Perth Academy

Mathematics Department

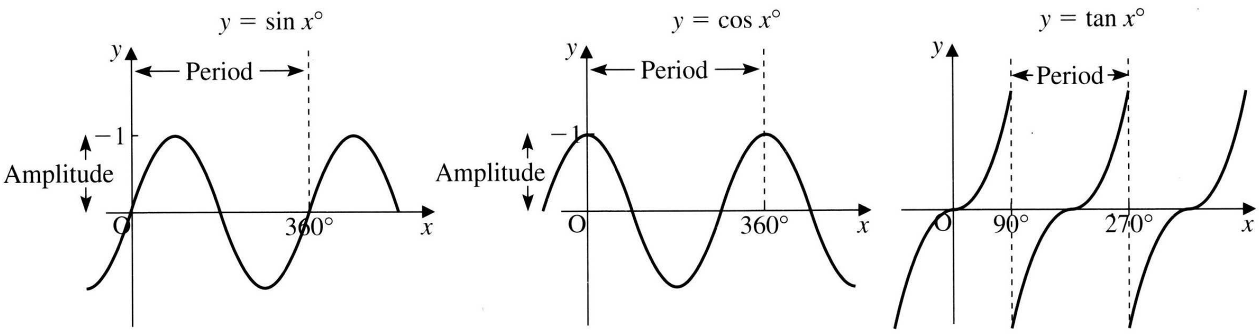
Higher

Key Points

Graphs and Equations

Graphs and Equations

1 A graph which consists of a repeated pattern is described as **periodic**.



Period = 360°
Amplitude = 1

Period = 360°
Amplitude = 1

Period = 180°
Amplitude cannot be measured

2 The horizontal extent of the basic pattern is called the **period**.
Half of the vertical extent is called the **amplitude**.

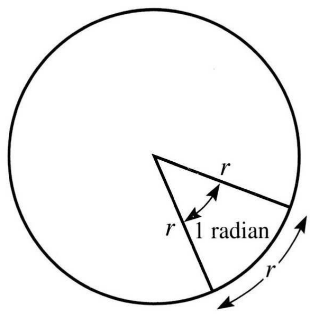
3 For $y = a \sin bx°$ and $y = a \cos bx°$

Amplitude $= a$ and period $= \dfrac{360°}{b}$

For $y = a \tan bx°$

Amplitude cannot be measured and period $= \dfrac{180°}{b}$

4 The angle subtended at the centre of a circle by an arc equal in length to the radius is **1 radian**.



5 π radians $= 180°$

6

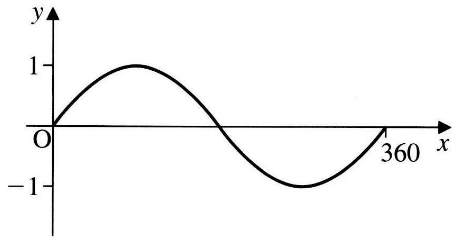
	0°	30°	45°	60°	90°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

Example 1

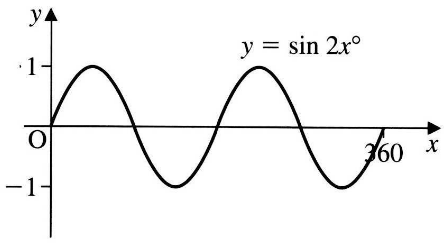
Sketch the graph of the function $y = 3\sin 2x° + 1$.

Solution

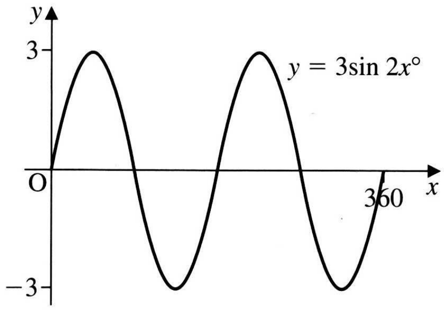
Start with the graph of $y = \sin x°$.



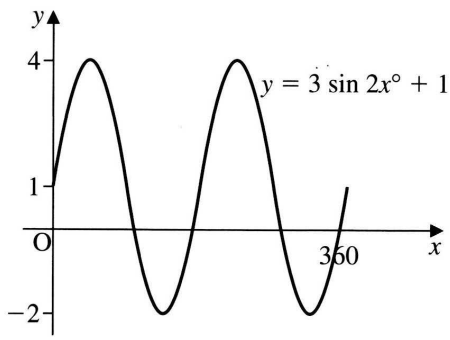
Squeeze horizontally by a factor of $\frac{1}{2}$



Stretch vertically by a factor of 3



Slide vertically 1 unit upwards

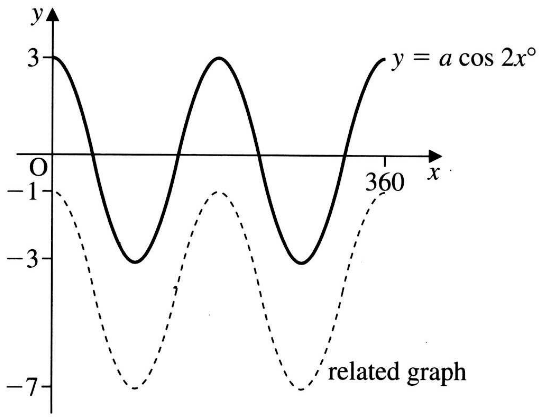


Example 2

The diagram shows part of the graph $y = a \cos 2x°$.

