Perth Academy

Mathematics Department

Higher

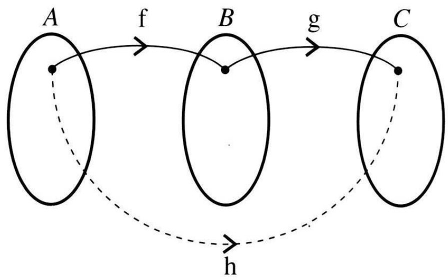
Key Points

Sets and Functions

Sets and Functions

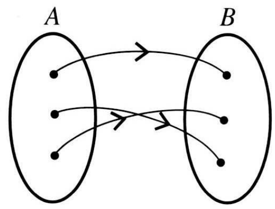
1 A **function** or mapping from a set A to a set B is a rule that relates each element in set A to one and only one element in set B.
The set of elements in set A is called the **domain**.
The set of images in set B is called the **range**.

2 A composite function can be written in the form $h(x) = g(f(x))$ and is read as 'g of f of x'.

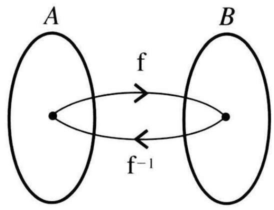


3 In general $f(g(x)) \neq g(f(x))$

4 A function in which the elements of two sets are paired so that each element of set A corresponds to one element of set B, and vice versa, is called a **one-to-one correspondence**.

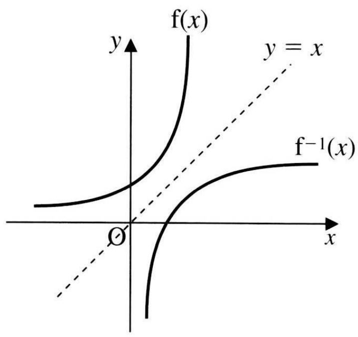


5 When a function f is a one-to-one correspondence from set A to set B, another function, f^{-1}, exists that maps from set B to set A. This function is called the **inverse** of f.



6 $f^{-1}(f(x)) = f(f^{-1}(x)) = x$

7 To find the graph of an inverse function reflect the graph of the function in the line $y = x$.



8 $f(x) = a^x$, $x \in \mathbf{R}$, is called an **exponential function** to base a, $a \in \mathbf{R}$, $a \neq 0$.

9 The inverse function of $f(x) = a^x$ is called the **logarithmic function** to base a, written as $\log_a x$.

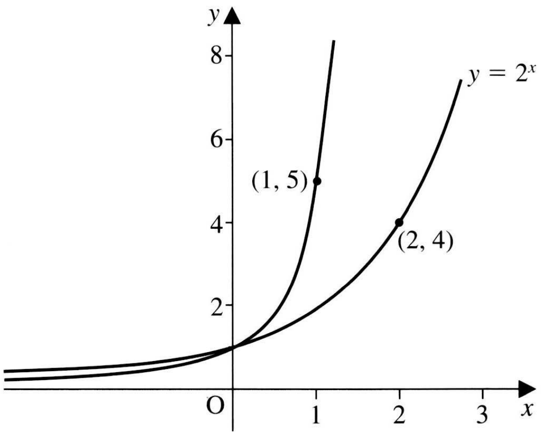
10 If $f(x) = a^x$, then $f^{-1}(x) = \log_a x$.
If $f(x) = \log_a x$, then $f^{-1}(x) = a^x$.

Example 1

The graph of $y = 2^x$ is shown in the diagram below.

(a) Write down the equation of the graph of the exponential function of the form $y = a^x$ which passes through the point (1, 5) as shown in the diagram.

(b) On a similar diagram, draw the graph of the function $y = 4^x$.

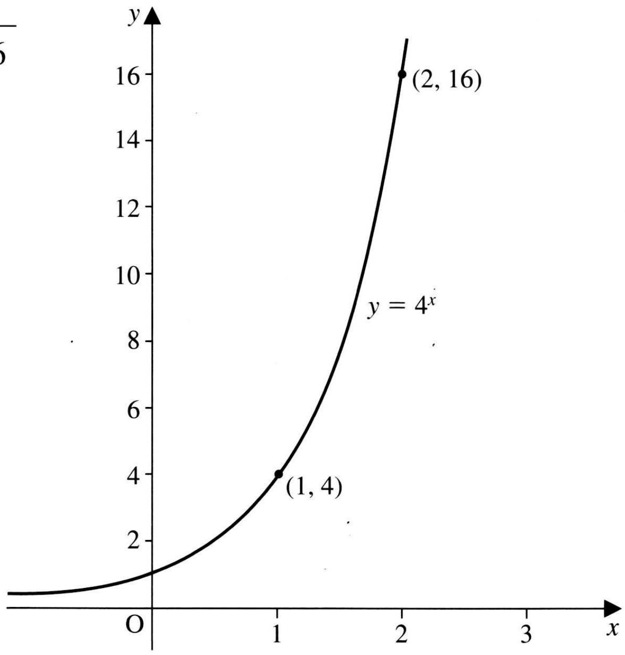


Solution

(a) For the function $y = a^x$, when $x = 1$ and $y = 5$ then $5 = a^1$.
So $a = 5$ and the equation of the graph is $y = 5^x$

