happen when working has been eased.

Bullets showing where marks are being allotted may be shown on scripts

- \( \frac{dy}{dx} = 4x - 7 \) ✓ •
- \( 4x - 7 = 0 \) ×
- \( x = \frac{7}{4} \)
- \( y = 3 \frac{7}{8} \) × •
- \( C = (1, -1) \) ×
- \( m = \frac{3 - (-1)}{4 - 1} \)
- \( m_{\text{rad}} = \frac{1}{3} \) × •
- \( m_{\text{deg}} = \frac{-1}{4} \)
- \( m_{\text{gt}} = \frac{-2}{7} \)
- \( y - 3 = -\frac{3}{7}(x - 2) \) × •
- \( x^2 - 3x = 28 \) ✓ •
- \( x = 7 \) ∧ × •
- \( \sin(x) = 0.75 = \arcsin(0.75) = 48.6^\circ \) ✓ •

Remember - No comments on the scripts. No abbreviations. No new signs.
Please use the above and nothing else.

All of these are to help us be more consistent and accurate.

Note: There is no such thing as a transcription error, a trivial error, a casual error or an insignificant error. These are all mistakes and as a consequence a mark is lost.

Page 5 lists the syllabus coding for each topic. This information is given in the legend underneath the question. The calculator classification is CN(calculator neutral), CR(calculator required) and NC(non-calculator).
### UNIT 1
- **A1** determine range/domain
- **A2** recognise general features of graphs; poly, exp, log
- **A3** sketch and annotate related functions
- **A4** obtain a formula for composite function
- **A5** complete the square
- **A6** interpret equations and expressions
- **A7** determine function (poly, exp, log) from graph & vv
- **A8** sketch/annotate graph given critical features
- **A9** interpret loci such as st. lines, parabola, circle

- **A10** use the notation un, for the nth term
- **A11** evaluate successive terms of a RR
- **A12** decide when RR has limit/interpret limit
- **A13** evaluate limit
- **A14** apply A10-A14 to problems

### UNIT 2
- **A15** use the general equation of a parabola
- **A16** solve a quadratic inequality
- **A17** find nature of roots of a quadratic
- **A18** given nature of roots, find a condition on coeffs.
- **A19** form an equation with given roots
- **A20** apply A15-A19 to solve problems

### UNIT 3
- **A21** use Rem Th. For values, factors, roots
- **A22** solve cubic and quartic equations
- **A23** find intersection of line and polynomial
- **A24** find if line is tangent to polynomial
- **A25** find intersection of two polynomials
- **A26** confirm and improve on approx roots
- **A27** apply A21-A26 to problems

- **G1** use the distance formula
- **G2** find gradient from 2 pts/angle/equ. of line
- **G3** find equation of a line
- **G4** interpret all equations of a line
- **G5** use property of perpendicular lines
- **G6** calculate mid-point
- **G7** find equation of median, altitude, perp. bisector

- **G8** apply G1-G7 to problems eg intersect., concur., collin.

- **G9** find C/R of a circle from its equation/other data
- **G10** find the equation of a circle
- **G11** find equation of a tangent to a circle
- **G12** find if/when line is tangent to circle
- **G13** find if/when line is tangent to polynomial
- **G14** find if two circles touch

- **G15** apply G9-G14 to problems

- **C1** differentiate sums, differences
- **C2** differentiate negative & fractional powers
- **C3** express in differentiable form and differentiate
- **C4** find gradient at point on curve & vv
- **C5** find equation of a tangent to a polynomial/trig curve
- **C6** find rate of change
- **C7** find when curve strictly increasing etc
- **C8** find stationary points/values
- **C9** determine nature of stationary points
- **C10** sketch curve/given the equation

- **C11** apply C1-C10 to problems eg optimise, greatest/least

- **T1** use gen. features of graphs of \( f(x) = \sin(ax + b) \), \( f(x) = \cos(ax + b) \);
- **T2** use radians inc conversion from degrees & vv
- **T3** know and use exact values
- **T4** recognise form of trig. function from graph
- **T5** interpret trig. equations and expressions
- **T6** apply T2-T5 to problems

- **T7** solve linear & quadratic equations in radians
- **T8** apply compound and double angle (c & da) formulae in numerical & literal cases
- **T9** know and use exact values
- **T10** use c & da formulae when solving equations
- **T11** apply T7-T10 to problems

- **T12** solve sin. equs of form \( \cos(a) = p, \sin(a) = q \)
- **T13** express \( \cos(z) + \sin(z) \) in form \( \cos(x) + \sin(x) \)
- **T14** find max/min/roots of \( \cos(z) + \sin(z) \)
- **T15** sketch graph of \( y = \cos(x) + \sin(x) \)
- **T16** solve equs of the form \( y = \cos(z) + \sin(z) \)
- **T17** apply T12-T16 to problems
1. Triangle ABC has vertices A(−1, 12), B(−2, −5) and C(7, −2).
   (a) Find the equation of the median BD.
   (b) Find the equation of the altitude AE.
   (c) Find the coordinates of the point of intersection of BD and AE.

### Notes
1. For candidates who find two medians
   - \( \cdot 1^1 \), \( \cdot 2^2 \), \( \cdot 3^3 \) and \( \cdot 7^7 \), \( \cdot 8^8 \), \( \cdot 9^9 \) are available.
2. For candidates who find two altitudes
   - \( \cdot 4^4 \), \( \cdot 5^5 \), \( \cdot 6^6 \) and \( \cdot 7^7 \), \( \cdot 8^8 \), \( \cdot 9^9 \) are available.
3. For candidates who find (a) altitude and (b) median
   - See common error box number 3.
4. In (a) note that (4, 7) happens to lie on the median but does not qualify as a point to be used in \( \cdot 3^3 \).

### Common Error 1
Finding two medians

| \( \cdot 1^1 \) | \( D = (3, 5) \) |
| \( \cdot 2^2 \) | \( m_{BD} = 2 \) |
| \( \cdot 3^3 \) | \( y - 5 = 2(x - 3) \) or \( y + 5 = 2(−x - 2) \) etc |
| \( \cdot 4^4 \) | \( m_{BC} = \frac{1}{2} \) |
| \( \cdot 5^5 \) | \( m_{alt} = -3 \) |
| \( \cdot 6^6 \) | \( y - 12 = -3(x - 1) \) |
| \( \cdot 7^7 \) | \( y - 5 = 2(x - 3) \) and \( y - 12 = -3(x - 1) \) |
| \( \cdot 8^8 \) | \( x = 2 \) |
| \( \cdot 9^9 \) | \( y = 3 \) |

### Common Error 2
Finding two altitudes

| \( \cdot 1^1 \) | \( X \) |
| \( \cdot 2^2 \) | \( X \) |
| \( \cdot 3^3 \) | \( X \) |
| \( \cdot 4^4 \) | \( m_{BC} = \frac{1}{2} \) |
| \( \cdot 5^5 \) | \( m_{alt} = -3 \) |
| \( \cdot 6^6 \) | \( y - 12 = -3(x - (-1)) \) |
| \( \cdot 7^7 \) | \( 4x - 7y = 27 \) and \( y = -3x + 9 \) |
| \( \cdot 8^8 \) | \( x = \frac{18}{5} \) |
| \( \cdot 9^9 \) | \( y = -\frac{9}{5} \) |

### Common Error 3
Finding (a) altitude and (b) median

| \( \cdot 1^1 \) | \( m_{AC} = -\frac{7}{1} \) |
| \( \cdot 2^2 \) | \( X \) |
| \( \cdot 3^3 \) | \( y = -y = -\frac{5}{7}(x - -2) \) |
| \( \cdot 4^4 \) | \( \text{midpt of } BC = \left( \frac{7}{7}, -\frac{1}{7} \right) \) |
| \( \cdot 5^5 \) | \( m_{AC} = -\frac{7}{1} \) |
| \( \cdot 6^6 \) | \( y - 12 = -\frac{2}{7}(x - (-1)) \) |
| \( \cdot 7^7 \) | \( 4x - 7y = 27 \) and \( 31x + 7y = 53 \) |
| \( \cdot 8^8 \) | \( x = \frac{16}{9} \) |
| \( \cdot 9^9 \) | \( y = -\frac{125}{49} \) |

maximum of 6 marks

maximum of 6 marks

maximum of 5 marks

maximum of 6 marks
A circle has centre \( C(-2, 3) \) and passes through \( P(1, 6) \).

