Perth Academy

Mathematics Department

Higher

Key Points

Further Calculus

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1 $\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$

2 $\int \cos x \, dx = \sin x + C \qquad \int \sin x \, dx = -\cos x + C$

3 The **chain rule**:

If $h(x) = f(g(x))$ then $h'(x) = f'(g(x)).g'(x)$ or $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

4 $\int (ax + b)^n \, dx = \frac{(ax + b)^{n+1}}{a(n + 1)} + C$

5 $\int \cos(ax + b) \, dx = \frac{1}{a}\sin(ax + b) + C$

$\int \sin(ax + b) \, dx = -\frac{1}{a}\cos(ax + b) + C$

Example 1

Differentiate $2 \cos x + \frac{2}{3} \sin x$ with respect to x.

Solution

$$f(x) = 2 \cos x + \tfrac{2}{3} \sin x$$
$$f'(x) = 2(-\sin x) + \tfrac{2}{3} \cos x$$
$$= -2 \sin x + \tfrac{2}{3} \cos x$$

Example 2

Given that $f(x) = (x^2 + 3)^8$ find $f'(x)$.

Solution

$$f(x) = (x^2 + 3)^8$$
$$f'(x) = 8(x^2 + 3)^7 \times 2x$$
$$= 16x(x^2 + 3)^7$$

Example 3

Find $\int 3 \sin x - \frac{1}{2} \cos x \, dx$.

Solution

$$\int 3 \sin x - \tfrac{1}{2} \cos x \, dx = 3 \times (-\cos x) - \tfrac{1}{2} \sin x + C$$
$$= -3 \cos x - \tfrac{1}{2} \sin x + C$$

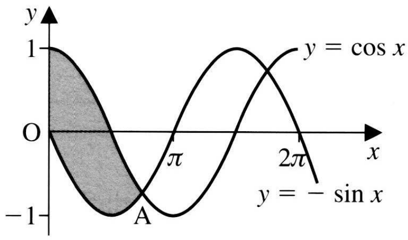
Example 4

Evaluate $\int_1^2 (2x + 1)^3 \, dx$.

Solution

$$\int_1^2 (2x + 1)^3 \, dx = \left[\frac{(2x + 1)^4}{4 \times 2}\right]_1^2$$
$$= \left[\tfrac{1}{8}(2x + 1)^4\right]_1^2$$
$$= \tfrac{1}{8}((2 \times 2 + 1)^4 - (2 \times 1 + 1)^4)$$
$$= \tfrac{1}{8}(625 - 81)$$
$$= 68$$

Example 5



The diagram shows the graphs of $y = -\sin x$ and $y = \cos x$.
(a) Find the coordinates of A.
(b) Hence find the shaded area.

Solution

(a) Solve simultaneously $y = -\sin x$ and $y = \cos x$

$$-\sin x = \cos x$$

$$\frac{-\sin x}{\cos x} = 1$$

$$\tan x = -1$$

$$x = \frac{5\pi}{4} \quad \text{or} \quad \frac{7\pi}{4}$$

when $x = \frac{5\pi}{4}$, $y = \frac{-1}{\sqrt{2}}$

The coordinates of A are $\left(\frac{5\pi}{4}, \frac{-1}{\sqrt{2}}\right)$

(b) Shaded area $= \int_0^{\frac{5\pi}{4}} \cos x - (-\sin x)\, dx$

$$= \int_0^{\frac{5\pi}{4}} \cos x + \sin x\, dx$$

$$= \left[\sin x - \cos x\right]_0^{\frac{5\pi}{4}}$$

$$= \left(\sin\frac{5\pi}{4} - \cos\frac{5\pi}{4}\right) - (\sin 0 - \cos 0)$$

$$= \left(-\frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right)\right) - (-1)$$

$$= 1$$

Example 6

Find $\int \sin 2x - \cos\left(3x - \frac{\pi}{4}\right) dx$.

Solution

$$\int \sin 2x - \cos\left(3x - \frac{\pi}{4}\right) dx = -\frac{1}{2}\cos 2x - \frac{1}{3}\sin\left(3x - \frac{\pi}{4}\right) + C$$