Perth Academy

Mathematics Department

Higher

Key Points

Straight Line

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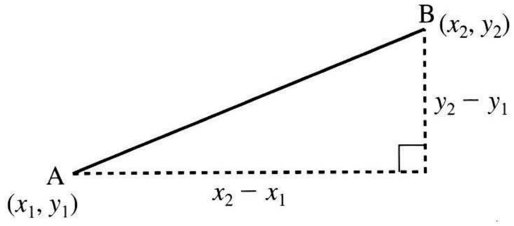
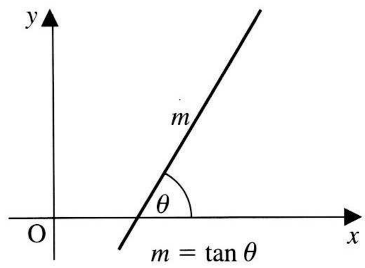
1 The gradient of AB $= m_{AB} = \dfrac{y_2 - y_1}{x_2 - x_1}$



2 The gradient of a line is the tangent of the angle it makes with the positive direction of the x-axis.



3 Lines parallel to the

- x-axis have gradient zero and equations of the form $y = a$
- y-axis have gradient undefined and equations of the form $x = b$.

4 Parallel lines have equal gradients.

5 If two lines with gradients m_1 and m_2 are perpendicular then $m_1 \times m_2 = -1$.

If $m_1 \times m_2 = -1$ then the lines with gradients m_1 and m_2 are perpendicular.

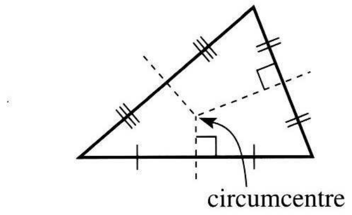
6 The equation of a straight line with gradient m and y-intercept c is $y = mx + c$.

7 The equation of a straight line with gradient m and passing through (a, b) is $y - b = m(x - a)$.

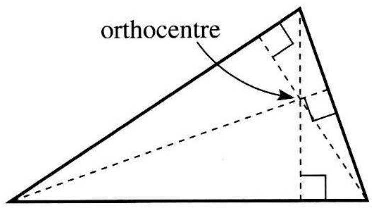
8 $Ax + By + C = 0$ is the **general equation** of a straight line.

9 In any triangle

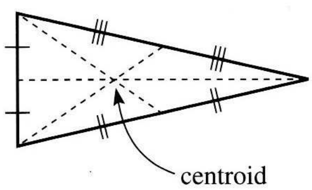
(i) the perpendicular bisectors of the sides are concurrent



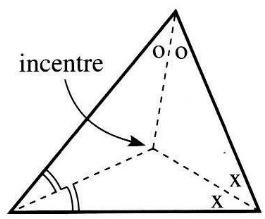
(ii) the altitudes are concurrent



(iii) the medians are concurrent



(iv) the angle bisectors are concurrent.



10 The distance between (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
This is known as the **distance formula**.

Example 1

A line passes through the points P(−2, 5) and Q(1, −1).
Find the equation of this line.

Solution

$$m_{PQ} = \frac{-1-5}{1-(-2)} = \frac{-6}{3} = -2$$

Equation of PQ is $y - b = m(x - a)$

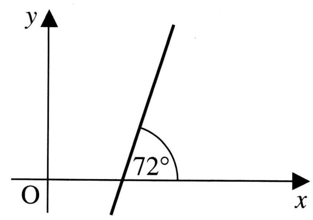
$$y - 5 = -2(x - (-2))$$

$$y - 5 = -2x - 4$$

$$2x + y - 1 = 0$$

Example 2

A line passes through an angle of 72° with the positive direction of the x-axis.
Find the gradient of the line.



Solution

Gradient of line, $m = \tan 72° = 3.1$

Example 3

(a) The line AB is parallel to a line with equation $5x - y - 2 = 0$. Write down the gradient of AB.

(b) The line PQ is perpendicular to a line with equation $5x - y - 2 = 0$. Write down the gradient of PQ.