(a) Find the equation of the circle.

(b) \( PQ \) is a diameter of the circle. Find the equation of the tangent to this circle at \( Q \).

Notes

1. In (a) \((\sqrt{18})^2\) is not acceptable for \( r^2 \).

2. In (b) if the coordinates of \( Q \) are estimated (i.e. guessed) then \( c \) can only be awarded if the coordinates are of the form \((a, 0)\) where \( a < -2 \).

3. In (b) \( d \) is only available if an attempt has been made to find a perpendicular gradient.

Primary Method : Give 1 mark for each •

- \( (x-a)^2 + (y-b)^2 = r^2 \)
- \( (x-(-2))^2 + (y-3)^2 = 18 \) \( \star \) \( 2 \) marks
- \( Q = (-5, 0) \) \( \star \)
- \( m_{diameter} = 1 \) stated or implied by \( \star 5 \)
- \( m_{tangent} = -1 \) \( \star \)
- \( y - 0 = -\left( x + 5 \right) \) \( \star \)

Alternative Method for (a)

For answers of the form \( x^2 + y^2 + 2gx + 2fy + c = 0 \)

- \( x^2 + y^2 + 4x - 6y + c = 0 \)
- \( c = -5 \)

General Notes applicable throughout the marking scheme

There are many instances when follow throughs come into play and these will not always be highlighted for you. The following example is a reminder of what you have to look out for when you are marking.

example

At the \( \star 3 \) stage a candidate start with the wrong coordinates for \( Q \). Then

\[
\begin{align*}
X & \quad \star 3 \quad Q = (-4, 0) \\
X \sqrt{ } & \quad \star 4 \quad m_{diameter} = \frac{6}{5} \\
X \sqrt{ } & \quad \star 5 \quad m_{tangent} = -\frac{5}{6} \\
X \sqrt{ } & \quad \star 6 \quad y - 0 = -\frac{5}{6} \left( x + 4 \right)
\end{align*}
\]

so the candidate loses \( \star 3 \) but gains \( \star 4, \star 5 \) and \( \star 6 \) as a consequence of following through.

Any error can be followed through and the subsequent marks awarded provided the working has not been eased.

Any deviation from this will be noted in the marking scheme.
Two functions \( f \) and \( g \) are defined on the set of real numbers by
\[
\begin{align*}
f(x) &= 2x + 3 \\
g(x) &= 2x - 3
\end{align*}
\]

(a) Find an expression for \( f(g(x)) \) or \( g(f(x)) \) and the third mark is for the other one.

(b) Determine the least possible value of \( f(g(x)) \times g(f(x)) \).

Notes

1. In (a) 2 marks are available for finding one of \( f(g(x)) \) or \( g(f(x)) \) and the third mark is for the other one.

2. In (a) the finding of \( f(g(x)) \) and \( g(f(x)) \) earns no marks.

3. \( \cdot^5 \) is only available if \( \cdot^4 \) has been awarded.

4. In (b) for \( \cdot^5 \), no justification is necessary. Ignore any comments, rational or irrational.
A sequence is defined by the recurrence relation \( u_{n+1} = 0.8u_n + 12 \) \( u_0 = 4 \).

(a) State why the recurrence relation has a limit.

(b) Find this limit.

The primary method m/s is based on the following generic m/s:

THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

Primary Method: Give 1 mark for each •

<table>
<thead>
<tr>
<th>Primary Method</th>
<th>1 mark</th>
<th>2 marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>•1 ic state limit condition</td>
<td></td>
<td></td>
</tr>
<tr>
<td>•2 ss know how to find L</td>
<td></td>
<td></td>
</tr>
<tr>
<td>•3 pr process limit</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes

- For (a)
  1 Accept
  
  \[ 0.8 < 1 \]
  
  \[ 0 < 0.8 < 1 \]
  
  0.8 lies between −1 and 1
  
  0.8 is a proper fraction

2 Do NOT accept

\[-1 \leq 0.8 \leq 1 \]

\[-1 < a < 1 \] unless a is clearly identified/replaced by 0.8 anywhere in the answer.

\[ 0.8 < 1 \]

In (b)

3 \[ L = \frac{b}{1 - a} \] and nothing else gains no marks.

4 \[ L = \frac{12}{0.2} \] or \( \frac{120}{2} \) or \( \frac{60}{1} \) etc does NOT gain •3.

5 An answer of 60 without any working gains NO marks.

6 Any calculations based on “wrong” formulae gain NO marks.
5 A function \( f \) is defined by \( f(x) = (2x - 1)^3 \). Find the coordinates of the stationary point on the graph with equation \( y = f(x) \) and determine its nature.

### Primary Method : Give 1 mark for each •

- \(^4\) \( f'(x) = \ldots \)
- \(^2\) \( 5(2x - 1)^4 \times 2 \)
- \(^3\) \( f(x) = 0 \)
- \(^4\) \( x = \frac{1}{2} \)
- \(^5\) \( f\left(\frac{1}{2}\right) = 0 \)
- \(^6\) nature table
- \(^7\) pt of inflexion at \( \left(\frac{1}{2}, 0\right) \) 7 marks

### Common Error No.1

\[
\begin{align*}
\sqrt{ } & \quad \cdot^1 \quad f'(x) = \ldots \\
X & \quad \cdot^2 \quad 5(2x - 1)^4 \\
\sqrt{ } & \quad \cdot^3 \quad f'(x) = 0 \\
X \sqrt{ } & \quad \cdot^4 \quad x = \frac{1}{2} \\
\cdot^5, \cdot^6 \text{ and } \cdot^7 \text{ are still available}
\end{align*}
\]

### Common Error No.2

\[
\begin{align*}
\sqrt{ } & \quad \cdot^1 \quad f'(x) = \ldots \\
X & \quad \cdot^2 \quad \frac{1}{12} (2x - 1)^6 \\
\sqrt{ } & \quad \cdot^3 \quad f'(x) = 0 \\
X \sqrt{ } & \quad \cdot^4 \quad x = \frac{1}{2} \\
\cdot^5, \cdot^6 \text{ and } \cdot^7 \text{ are still available}
\end{align*}
\]

Notes:
1. The “= 0” shown at \(^3\) must appear at least once somewhere in the working between \(^1\) and \(^4\) (but not necessarily at \(^3\)).
2. \(^4\) is only available as a consequence of solving \( f'(x) = 0 \).
3. A wrong derivative which eases the working will preclude at least \(^4\) from being awarded.
4. For marks \(^6\) and \(^7\), a nature table is mandatory. The minimum amount of detail that is required is shown here:

\[
\begin{array}{c|cccc}
& < \frac{1}{2} & \frac{1}{2} & > \frac{1}{2} \\
f'(x) & + & 0 & + \\
\end{array}
\]

Candidates who use only \( f''(x) = 0 \) and try to draw conclusions from this cannot gain \(^6\) or \(^7\). [ \( f''(x) = 0 \) is a necessary but not sufficient condition for identifying points of inflexion].

