Degrees, Radians and Exact Values

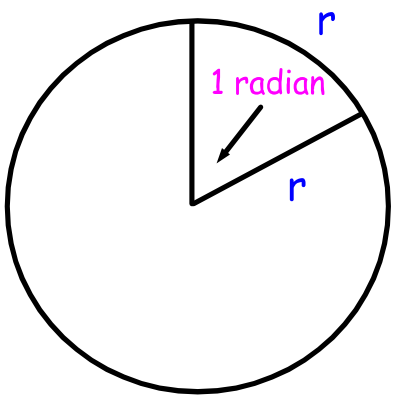
LI

- Convert angles from degrees to Radians (and vice versa).
- Know and work out Exact Values of sin, cos and tan.

SC

- Exact Value Table or Triangles.
- Trig. Graphs.

1 radian is the angle made by an arc of length equal to the radius of the circle



$$\boxed{\pi \text{ radians} = 180^\circ}$$

$$1 \text{ radian} \approx 57.3^\circ$$

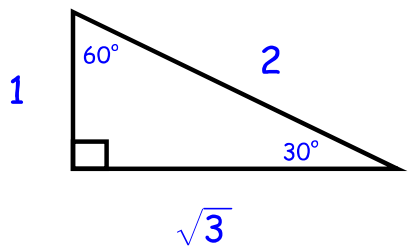
Radians are used to make equations in maths and physics look simpler

Must know these 8 equivalents :

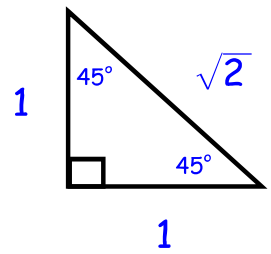
Degrees		Radians
0°	◄ ▪▪▪▪▪▪ ►	0
30°	◄ ▪▪▪▪▪▪ ►	$\frac{\pi}{6}$
45°	◄ ▪▪▪▪▪▪ ►	$\frac{\pi}{4}$
60°	◄ ▪▪▪▪▪▪ ►	$\frac{\pi}{3}$
90°	◄ ▪▪▪▪▪▪ ►	$\frac{\pi}{2}$
180°	◄ ▪▪▪▪▪▪ ►	π
270°	◄ ▪▪▪▪▪▪ ►	$\frac{3\pi}{2}$
360°	◄ ▪▪▪▪▪▪ ►	2π

Exact Values are numerical answers that are not evaluated with a calculator, but left as surds, fractions, expressions with π etc.

Must know standard sin, cos and tan exact values :



Exact Value Triangles



$$\sin 30^\circ = \sin\frac{\pi}{6} = \frac{1}{2}$$

$$\cos 30^\circ = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

$$\sin 60^\circ = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \cos\frac{\pi}{3} = \frac{1}{2}$$

$$\tan 60^\circ = \tan\frac{\pi}{3} = \sqrt{3}$$

$$\sin 45^\circ = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\tan 45^\circ = \tan\frac{\pi}{4} = 1$$

S	Angle	C	T
	0°		
	30°		
	45°		
	60°		
	90°		

Example 1

Change 225° to radians.

$$225^\circ = 180^\circ + 45^\circ$$
$$= \pi + \frac{\pi}{4}$$
$$= \boxed{\frac{5\pi}{4}}$$

Alternative

$$225^\circ = 5 \times 45^\circ$$
$$= 5 \times \frac{\pi}{4}$$
$$= \boxed{\frac{5\pi}{4}}$$

Example 2

Change $\frac{4\pi}{3}$ radians into degrees.

$$\frac{4\pi}{3} = \frac{3\pi + \pi}{3}$$

$$= \pi + \frac{\pi}{3}$$

$$= 180^\circ + 60^\circ$$

$$= \boxed{240^\circ}$$

Alternative

$$\frac{4\pi}{3} = 4 \times \frac{\pi}{3}$$

$$= 4 \times 60^\circ$$

$$= \boxed{240^\circ}$$

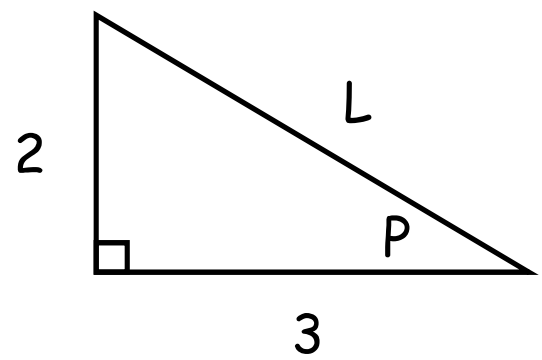
Example 3

If P is an acute angle with $\tan P = \frac{2}{3}$,

find the exact values of $\sin P$ and $\cos P$.

