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Unit 2 : Properties of Functions - Lesson 1

Even, Odd and Neither Functions

LI

- Determine whether a function is Even, Odd or Neither.

SC

- Algebra.

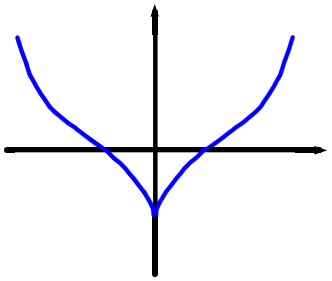
Jan 10-12:11

Even Functions

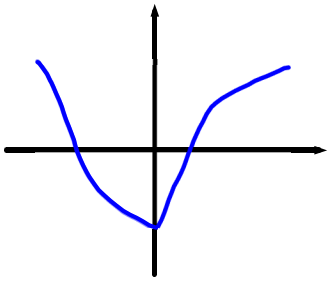
A function is even if $f(-x) = f(x)$ $(\forall x \in \text{dom } f)$

An even function is one whose graph is symmetrical about the y - axis

EVEN



NOT EVEN

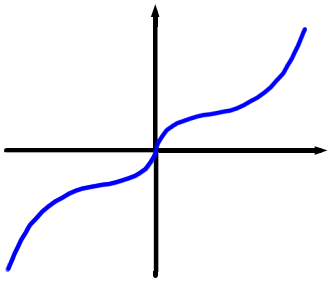


Odd Functions

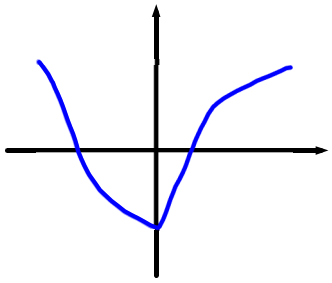
A function is odd if $f(-x) = -f(x)$ $(\forall x \in \text{dom } f)$

An odd function is one whose graph, when reflected in the x - axis and then the y - axis (or vice versa) results in a graph that is identical with the original graph; this is the same as rotating the graph 180° about the origin

ODD



NOT ODD



WARNING : 'Not even' doesn't necessarily mean 'odd' and 'not odd' doesn't necessarily mean 'even'

Aug 25-13:18

Neither (even nor odd) Functions

A function is Neither (even nor odd) if it is not even and not odd

To show that a function is Neither, assuming $\exists\ f(-x)$:

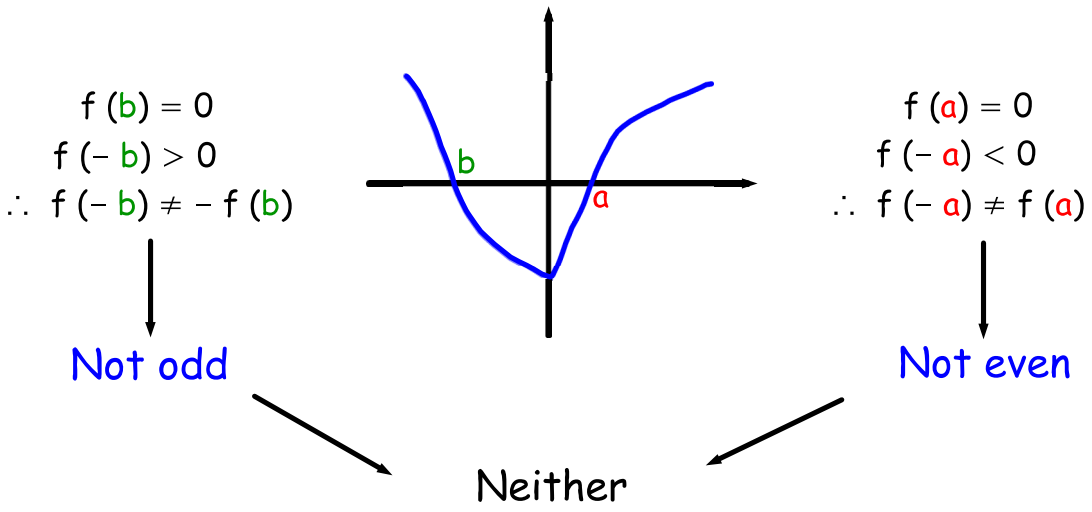
- Show there is an x - value in dom f satisfying $f(-x) \neq f(x)$.

and

- Show there is an x - value in dom f satisfying $f(-x) \neq -f(x)$.

(could be the same x - value)

If $\nexists\ f(-x)$, then both conditions are automatically satisfied and hence the function is Neither.



Sep 7-11:11

Standard Examples of Even Functions

- $f(x) = x^2$
- $f(x) = \cos x$

Standard Examples of Odd Functions

- $f(x) = x^3$
- $f(x) = \sin x$
- $f(x) = \tan x$

Dec 4-14:58

Example 1

Determine whether the function $f(x) = x^3 \sin 2x$ is even, odd or neither.