5. \(^7\) is ONLY available subsequent to a correct nature table for the candidate’s own derivative.

6. \(^4\) is lost in each of the following cases for the candidate’s solution to the equation at \(^3\).
   (i) \( x = \frac{1}{2} \) and \( x = \text{something else} \)
   (ii) two wrong values for \( x \)
   (iii) guess a value for \( x \)

Only one value for \( x \) needs to be followed through for \(^5, \cdot^6\) and \(^7\).
6 The graph shown has equation \( y = x^3 - 6x^2 + 4x + 1 \).

The shaded area is bounded by the curve, the \( x \)-axis, the \( y \)-axis and the line \( x = 2 \).

(a) Calculate the shaded area labelled \( S \).
(b) Hence find the total shaded area.

<table>
<thead>
<tr>
<th>Qu. part</th>
<th>marks</th>
<th>Grade</th>
<th>Syllabus Code</th>
<th>Calculator class</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>4</td>
<td>C</td>
<td>C16</td>
<td>NC</td>
<td>06/40</td>
</tr>
<tr>
<td>b</td>
<td>3</td>
<td>B</td>
<td>C16</td>
<td>NC</td>
<td></td>
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</tbody>
</table>

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- **1** ss know to integrate
- **2** pr integrate
- **3** ic substitute limits
- **4** pr evaluate
- **5** ic use result from **2** with new limits
- **6** pr evaluate
- **7** ss deal with the “−ve” sign and evaluate total area

**Primary Method : Give 1 mark for each •**

1 \[ \int _0 ^1 (x^3 - 6x^2 + 4x + 1) \, dx \] stated or implied by \( y^2 \)

2\[ \int [x^4 - \frac63 x^3 + \frac12 x^2 + x] \, dx \]

3\[ \left( \frac14 - 2.1^3 + 2.1^2 + 1 \right) - 0 \]

4\[ \int _0 ^1 \ ... \ dx \]

- **5** \[ \int _1 ^2 \ ... \ dx \]

- **6** \[ \left( \frac14 - 2.2^3 + 2.2^2 + 2 \right) - \left( \frac14 - 2.1^3 + 2.1^2 + 1 \right) = -\frac{13}{4} \]

- **7** \[ \frac{9}{2} \] or equivalent

**Alternative Method 1 for (b)**

- **5** \[ \int _0 ^1 \ ... \ dx \]

- **6** \[ \left( \frac14 - 2.1^3 + 2.1^2 + 1 \right) - \left( \frac14 - 2.2^3 + 2.2^2 + 2 \right) \]

- **7** \[ \frac{9}{2} \]

**Alternative Method 2 for (b)**

- **5** \[ \int _1 ^2 \ ... \ dx \]

- **6** \[ \left( \frac14 - 2.2^3 + 2.2^2 + 2 \right) + \left( \frac14 - 2.1^3 + 2.1^2 + 1 \right) \]

- **7** \[ \frac{9}{2} \]

**Alternative Method 3 for (b)**

- **5** \[ \int _1 ^2 \ ... \ dx \]

- **6** \[ \left( \frac14 - 2.2^3 + 2.2^2 + 2 \right) - \left( \frac14 - 2.1^3 + 2.1^2 + 1 \right) \]

- **7** \[ \frac{9}{2} \]

**Notes for (a)**

1 Only a limited number of marks are available to candidates who differentiate – see Common Error No. 1.

2 In (a) candidates who transpose the limits can still earn \( \frac{4}{9} \) if they deal with the “−ve” sign appropriately.

3 In (b) \( -\frac{3}{4} \) is lost for such statements as \( -3 \frac{1}{4} = 3 \frac{1}{4} \).

4 In (b) using \( \int _0 ^2 \ ... \ dx \) earns no marks.

**Common Error No.1**

- \( \sqrt{ \int _0 ^1 (x^3 - 6x^2 + 4x + 1) \, dx } \)
  - \( X \cdot 2 \)
  - \( 3x^2 - 12x + 4 \)
  - \( 3.1^2 - 12.1 + 4 \) - \( 4 \)
  - \( X \cdot 4 \)
  - \(-9 \)

- \( \sqrt{ \int _0 ^1 (x^3 - 6x^2 + 4x + 1) \, dx } \)
  - \( \sqrt{ \int _0 ^1 \ ... \ dx } \) or equivalent
  - \( X \sqrt{ \int _0 ^1 (3.2^2 - 12.2 + 4) - (3.1^2 - 12.1 + 4) = -3 } \)
  - \( X \sqrt{ \int _0 ^1 12 } \)
7 Solve the equation \( \sin x^\circ - \sin 2x^\circ = 0 \) in the interval \( 0 \leq x \leq 360^\circ \).

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<td>C</td>
<td>T10</td>
<td>NC</td>
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**Primary Method**: Give 1 mark for each •

- **1** ss know to use double angle formula
- **2** pr factorise
- **3** pr solve
- **4** ic know exact values

### Notes

1. An “= 0” must appear somewhere between the start and evidence.
2. The inclusion of extra answers which would have been correct with a larger interval should be treated as bad form and NOT penalised.
3. The omission of a correct answer (e.g. 0) means the candidates loses a mark (•4 in the Primary Method).
4. Candidates may embark on a journey with the wrong formula for \( \sin(2x^\circ) \). With an equivalent level of difficulty it may still be worth a maximum of 3 marks. See Common Error No.1.
5. Candidates who draw a sketch of \( y = \sin(x^\circ) \) and \( y = \sin(2x^\circ) \) giving 0,180,360 may be awarded •1 and •3.

### Common Error No.1

\[
X \quad \bullet \quad \sin(x^\circ) - \left( 1 - 2 \sin^2(x^\circ) \right) = 0 \\
2 \sin^2(x^\circ) + \sin(x^\circ) - 1 = 0 \\
X \sqrt{2} \left( 2 \sin(x^\circ) - 1 \right) \left( \sin(x^\circ) + 1 \right) = 0 \\
X \sqrt{3} \sin(x^\circ) = \frac{1}{2} \text{ or } \sin(x^\circ) = -1 \\
X \sqrt{4} \quad x = 30,150^\circ, \quad x = 270^\circ \\
award 3 marks
\]

### Alternative Marking Method (Cross marking for •3 and •4)

- **1** \( \sin(x^\circ) - 2 \sin(x^\circ) \cos(x^\circ) = 0 \)
- **2** \( \sin(x^\circ) \left( 1 - 2 \cos(x^\circ) \right) = 0 \)
- **3** \( \sin(x^\circ) = 0 \text{ and } x = 0,180,360 \)
- **4** \( \cos(x^\circ) = 0.5 \text{ and } x = 60,300 \)

### Common Error No.2

\[
\sin(x^\circ) - \sin^2(x^\circ) = 0 \\
X \quad \bullet \quad \sin(x^\circ) \quad (1 - \sin(x^\circ)) = 0 \\
X \sqrt{1} \quad \sin(x^\circ) = 0 \text{ or } \sin(x^\circ) = 1 \\
X \sqrt{4} \quad x = 0,180,360, \quad 90 \\
award 2 marks
\]

### Common Error No.3

\[
sin(x^\circ) - \sin(2x^\circ) = 0 \\
sin(x^\circ) = 0, \quad \sin(2x^\circ) = 0 \\
etc \\
gains NO marks
\]
8  (a) Express $2x^2 + 4x - 3$ in the form $a(x + b)^2 + c$.