Solution

(a) $5x - y - 2 = 0$

$y = 5x - 2$

$m_{AB} = 5$

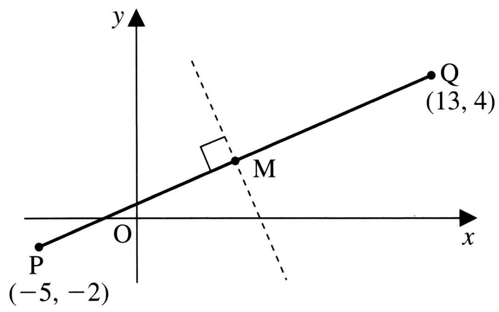
(b) $m_{AB} = 5$

$m_{PQ} = \frac{-1}{5}$

Example 4

Find the equation of the perpendicular bisector of the line joining P(−5, −2) and Q(13, 4).

Solution



$$m_{PQ} = \frac{4-(-2)}{13-(-5)} = \frac{6}{18} = \frac{1}{3}$$

$$m_{\text{perp. bis.}} = -3$$

Mid-point of PQ is M $\left(\frac{-5+13}{2}, \frac{-2+4}{2}\right)$

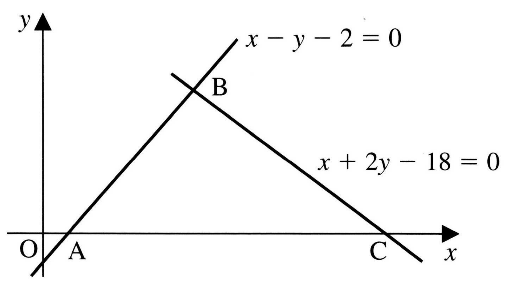
The perpendicular bisector passes through M(4, 1).

Equation of perpendicular bisector is

$$y - b = m(x - a)$$
$$y - 1 = -3(x - 4)$$
$$y + 3x - 13 = 0$$

Example 5

AB has equation $x - y - 2 = 0$.
CB has equation $x + 2y - 18 = 0$.
Calculate the size of angle ABC.



Solution

For line AB: $x - y - 2 = 0$

$y = x - 2$

$m_{AB} = 1$

$\tan x\text{AB} = 1$

angle CAB = 45°

For line CB: $x + 2y - 18 = 0$

$y = -\frac{1}{2}x + 9$

$m_{CB} = -\frac{1}{2}$

$\tan x\text{CB} = -0.5$

angle xCB = 153.4°

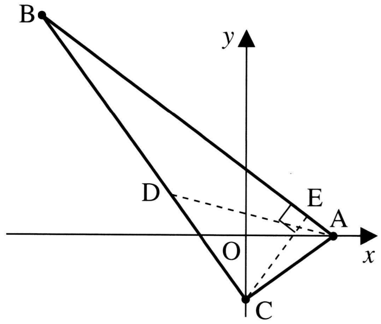
So angle ACB = 26.6°

Hence angle ABC = 108.4°

Example 6

Triangle ABC has vertices A(3, 0), B(−12, 9) and C(0, −3).

(a) Show that triangle ABC is right-angled at C.
(b) Find the equation of median AD.
(c) Find the equation of the altitude CE.
(d) Find the point of intersection of AD and CE.



Solution

(a) $m_{AC} = \frac{-3-0}{0-3} = \frac{-3}{-3} = 1$

$m_{BC} = \frac{-3-9}{0-(-12)} = \frac{-12}{12} = -1$

$m_{AC} \times m_{BC} = -1$

AC is perpendicular to BC, therefore triangle ABC is right-angled at C.

(b) The mid-point of BC is D(−6, 3).

$m_{AD} = \frac{3-0}{-6-3} = \frac{3}{-9} = \frac{-1}{3}$

The equation of AD is $y - 0 = \frac{-1}{3}(x-3)$

$$3y = -x + 3$$

$$x + 3y = 3$$

(c) $m_{AB} = \frac{9-0}{-12-3} = \frac{9}{-15} = \frac{-3}{5}$

$m_{CE} = \frac{5}{3}$

The equation of CE is $y - (-3) = \frac{5}{3}(x - 0)$

$$3y + 9 = 5x$$

$$5x - 3y = 9$$

(d) AD and CE meet where $x + 3y = 3$

and $5x - 3y = 9$

Solve these equations simultaneously to give

$x = 2$

$y = \frac{1}{3}$

The point of intersection of AD and CE is $(2, \frac{1}{3})$.