(a) State the value of a.

(b) Write the equation of the related graph.



Solution

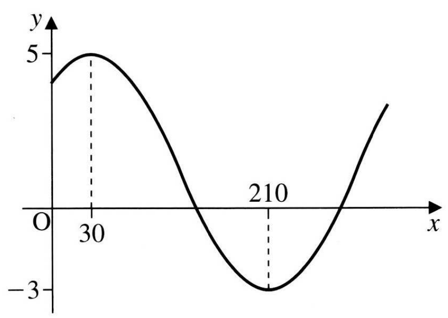
(a) The amplitude is 3, so $a = 3$.

(b) Since the related graph is 4 units below $y = 3 \cos 2x°$, its equation is $y = 3 \cos 2x° - 4$.

Example 3

The diagram shows part of the graph of $y = a \cos (x - b)° + c$.

Find the values of a, b and c.



Solution

$$\text{Period} = 360°$$

$$\text{Amplitude} = \frac{5 - (-3)}{2} = 4$$

$$\text{Horizontal shift} = 30 \text{ to the right}$$

$$\text{Vertical shift} = 1 \text{ up}$$

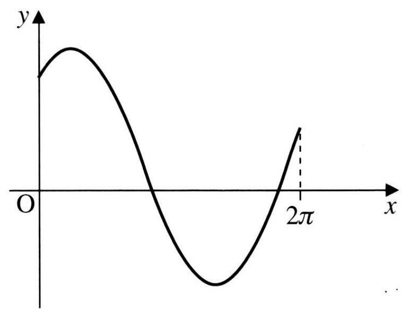
The graph has equation $y = 4 \cos (x - 30)° + 1$.

Example 4

Find the coordinates of the maximum turning point of the graph of $y = 3\sin\left(x + \frac{\pi}{3}\right)$ for $0 \leqslant x \leqslant 2\pi$.

Solution

Since $-1 \leqslant \sin x \leqslant 1$ then $-3 \leqslant 3\sin\left(x + \frac{\pi}{3}\right) \leqslant 3$



The maximum value of the function is 3, when

$$3\sin\left(x + \frac{\pi}{3}\right) = 3$$

$$\sin\left(x + \frac{\pi}{3}\right) = 1$$

$$x + \frac{\pi}{3} = \frac{\pi}{2}$$

$$x = \frac{\pi}{6}$$

The maximum turning point is at $\left(\frac{\pi}{6}, 3\right)$.

Example 5

Solve $4\cos^2 x - 1 = 0$ for $0 \leqslant x \leqslant 2\pi$.

Solution

$4\cos^2 x - 1 = 0$

$4\cos^2 x = 1$

$\cos^2 x = \frac{1}{4}$

$\cos x = \pm\frac{1}{2}$

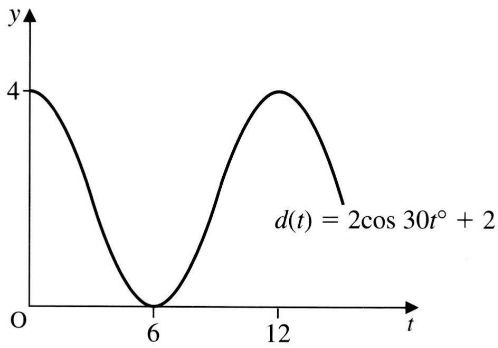
When $\cos x = \frac{1}{2}$, $x = \frac{\pi}{3}$ or $\frac{5\pi}{3}$

When $\cos x = -\frac{1}{2}$, $x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

$x = \frac{\pi}{3}$ or $\frac{2\pi}{3}$ or $\frac{4\pi}{3}$ or $\frac{5\pi}{3}$.

Example 6

The graph shows the depth, d metres, of water in a harbour t hours after midnight. The depth of water can be modelled by the function $d(t) = 2\cos 30t° + 2$. A boat anchored in the harbour has a draught of 3 metres. Between which hours will it be grounded?



Solution

The boat will float if $d(t) > 3$

so $2\cos 30t° + 2 > 3$

$\cos 30t° > \frac{1}{2}$

When $\cos 30t° = \frac{1}{2}$ then $30t = 60$ or 300

$t = 2$ or 10

The boat will be grounded between 0200 hours and 1000 hours.