(b) Write down the coordinates of the turning point on the parabola with equation $y = 2x^2 + 4x - 3$.

The primary method m/s is based on the following generic m/s.

THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

Primary Method: Give 1 mark for each:

- $a = 2$
- $b = 1$
- $c = -5$
- $(-1, -5)$

Alternative Method 1 for (a)

- $2(x^2 + 2x)$
- $2(x + 1)^2$
- $2(x + 1)^2 - 5$
- $(-1, -5)$

Alternative Method 2 for (a): Comparing coefficients

- $2x^2 + 4x - 3 = ax^2 + 2abx + ab^2 + c$  $\Rightarrow a = 2$
- $2ab = 4$  $\Rightarrow b = 1$
- $ab^2 + c = -3$  $\Rightarrow c = -5$
- $(-1, -5)$

Note

1. Alternative Method 1 should be used for assessing part marks/follow throughs.
2. For $x^4$, no justification is required.
   Candidates may choose to differentiate etc. but may still earn only one mark for the correct answer.
3. For $x^4$, accept $(-b, c)$. 
9 \( \mathbf{u} \) and \( \mathbf{v} \) are vectors given by
\[
\mathbf{u} = \begin{pmatrix} k^3 \\ 1 \\ k^2 \end{pmatrix} \quad \text{and} \quad \mathbf{v} = \begin{pmatrix} 1 \\ 3k^2 \\ -1 \end{pmatrix}, \quad \text{where} \quad k > 0.
\]

(a) If \( \mathbf{u} \cdot \mathbf{v} = 1 \) show that \( k^3 + 3k^2 - k - 3 = 0 \). 

(b) Show that \( (k + 3) \) is a factor of \( k^3 + 3k^2 - k - 3 \) and hence factorise \( k^3 + 3k^2 - k - 3 \) fully.

(c) Deduce the only possible value of \( k \).

(d) The angle between \( \mathbf{u} \) and \( \mathbf{v} \) is \( \theta \). Find the exact value of \( \cos \theta \).

Notes

1. No explanation is required for \( k \) but the chosen value must follow from the working for •6 or •7. \textbf{Do not accept} \( \sqrt{1} \).

2. In primary method (•4) and alternative (•5) candidates must show some acknowledgement of the resulting “zero”. Although a statement w.r.t. the zero is preferable, accept something as simple as “underlining” the zero.

3. Only numerical values are acceptable for •9, •10 and •11; answers are acceptable in unsimplified form e.g.
\[
\cos \theta = \frac{1}{\sqrt{11} \times \sqrt{11}}
\]

Alternative method 1 (marks 3–7) Long Division

\[
k^2 \\
\underline{3k^3 + 3k^2 - k - 3}
\]

...... ...... ...... 

\( k^2 - 1 \)

\( (k + 3)(k + 1)(k - 1) \)

stated explicitly
10 Two variables, $x$ and $y$, are connected by the law $y = a^x$. A graph of $\log_4(y)$ against $x$ is a straight line passing through the origin and the point A(6,3). Find the value of $a$.

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME.

**Primary Method:** Give 1 mark for each •

- **1** ss know to take logarithms
- **2** ic substitute known point
- **3** pr solve
- **4** pr solve

**Alternative Method 1**

- **1** $\log_4(y) = \log_4(a^x)$
- **2** $3 = \log_4(a^6)$
- **3** $a^6 = 4^3$
- **4** $a = 2$

**Alternative Method 2**

- **1** $\log_4(y) = mx + c$
- **2** $m = \frac{1}{2}$, $c = 0$
- **3** $y = 4^{\frac{x}{2}}$
- **4** $y = \left(4^{\frac{1}{2}}\right)^x = 2^x \Rightarrow a = 2$

**Alternative Method 3**

- **1** At A, $\log_4(y) = 3$
- **2** $y = 4^3$
- **3** $a^6 = 4^3$
- **4** $a = 2$

**Alternative Method 4**

- **1** $\log_4(y) = \log_4(a^y)$
- **2** $\log_4(y) = x \log_4(a)$
- **3** $\log_4(a) = \frac{1}{2}$
- **4** $a = 4^{\frac{1}{2}} = 2$
2006 Mathematics

Higher – Paper 2

Finalised Marking Instructions

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Mathematics Higher: Instructions to Markers

1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.

2. Award one mark for each ‘bullet’ point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.

3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of mark(s) should be made. This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (✓). This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (✗ or X✓). In appropriate cases attention may be directed to work which is not quite correct (e.g. bad form) but which has not been penalised, by underlining with a dotted or wavy line. Work which is correct but inadequate to score any marks should be corrected with a double cross tick (✗ ×).

5. • The total mark for each section of a question should be entered in red in the outer right hand margin, opposite the end of the working concerned.
   • Only the mark should be written, not a fraction of the possible marks.
   • These marks should correspond to those on the question paper and these instructions.

6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked. Where a candidate has scored zero marks for any question attempted, “0” should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

8. Do not penalise:
   • working subsequent to a correct answer
   • legitimate variations in numerical answers
   • correct working in the “wrong” part of a question
   • omission of units
   • bad form
9. No piece of work should be scored through without careful checking - even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme - answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).

10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referral to the P.A. Please see the general instructions for P.A. referrals.

12. No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.

13. Transcription errors: In general, as a consequence of a transcription error, candidates lose the opportunity of gaining either the first ic mark or the first pr mark.

14. Casual errors: In general, as a consequence of a casual error, candidates lose the opportunity of gaining the appropriate ic mark or pr mark.

15. **Do not write any comments on the scripts.** A revised summary of acceptable notation is given on page 4.

16. Working that has been crossed out by the candidate cannot receive any credit. If you feel that a candidate has been disadvantaged by this action, make a P.A. Referral.

17. Throughout this paper, unless specifically mentioned, a correct answer with no working receives no credit.

**Summary**

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1. **Tick** correct working.

2. Put a mark in the outer right-hand margin to match the marks allocations on the question paper.

3. Do **not** write marks as fractions.

4. Put each mark at the end of the candidate’s response to the question.

5. **Follow through** errors to see if candidates can score marks subsequent to the error.

6. Do **not** write any comments on the scripts.
Higher Mathematics: A Guide to Standard Signs and Abbreviations

Remember - No comments on the scripts. Please use the following and nothing else.

Signs

✔️ The tick. You are not expected to tick every line but of course you must check through the whole of a response.

❌ The cross and underline. Underline an error and place a cross at the end of the line.

❌ The tick-cross. Use this to show correct work where you are following through subsequent to an error.

∧ The roof. Use this to show something is missing such as a crucial step in a proof or a 'condition' etc.

~ The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).

❌ The double cross-tick. Use this to show correct work but which is inadequate to score any marks. This may happen when working has been eased.