(b) $y = 4^x$

x	0	1	2
y	1	4	16

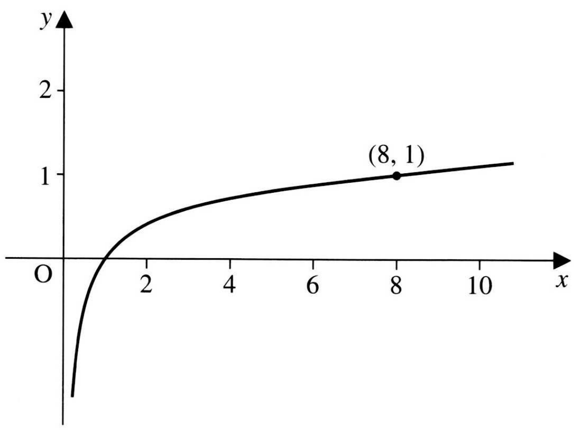


Example 2

The diagram below shows part of the graph of a logarithmic function.

(a) Write down the equation of the function.

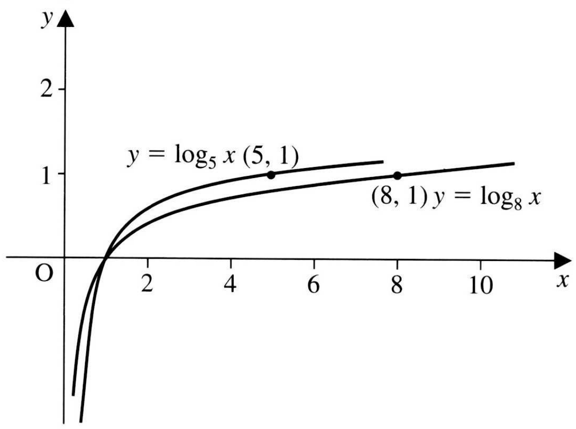
(b) Make a copy of the diagram and on it draw the graph of the function $y = \log_5 x$, showing clearly where it crosses the x-axis and marking in the coordinates of one other point that it passes through.



Solution

(a) $y = \log_8 x$

(b)



Example 3

(a) Two functions f and g are given by $f(x) = -2x^2$ and $g(x) = 5 - 3x$. Obtain an expression for $f(g(x))$ and for $g(f(x))$.

(b) Functions h and k, defined on suitable domains, are given by $h(x) = 5x$ and $k(x) = \cos x°$. Find $k(h(x))$ and $h(k(x))$.

Solution

(a)
$$\begin{aligned} f(g(x)) &= f(5 - 3x) \\ &= -2(5 - 3x)^2 \\ &= -50 + 60x - 18x^2 \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(-2x^2) \\ &= 5 - 3(-2x^2) \\ &= 5 + 6x^2 \end{aligned}$$

(b)
$$\begin{aligned} k(h(x)) &= k(5x) \\ &= \cos 5x° \end{aligned}$$

$$\begin{aligned} h(k(x)) &= h(\cos x°) \\ &= 5 \cos x° \end{aligned}$$

Example 4

A function f is defined by $f(x) = 2x + 3$ where $x \in \mathbf{R}$ and a second function g is defined by $g(x) = \dfrac{x^2 + 25}{x^2 - 25}$ where $x \in \mathbf{R}$, $x \neq \pm 5$.
The function H is defined by $H(x) = g(f(x))$. For which real values of x is the function H undefined? [Higher]

Solution

$$H(x) = g(f(x)) = g(2x + 3)$$
$$= \frac{(2x + 3)^2 + 25}{(2x + 3)^2 - 25}$$
$$= \frac{4x^2 + 12x + 9 + 25}{4x^2 + 12x + 9 - 25}$$
$$= \frac{4x^2 + 12x + 34}{4x^2 + 12x - 16}$$
$$= \frac{2(2x^2 + 6x + 17)}{4(x + 4)(x - 1)}$$

The function H is undefined when the denominator is zero.
Hence, the function is undefined for $x = -4$ and $x = 1$.