Let $x \in \text{dom } f$. Then,

$$f(x) = x^3 \sin 2x$$

$$\therefore \quad f(-x) = (-x)^3 \sin 2(-x)$$

$$\Rightarrow \quad f(-x) = -x^3 \sin(-2x)$$

$$\Rightarrow \quad f(-x) = -x^3(-\sin 2x)$$

$$\Rightarrow \quad f(-x) = x^3 \sin 2x$$

$$\Rightarrow \quad \underline{f(-x) = f(x)}$$

As $f(-x) = f(x) \ \forall \ x \in \text{dom } f$, f is even

Oct 2-23:10

Example 2

Determine whether the function $f(x) = x^5 \cos 6x$ is even, odd or neither.

Let $x \in \text{dom } f$. Then,

$$f(x) = x^5 \cos 6x$$

$$\therefore \quad f(-x) = (-x)^5 \cos 6(-x)$$

$$\Rightarrow \quad f(-x) = -x^5 \cos(-6x)$$

$$\Rightarrow \quad f(-x) = -x^5 \cos 6x$$

$$\Rightarrow \quad \underline{f(-x) = -f(x)}$$

As $f(-x) = -f(x) \ \forall \ x \in \text{dom } f$, f is odd

Oct 2-23:10

Example 3

Determine whether the function $f(x) = e^x$ is even, odd or neither.

Let $x \in \text{dom } f$. Then,

$$f(x) = e^x$$

$$\therefore \quad f(-x) = e^{-x}$$

$$f(1) = e^1 \Rightarrow \underline{f(1) = e}$$

$$f(-1) = e^{-1} \Rightarrow \underline{f(-1) = 1/e}$$

As $1/e \neq e$, $f(-1) \neq f(1)$; as $1/e \neq -e$, $f(-1) \neq -f(1)$.

As $f(-x) \neq f(x)\ \forall\, x \in \text{dom } f$ and $f(-x) \neq -f(x)\ \forall\, x \in \text{dom } f$, f is Neither.

Oct 2-23:10

Example 4

Determine whether the function $f(x) = \ln x$ is even, odd or neither.

Let $x \in$ dom f (= all real numbers $x > 0$). Then,

$$f(x) = \ln x$$

$$\therefore \quad f(-x) = \ln(-x)$$

As $x > 0$, $\ln(-x) = f(-x)$ does not exist.

f is Neither

Oct 2-23:10

Example 5

Determine whether the function $f(x) = x^2 + e^{-x}$ is even, odd or neither.

Let $x \in \text{dom } f$. Then,

$$f(x) = x^2 + e^{-x}$$

$$\therefore \quad f(-x) = (-x)^2 + e^{-(-x)}$$

$$\Rightarrow \quad f(-x) = x^2 + e^{x}$$

$$\underline{f(1) = 1 + e^{-1}}$$

$$\underline{f(-1) = 1 + e}$$

As $1 + e \neq 1 + e^{-1}$ (as $e \neq 1/e$), $f(-1) \neq f(1)$;
as $1 + e \neq -(1 + e^{-1})$, $f(-1) \neq -f(1)$.

As $f(-x) \neq f(x)\ \forall\, x \in \text{dom } f$ and $f(-x) \neq -f(x)$ $\forall\, x \in \text{dom } f$, f is Neither.

Oct 2-23:10

Questions

Determine whether the following funtions are even, odd or neither.

1) $f(x) = x^4 \sin 3x$

2) $g(x) = x^2 \cos 5x$

3) $h(x) = e^x - e^{-x}$

4) $L(x) = \ln(x - 5)$

5) $P(x) = \sec x$

6) $Q(x) = \operatorname{cosec} x$

7) $R(x) = \cot x$

8) $A(x) = \sin x \cos x$

9) $B(x) = \sin x + \cos x$

10) $C(x) = x + x^3$

11) $D(x) = 1/x$

12) $E(x) = x + 1/x^2$

13) $M(x) = 1/(6 + x^8)$

14) $Z(x) = \sin^2 x$

Answers

Determine whether the following funtions are even, odd or neither.

1) $f(x) = x^4 \sin 3x$ O

2) $g(x) = x^2 \cos 5x$ E

3) $h(x) = e^x - e^{-x}$ O

4) $L(x) = \ln(x - 5)$ N

5) $P(x) = \sec x$ E

6) $Q(x) = \operatorname{cosec} x$ O

7) $R(x) = \cot x$ O

8) $A(x) = \sin x \cos x$ O

9) $B(x) = \sin x + \cos x$ N

10) $C(x) = x + x^3$ O

11) $D(x) = 1/x$ O

12) $E(x) = x + 1/x^2$ N

13) $M(x) = 1/(6 + x^8)$ E

14) $Z(x) = \sin^2 x$ E