Bullets showing where marks are being allotted may be shown on scripts

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>margins</th>
</tr>
</thead>
<tbody>
<tr>
<td>dy/dx = 4x - 7</td>
<td>✔️</td>
<td>✔️</td>
<td>2</td>
</tr>
<tr>
<td>4x - 7 = 0</td>
<td>❌</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x = 7/4</td>
<td></td>
<td>✔️</td>
<td></td>
</tr>
<tr>
<td>y = 3 7/8</td>
<td>❌</td>
<td>✔️</td>
<td></td>
</tr>
<tr>
<td>C = (1,-1)</td>
<td>❌</td>
<td></td>
<td></td>
</tr>
<tr>
<td>m = 3 / (-1)</td>
<td>4 + 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>m_red = 1/3</td>
<td></td>
<td>✔️</td>
<td></td>
</tr>
<tr>
<td>m_90 = -1/3</td>
<td></td>
<td>✔️</td>
<td></td>
</tr>
<tr>
<td>m_90 = -2/3</td>
<td></td>
<td>✔️</td>
<td></td>
</tr>
<tr>
<td>y - 3 = -2/3 (x - 2)</td>
<td>❌</td>
<td>✔️</td>
<td>3</td>
</tr>
<tr>
<td>x^2 - 3x = 28</td>
<td>✔️</td>
<td>✔️</td>
<td></td>
</tr>
<tr>
<td>x = 7</td>
<td>∧</td>
<td>❌</td>
<td>1</td>
</tr>
<tr>
<td>sin(x) = 0.75 = invsin(0.75) = 48.6°</td>
<td>✔️</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Remember - No comments on the scripts. No abbreviations. No new signs.
Please use the above and nothing else.

All of these are to help us be more consistent and accurate.

Note: There is no such thing as a transcription error, a trivial error, a casual error or an insignificant error. These are all mistakes and as a consequence a mark is lost.

Page 5 lists the syllabus coding for each topic. This information is given in the legend underneath the question. The calculator classification is CN(calculator neutral), CR(calculator required) and NC(non-calculator).
<table>
<thead>
<tr>
<th>UNIT 1</th>
<th>UNIT 2</th>
<th>UNIT 3</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A1</strong> determine range/domain</td>
<td><strong>A15</strong> use the general equation of a parabola</td>
<td><strong>A28</strong> use the laws of logs to simplify/find equiv. expression</td>
<td><strong>Year</strong></td>
</tr>
<tr>
<td><strong>A2</strong> recognise general features of graphs; poly, exp, log</td>
<td><strong>A16</strong> solve a quadratic inequality</td>
<td><strong>A29</strong> sketch associated graphs</td>
<td></td>
</tr>
<tr>
<td><strong>A3</strong> sketch and annotate related functions</td>
<td><strong>A17</strong> find nature of roots of a quadratic</td>
<td><strong>A30</strong> solve eqns of the form $A = B e^{kt}$ for $A, B, k$ or $t$</td>
<td></td>
</tr>
<tr>
<td><strong>A4</strong> obtain a formula for composite function</td>
<td><strong>A18</strong> given nature of roots, find a condition on coeffs</td>
<td><strong>A31</strong> solve eqns of the form $\log_b(a) = c$ for $a, b$ or $c$</td>
<td></td>
</tr>
<tr>
<td><strong>A5</strong> complete the square</td>
<td><strong>A19</strong> form an equation with given roots</td>
<td><strong>A32</strong> solve equations involving logarithms</td>
<td></td>
</tr>
<tr>
<td><strong>A6</strong> interpret equations and expressions</td>
<td><strong>A20</strong> apply A15-A19 to solve problems</td>
<td><strong>A33</strong> use relationships of the form $y = ax^2$ or $y = ab^x$</td>
<td></td>
</tr>
<tr>
<td><strong>A7</strong> determine function (poly, exp, log) from graph &amp; vv</td>
<td><strong>A8</strong> sketch/annotate graph given critical features</td>
<td><strong>A34</strong> apply A38-A33 to problems</td>
<td></td>
</tr>
<tr>
<td><strong>A9</strong> interpret loci such as sl. lines, para, poly, circle</td>
<td><strong>A10</strong> apply the notation $u_n$, for the nth term</td>
<td><strong>A21</strong> use Rem Th. For values, factors, roots</td>
<td></td>
</tr>
<tr>
<td><strong>A11</strong> evaluate successive terms of a RR</td>
<td><strong>A11</strong> solve cubic and quartic equations</td>
<td><strong>A22</strong> calculate the length of a vector</td>
<td></td>
</tr>
<tr>
<td><strong>A12</strong> decide when RR has limit/interpret limit</td>
<td><strong>A12</strong> find intersection of line and polynomial</td>
<td><strong>A23</strong> calculate the 3rd given two from $A, B$ and vector $AB$</td>
<td></td>
</tr>
<tr>
<td><strong>A13</strong> evaluate limit</td>
<td><strong>A13</strong> find if line is tangent to polynomial</td>
<td><strong>A24</strong> use unit vectors</td>
<td></td>
</tr>
<tr>
<td><strong>A14</strong> apply A10-A14 to problems</td>
<td><strong>A14</strong> find intersection of two polynomials</td>
<td><strong>A25</strong> use: if $\mathbf{u, v}$ are parallel then $\mathbf{v} = k \mathbf{u}$</td>
<td></td>
</tr>
<tr>
<td><strong>A15</strong> apply G1-G7 to problems</td>
<td><strong>A15</strong> confirm and improve on approx roots</td>
<td><strong>A26</strong> simplify vector pathways</td>
<td></td>
</tr>
<tr>
<td><strong>A16</strong> apply G1-G7 to problems eg intersect., concur., collin.</td>
<td><strong>A16</strong> apply A21-A36 to problems</td>
<td><strong>A27</strong> interpret 2D sketches of 3D situations</td>
<td></td>
</tr>
<tr>
<td><strong>A17</strong> use the distance formula</td>
<td><strong>A17</strong> find C/R of a circle from its equation/other data</td>
<td><strong>A28</strong> find if 2 points in space are collinear</td>
<td></td>
</tr>
<tr>
<td><strong>A18</strong> find gradient from 2 pts./angle/equ. of line</td>
<td><strong>A18</strong> find the equation of a circle</td>
<td><strong>A29</strong> find ratio which one point divides two others</td>
<td></td>
</tr>
<tr>
<td><strong>A19</strong> find equation of a line</td>
<td><strong>A19</strong> find equation of a tangent to a circle</td>
<td><strong>A30</strong> calculate the scalar product</td>
<td></td>
</tr>
<tr>
<td><strong>A20</strong> find equation of a line &amp; vv</td>
<td><strong>A20</strong> find if line is tangent to polynomial</td>
<td><strong>A31</strong> use: if $\mathbf{u, v}$ are perpendicular then $\mathbf{u} \cdot \mathbf{v} = 0$</td>
<td></td>
</tr>
<tr>
<td><strong>A21</strong> interpret all equations of a line</td>
<td><strong>A21</strong> find intersection of line &amp; circle</td>
<td><strong>A32</strong> calculate the angle between two vectors</td>
<td></td>
</tr>
<tr>
<td><strong>A22</strong> find property of perpendicular lines</td>
<td><strong>A22</strong> find if line is parallel to a circle</td>
<td><strong>A33</strong> use the distributive law</td>
<td></td>
</tr>
<tr>
<td><strong>A23</strong> calculate mid-point</td>
<td><strong>A23</strong> find if line is tangent to circle</td>
<td><strong>A34</strong> apply G16-G29 to problems eg geometry probs.</td>
<td></td>
</tr>
<tr>
<td><strong>A24</strong> find equation of median, altitude, perp. bisector</td>
<td><strong>A24</strong> apply A5-G14 to problems</td>
<td><strong>A35</strong> differentiate $\sin(ax+b), \cos(ax+b)$</td>
<td></td>
</tr>
<tr>
<td><strong>A25</strong> apply G1-G7 to problems eg intersect., concur., collin.</td>
<td><strong>A25</strong> find if/when line is tangent to circle</td>
<td><strong>A36</strong> find integrals of $pq\sin(ax+b), pq\cos(ax+b)$</td>
<td></td>
</tr>
<tr>
<td><strong>A26</strong> apply G1-G7 to problems eg intersect., concur., collin.</td>
<td><strong>A26</strong> find the equation of a circle</td>
<td><strong>A37</strong> find integrals of $p\sin(ax+b), p\cos(ax+b)$</td>
<td></td>
</tr>
<tr>
<td><strong>A27</strong> use the distance formula</td>
<td><strong>A27</strong> find intersection of lines &amp; circle</td>
<td><strong>A38</strong> find integrals of $\sin^2(x), \cos^2(x)$</td>
<td></td>
</tr>
<tr>
<td><strong>A28</strong> use the general equation of a parabola</td>
<td><strong>A28</strong> find the equation of a tangent to a circle</td>
<td><strong>A39</strong> use the general equation of a parabola</td>
<td></td>
</tr>
<tr>
<td><strong>A29</strong> use Rem Th. For values, factors, roots</td>
<td><strong>A29</strong> use the distance formula</td>
<td><strong>A40</strong> use the general equation of a parabola</td>
<td></td>
</tr>
<tr>
<td><strong>A30</strong> use the general equation of a parabola</td>
<td><strong>A30</strong> use the distance formula</td>
<td><strong>A41</strong> use the general equation of a parabola</td>
<td></td>
</tr>
<tr>
<td><strong>A31</strong> calculate the length of a vector</td>
<td><strong>A31</strong> use the distance formula</td>
<td><strong>A42</strong> use the general equation of a parabola</td>
<td></td>
</tr>
<tr>
<td><strong>A32</strong> sketch associated graphs</td>
<td><strong>A32</strong> use the distance formula</td>
<td><strong>A43</strong> use the general equation of a parabola</td>
<td></td>
</tr>
<tr>
<td><strong>A33</strong> use relationships of the form $y = ax^2$ or $y = ab^x$</td>
<td><strong>A33</strong> use relationships of the form $y = ax^2$ or $y = ab^x$</td>
<td><strong>A44</strong> use the general equation of a parabola</td>
<td></td>
</tr>
</tbody>
</table>
1 PQRS is a parallelogram. P is the point (2, 0), S is (4, 6)
and Q lies on the x-axis, as shown.
The diagonal QS is perpendicular to the side PS.
(a) Show that the equation of QS is $x + 3y = 22$.
(b) Hence find the coordinates of Q and R.