$$L = \sqrt{2^2 + 3^2}$$

$$\underline{L = \sqrt{13}}$$



$$\sin P = \frac{O}{H}$$

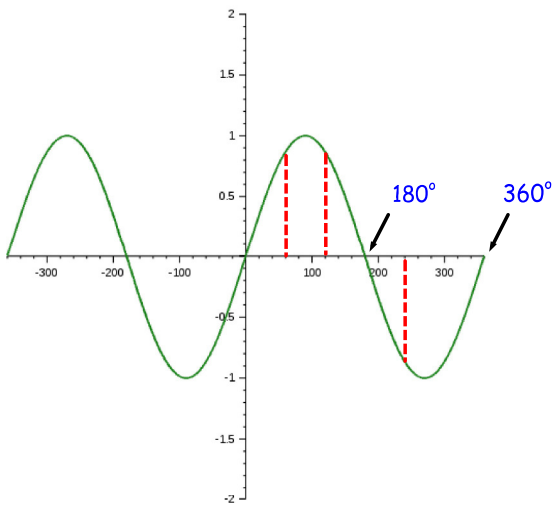
$$\sin P = \boxed{\frac{2}{\sqrt{13}}}$$

$$\cos P = \frac{A}{H}$$

$$\cos P = \boxed{\frac{3}{\sqrt{13}}}$$

Example 4

Find the exact value of $\sin 240^\circ$.



$$\sin 240^\circ$$
$$= -\sin 120^\circ$$
$$= -\sin 60^\circ$$
$$= \boxed{-\frac{\sqrt{3}}{2}}$$

CfE Higher Maths

pg. 28 Ex. 2A Q 1 - 3

pg. 31 Ex. 2B All Q

Questions - Ex. 2A

1 Find exact values.

a $\cos 30°$	**b** $\sin 150°$	**c** $\cos 330°$	**d** $\tan 180°$
e $\sin 240°$	**f** $\cos 135°$	**g** $\tan 225°$	**h** $\cos 300°$
i $\sin 540°$	**j** $\cos 510°$	**k** $\tan(-30)°$	**l** $\sin(-240)°$
m $\cos 270°$	**n** $\tan 750°$	**o** $\sin(-330)°$	**p** $\sin 180°$

2 Given that A is an acute angle with $\tan A = \frac{3}{2}$ calculate the exact values of $\sin A$ and $\cos A$.

3 If P is an acute angle with $\sin P = \frac{5}{6}$ calculate the exact values of $\cos P$ and $\tan P$.

Answers

1
- **a** $\frac{\sqrt{3}}{2}$
- **b** $\frac{1}{2}$
- **c** $\frac{\sqrt{3}}{2}$
- **d** 0
- **e** $-\frac{\sqrt{3}}{2}$
- **f** $-\frac{1}{\sqrt{2}}$
- **g** 1
- **h** $\frac{1}{2}$
- **i** 0
- **j** $-\frac{\sqrt{3}}{2}$
- **k** $-\frac{1}{\sqrt{3}}$
- **l** $\frac{\sqrt{3}}{2}$
- **m** 0
- **n** $\frac{1}{\sqrt{3}}$
- **o** $\frac{1}{2}$
- **p** 0

2 $\sin A = \frac{3}{\sqrt{13}}$ $\cos A = \frac{2}{\sqrt{13}}$

3 $\cos P = \frac{\sqrt{11}}{6}$ $\tan P = \frac{5}{\sqrt{11}}$

Questions - Ex. 2B

1 Convert these angles to radians giving your answer as a fraction or multiple of π.

a 30° **b** 150° **c** 300° **d** 135° **e** 315°

f 120° **g** 240° **h** 450° **i** −60° **j** 720°

2 Convert these angles to degrees.

a $\frac{\pi}{4}$ **b** $\frac{\pi}{12}$ **c** $\frac{\pi}{5}$ **d** $\frac{3\pi}{2}$ **e** $\frac{5\pi}{4}$

f $\frac{3\pi}{4}$ **g** $\frac{7\pi}{6}$ **h** $\frac{7\pi}{3}$ **i** $-\frac{\pi}{6}$ **j** 3π

3 Use a calculator to complete these conversions, rounding to 1 decimal place for degrees and 2 for radians.

a Convert to radians: **i** 47° **ii** 324°

b Convert to degrees: **i** 4.36 radians **ii** 1.57 radians

4 Give exact value ratios for each of these ratios.

a $\cos\frac{\pi}{4}$ **b** $\cos\frac{3\pi}{4}$ **c** $\sin\frac{5\pi}{3}$ **d** $\tan\frac{11\pi}{6}$

e $\cos(-\pi)$ **f** $\tan\frac{7\pi}{3}$ **g** $\sin\frac{5\pi}{6}$ **h** $\sin\frac{3\pi}{2}$

Answers

1
- **a** $\frac{\pi}{6}$
- **b** $\frac{5\pi}{6}$
- **c** $\frac{5\pi}{3}$
- **d** $\frac{3\pi}{4}$
- **e** $\frac{7\pi}{4}$
- **f** $\frac{2\pi}{3}$
- **g** $\frac{4\pi}{3}$
- **h** $\frac{5\pi}{2}$
- **i** $-\frac{\pi}{3}$
- **j** 4π

2
- **a** 45°
- **b** 15°
- **c** 36°
- **d** 270°
- **e** 225°
- **f** 135°
- **g** 210°
- **h** 420°
- **i** −30°
- **j** 540°

3
- **a** **(i)** 0.82 **(ii)** 5.65
- **b** **(i)** 249.8 **(ii)** 90.0

4
- **a** $\frac{1}{\sqrt{2}}$
- **b** $-\frac{1}{\sqrt{2}}$
- **c** $-\frac{\sqrt{3}}{2}$
- **d** $-\frac{1}{\sqrt{3}}$
- **e** -1
- **f** $\sqrt{3}$
- **g** $\frac{1}{2}$
- **h** -1