Notes

In (a)

1 In the Primary method, $^1$ is only available if an attempt has been made to find and use a perpendicular gradient.

2 In the Primary method and the Alt. method $^1$, $^4$ is only available for reaching the required equation.

3 To gain $^4$, some evidence of completion needs to be shown

\[ y - 6 = -\frac{1}{3}(x - 4) \]
\[ 3(y - 6) = -(x - 4) \]
\[ x + 3y = 22 \]

4 Sometimes candidates manage to find R first. Provided the coordinates of R are of the form (? , 6), only then is $^6$ available as a follow through.

5 $^5$ and $^6$ are available to candidates who use their own erroneous equation for QS.

General Notes applicable throughout the marking scheme

There are many instances when follow throughs come into play and these will not always be highlighted for you. The following example is a reminder of what you have to look out for when you are marking.

example

At the $^5$ stage a candidate may switch the coordinates round so we have

\[ X \quad Q(0, 22) \]
\[ X\sqrt{ } \quad R(2.28) \quad \text{repeated error} \]

so the candidate loses $^5$ for switching the coordinates but gains $^6$ as a consequence of following through.

Any error can be followed through and the subsequent marks awarded provided the working has not been eased. Any deviation from this will be noted in the marking scheme.

Primary Method : Give 1 mark for each •

\begin{itemize}
  \item \( m_{PS} = 3 \)  
  \item \( m_{QS} = -\frac{1}{3} \)  
  \item \( y - 6 = -\frac{1}{3}(x - 4) \)  
  \item completes proof  
  \item \( Q = (22, 0) \)  
  \item \( R = (24, 6) \)
\end{itemize}

Alternative Method 1

\begin{itemize}
  \item \( m_{PS} = 3 \)  
  \item \( m_{QS} = -\frac{1}{3} \)  
  \item \( y = -\frac{1}{3}x + c \)  
  \item \( 6 = -\frac{1}{3} \times 4 + c \)  
  \item completes proof  
  \item \( Q = (22, 0) \)  
  \item \( R = (24, 6) \)
\end{itemize}

Alternative Method 2

Let \( Q = (q, 0) \)

\begin{itemize}
  \item \( (q - 2)^2 = 2^2 + 6^2 + (q - 4)^2 + 6^2 \)  
  \item \( q = 22 \)  
  \item \( Q = (22, 0) \) and \( R = (24, 6) \)  
  \item \( m_{QS} = -\frac{1}{3} \)  
  \item \( y - 0 = -\frac{1}{3}(x - 22) \)  
  \item leading to \( 3y + x = 22 \)
\end{itemize}

N.B.
The coordinates of Q can also be arrived at by right-angled trig.
Use the alt. method 2 marking scheme with $^1$ replaced by appropriate trig. work.
The only acceptable value for q is 22.
Find the value of \( k \) such that the equation \( kx^2 + kx + 6 = 0 \), \( k \neq 0 \), has equal roots.

### Notes

1. The evidence for •1 and/or •2 may not appear until the working immediately preceding the evidence for •3, i.e. a candidate may simply start:

   \[
   \sqrt{\text{•1}}, \sqrt{\text{•2}} \quad k^2 - 4 \times k \times 6 = 0 \\
   \sqrt{\text{•3}} \quad k(k - 24)
   \]

   or

   \[
   \sqrt{\text{•2}} \quad k^2 - 4 \times k \times 6 \\
   \sqrt{\text{•3}} \quad k(k - 24) = 0
   \]

2. The "= 0" has to appear at least once, at the •1 stage or at the •3 stage.

3. In the Primary method, candidates who do not deal with the root \( k = 0 \) cannot obtain •4. [see Common Errors 1 and 2]

   Minimum evidence for •4 would be scoring out "k = 0" or "k = 24" underlined.

4. Some candidates may start with the quadratic formula. Apply the marking scheme to the part underneath the square root sign.

5. The use of any expression masquerading as the discriminant can only gain •2 at most.

### Primary Method: Give 1 mark for each •

- **•1** "\( b^2 - 4ac \)" = 0
- **•2** \( a = k, b = k, c = 6 \)
- **•3** \( k(k - 24) \)
- **•4** \( k = 0 \) and \( k = 24 \)  
  \[\therefore k = 24\]

4 marks

### Alternative Method 1 (completing the square)

- **•1** \( (x + \frac{1}{2})^2 + \ldots \)
- **•2** \( (x + \frac{1}{2})^2 - \frac{1}{4} + \frac{k}{x} = 0 \)
- **•3** equal roots \( \Rightarrow -\frac{1}{4} + \frac{k}{x} = 0 \)
- **•4** \( k = 24 \)

### Acceptable alternative for •4

- **•1** "\( b^2 - 4ac \)" = 0
- **•2** \( a = k, b = k, c = 6 \)
- **•3** \( k(k - 24) \)
- **•4** \( k \neq 0 \) or \( 24 \)

### Common Error 1 at the •4 stage

- **•1** "\( b^2 - 4ac \)" = 0
- **•2** \( a = k, b = k, c = 6 \)
- **•3** \( k(k - 24) \)
- **•4** \( k = 0 \) or \( 24 \)

### Common Error 2 at the •4 stage

- **•1** "\( b^2 - 4ac \)" = 0
- **•2** \( a = k, b = k, c = 6 \)
- **•3** \( k(k - 24) \)
- **•4** \( k = 24 \)

### Common Error 3 Division by k

- **•1** "\( b^2 - 4ac \)" = 0
- **•2** \( a = k, b = k, c = 6 \)
- **•3** \( k^2 - 24k = 0 \)
  \( k^2 = 24k \)
- **•4** \( k = 24 \)
3. The parabola with equation \( y = x^2 - 14x + 53 \) has a tangent at the point \( P(8,5) \).

(a) Find the equation of this tangent.

(b) Show that the tangent found in (a) is also a tangent to the parabola with equation \( y = -x^2 + 10x - 27 \) and find the coordinates of the point of contact \( Q \).

**Primary Method : Give 1 mark for each •**

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1. ss know to differentiate
2. pr differentiate
3. pr evaluate gradient
4. ic state equation of tangent
5. ss arrange in standard form
6. ss substitute into quadratic
7. pr process
8. ic factorise & interpret
9. ic state coordinates

**Notes**

ln (a)

1. \( ^4 \) is only available if an attempt has been made to find the gradient from differentiation.

ln (b)

2. \( ^6 \) is only available for a numerical value of \( m \).

3. An “= 0” must occur somewhere in the working between \( ^7 \) and \( ^8 \).

4. \( ^8 \) is awarded for drawing a conclusion from the candidate’s quadratic equation.

5. Candidates may substitute the equation of the parabola into the equation of the line. This is a perfectly acceptable approach.

**Common Error 1**

\[
\sqrt{\bullet}^1 \frac{dy}{dx} = \\
\sqrt{\bullet}^2 2x - 14 \\
X \bullet^3 2x - 14 = 0 \text{ so } x = 7 \text{ so } m = 7 \\
X \bullet^4 y - 5 = 7(x - 8) \\
X \sqrt{\bullet}^5 y = 7x - 51 \\
X \sqrt{\bullet}^6 7x - 51 = -x^2 + 10x - 27 \\
X \sqrt{\bullet}^7 x^2 - 3x - 24 = 0 \\
X \sqrt{\bullet}^8 b^2 - 4ac = 105 \Rightarrow \text{line is not tgt} \\
X \bullet^9 - - \\
\text{so award 6 marks}
\]
The circles with equations \((x - 3)^2 + (y - 4)^2 = 25\) and \(x^2 + y^2 - kx - 8y - 2k = 0\) have the same centre. Determine the radius of the larger circle.

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<th>Syllabus Code</th>
<th>Calculator class</th>
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- **Primary Method**: Give 1 mark for each •

- **•1**, ic state centre of circle 1
- **•2**, ss equate \(x\)-coordinates, find \(k\).
- **•3**, ic find radius of circle 1
- **•4**, ic substitute into the radius formula
- **•5**, ic process radius formula and compare.

**Notes**

1. **•2** requires no justification.

2. Evidence for **•3** may appear for the first time at the \(•5\) stage.

3. If \(R_1 = 5\) is clearly stated at the \(•3\) stage, then it does not have to appear at the \(•5\) stage for the conclusion to be drawn.

4. For any formula masquerading as the radius formula (e.g. see Common Error 2), **•4** and **•5** are NOT available.

**Alternative Method 1**

- **•1** \(x^2 + y^2 - 6x - 8y + 25 = 25\)
- **•2** \(k = 6\)
- **•3** \(R_1 = 5\)
- **•4** \(R_2 = \sqrt{(-3)^2 + (-4)^2 - 12}\) or equivalent
- **•5** \(\sqrt{37} > 5\ or \ "2nd\ circle"\)

**Common Error 1**

\[
\begin{align*}
\sqrt{•1} & \quad C_1 = (3, 4) \\
\sqrt{•2} & \quad k = 6 \\
\sqrt{•3} & \quad R_1 = 5 \\
X & \quad R_2 = \sqrt{(-3)^2 + (-4)^2 - 12} \\
X\sqrt{•5} & \quad \sqrt{13} < 5 \ or \ "1st\ circle" \\
\end{align*}
\]

**Common Error 2**

\[
\begin{align*}
\sqrt{•1} & \quad C_1 = (3, 4) \\
\sqrt{•2} & \quad k = 6 \\
\sqrt{•3} & \quad R_1 = 5 \\
X & \quad R_2 = \sqrt{(-3)^2 + (-4)^2 + (12)^2} \\
X \sqrt{•5} & \quad 13 > 5 \ or \ "2nd\ circle" \\
\end{align*}
\]
The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point $(-1, 9)$. Express $y$ in terms of $x$.

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME.

Primary Method: Give 1 mark for each •

- •1 ss know to integrate
- •2 pr integrate
- •3 ic substitute values
- •4 pr process constant

Notes
1 The equation "y = ......." must appear somewhere in the solution.

Common Error 1 Missing “equation”

√ •1 $y = \int ...$
√ •2 $\frac{1}{2}x^2 - \frac{6}{3}x^3$
√ •3 $9 = 2(-1)^2 - 2(-1)^3 + c$
X •4 $c = 5$

award 3 marks

Common Error 2: Not using $(-1, 9)$

√ •1 $y = \int ...$
√ •2 $\frac{1}{2}x^2 - \frac{6}{3}x^3$
X •3 $2(-1)^2 - 2(-1)^3 + c = 0$
X •4 $y = 2x^2 - 2x^3 - 4$

award 2 marks

Alternative Marking

•1 $y = \int ...$
•2 $\frac{1}{2}x^2 - \frac{6}{3}x^3$

$\begin{align*}
y &= 2x^2 - 2x^3 + c \\
&\text{stated explicitly}
\end{align*}$

and

$\begin{align*}
9 &= 2(-1)^2 - 2(-1)^3 + c \\
c &= 5
\end{align*}$
6 P is the point (−1, 2, −1) and Q is (3, 2, −4).

(a) Write down \( \overrightarrow{PQ} \) in component form.

(b) Calculate the length of \( \overrightarrow{PQ} \).

(c) Find the components of a unit vector which is parallel to \( \overrightarrow{PQ} \).

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**THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME**

- **• 1 ic state vector components**
- **• 2 pr find the length of a vector**
- **• 3 ic state unit vector**

<table>
<thead>
<tr>
<th>Primary Method : Give 1 mark for each •</th>
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<tbody>
<tr>
<td>1 ( \overrightarrow{PQ} = \begin{pmatrix} 4 \ 0 \ -3 \end{pmatrix} )</td>
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<td>1 ( \begin{pmatrix} \frac{4}{5} \ 0 \ -\frac{3}{5} \end{pmatrix} )</td>
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**Note**

In (a)

1 It is perfectly acceptable to write the components as a row vector eg \( \overrightarrow{PQ} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} \).

Treat \( \overrightarrow{PQ} = (4, 0, -3) \) as bad form (i.e. not penalised).

In (b)

2 \( \cdot 2 \) is not awarded for an unsimplified \( \sqrt{25} \).

In (c)

3 Beware of misappropriate use of the scalar product where, by coincidence, \( \cdot q = 5 \).

In (c)

4 Accept \( \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} \) for \( \cdot 3 \).
The diagram shows the graph of a function \( y = f(x) \).

Copy the diagram and on it sketch the graphs of

(a) \( y = f(x - 4) \)
(b) \( y = 2 + f(x - 4) \)

Notes

For (a)
1. A translation of \( \begin{pmatrix} -4 \\ 0 \end{pmatrix} \) earns a maximum of 1 mark with both points clearly annotated and \( f(x) \) retaining its shape.
2. Any other translation gains no marks.

In the Primary method

For (b)
3. \( a^2 \) and \( a^4 \) are only available for applying the translation to the resultant graph from (a).
4. A translation of \( \begin{pmatrix} 0 \\ -2 \end{pmatrix} \) earns a maximum of 1 mark with both points clearly annotated and the resultant graph from (a) retaining its shape.
5. Any other translation gains no marks.

In the Alternative method

For (b)
6. A translation of \( \begin{pmatrix} 4 \\ -2 \end{pmatrix}, \begin{pmatrix} -4 \\ 2 \end{pmatrix} \) or \( \begin{pmatrix} -4 \\ -2 \end{pmatrix} \) applied to the original graph earns a maximum of 1 mark with both points clearly annotated and the resultant graph retaining its original shape.
7. Any other translation gains no marks.

In either method

For (a) and (b)
8. For the annotated points, accept a superimposed grid or clearly labelled axes.
9. A candidate may choose to use two separate diagrams. This is acceptable.
8 The diagram shows a right-angled triangle with height 1 unit, base 2 units and an angle of \( a^\circ \) at A.

(a) Find the exact values of

(i) \( \sin a^\circ \)

(ii) \( \sin 2a^\circ \).

(b) By expressing \( \sin 3a^\circ \) as \( \sin(2a + a)^\circ \), find the exact value of \( \sin 3a^\circ \).

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Primary Method: Give 1 mark for each •

1. ic interpret diagram for \( \sin(a^\circ) \)
2. ss use double angle formula for \( \sin(2A) \)
3. ic interpret diagram for \( \cos(a^\circ) \)
4. pr substitute and complete
5. ss use compound angle formula
6. pr use double angle formula for \( \cos(2A) \)
7. ic substitute
8. pr complete

Note

1 Calculating approximate angles using arcsin and arccos gains no credit.

2 There are 3 processing marks •4, •6 and •8. None of these are available for an answer > 1.

3 \( \sin(2a) = 0.8 \) and \( \cos(2a) = 0.6 \) are the only two decimal fractions which may receive any credit.

4 Some candidates may double the height of the triangle and then call the base angle \( 2a \). This error is equivalent to Common Error 1 illustrated on the right.

Common Error 2
An example based on a numerical error in Pythagoras

\[
\begin{array}{c|c|c|c|c|c}
\text{X} & \text{•1} & \text{•2} & \text{•3} & \text{•4} & \text{•5} \\
\text{sin}(a^\circ) & \frac{1}{\sqrt{3}} & 2\sin(a^\circ)\cos(a^\circ) & \frac{2}{\sqrt{3}} & \frac{4}{3} & 2\sin(2a^\circ)\cos(a^\circ) + \cos(2a^\circ)\sin(a^\circ) \\
\text{sin}(2a^\circ) & \frac{2}{\sqrt{3}} & \frac{4}{3} & \frac{1}{3} & \frac{5}{3} & 2\cos^2(a^\circ) - 1 = \frac{5}{3} \text{ or equivalent} \\
\text{sin}(3a^\circ) & \frac{4}{\sqrt{3}} & \frac{2}{\sqrt{3}} + \frac{5}{3} & \frac{1}{3} & \frac{13}{3} & \sqrt{3} \\
\text{sin}(3a^\circ) & \frac{8}{5} & \end{array}
\]

Common Error 1 An example of Incorrect formulae

\[
\begin{array}{c|c|c|c|c|c}
\text{X} & \text{•1} & \text{•2} & \text{•3} & \text{•4} & \text{•5} \\
\text{sin}(a^\circ) & \frac{1}{\sqrt{5}} & 2\sin(a^\circ) & \frac{2}{\sqrt{5}} & \frac{2}{\sqrt{5}} & \frac{2}{\sqrt{5}} \text{ or equivalent} \\
\text{sin}(2a^\circ) & \frac{2}{\sqrt{5}} & \frac{2}{\sqrt{5}} + \frac{4}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{8}{5} \\
\text{sin}(3a^\circ) & \frac{4}{\sqrt{5}} & \end{array}
\]
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- **1** ss express in differentiable form
- **2** pr differentiate a term with a negative power
- **3** pr start to process a compound derivative
- **4** pr complete compound derivative

Notes
1 For clearly integrating, correctly or otherwise, only **1** is available.
2 If you cannot decide whether a candidate has attempted to differentiate or integrate, assume they have attempted to differentiate.
A curve has equation \[ y = 7\sin x - 24\cos x. \]

(a) Express \( 7\sin x - 24\cos x \) in the form \( k\sin(x - a) \) where \( k > 0 \) and \( 0 \leq a \leq \frac{\pi}{2} \).

(b) Hence find, in the interval \( 0 \leq x \leq \pi \), the \( x \)-coordinate of the point on the curve where the gradient is 1.
It is claimed that a wheel is made from wood which is over 1000 years old. To test this claim, carbon dating is used. The formula \( A(t) = A_0 e^{-0.000124t} \) is used to determine the age of the wood, where \( A_0 \) is the amount of carbon in any living tree, \( A(t) \) is the amount of carbon in the wood being dated and \( t \) is the age of the wood in years. For the wheel it was found that \( A(t) \) was 88% of the amount of carbon in a living tree. Is the claim true?

### Notes

1. Candidates may choose a numerical value for \( A_0 \) at the start of their solution. Accept this situation.
2. \( \cdot^5 \) is only available if \( \cdot^4 \) has been awarded.
3. In following through from an error, \( \cdot^5 \) is only available for a positive value of \( t \).

### Primary Method: Give 1 mark for each •

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- 1: \( A(t) = 0.88A_0 \)
- 2: \( e^{-0.000124t} = 0.88 \)
- 3: \( \ln(e^{-0.000124t}) = \ln(0.88) \)
- 4: \( -0.000124t = \ln(0.88) \)
- 5: \( t = 1031 \) years so claim valid

### Alternative Method 1 Graph and Calculator Solution

- 1: \( A(1000) = A_0 e^{-0.000124 \times 1000} \)
- 2: 0.883 \( A_0 \) and 1000 year old piece of wood contains 88.3% carbon.
- 3: try a point where \( t > 1030 \)
  - e.g. \( A(1050) \) getting 0.878 \( A_0 \)
- 4: sketch of \( y = A_0 e^{-0.000124t} \) showing
  - 1. a monotonic decreasing function
  - 2. points representing eg (1000, 88.3%) etc
- 5: observation that the point lies between the two plotted values for \( t \) and so claim valid.
12 PQRS is a rectangle formed according to the following conditions:

- it is bounded by the lines $x = 6$ and $y = 12$
- P lies on the curve with equation $y = \frac{8}{x}$ between $(1, 8)$ and $(4, 2)$
- R is the point (6, 12).

(a) (i) Express the lengths of PS and RS in terms of $x$, the $x$-coordinate of P.

(ii) Hence show that the area, $A$ square units, of PQRS is given by $A = 80 - 12x - \frac{48}{x}$.

(b) Find the greatest and least possible values of $A$ and the corresponding values of $x$ for which they occur.

Notes

1. For $i^3$ there needs to be clear evidence that candidates have multiplied out the brackets in order to complete the proof.

2. An “$= 0$” must appear somewhere in the working between $i^4$ and $i^7$.

3. At the $i^7$ stage, ignore the omission or inclusion of $x = -2$.

4. $i^8$ has to be as a consequence of solving $\frac{dA}{dx} = 0$.

5. $i^{11}$ is only available if both end points have been considered.