*Trigonometric Graphs - Lesson 1*

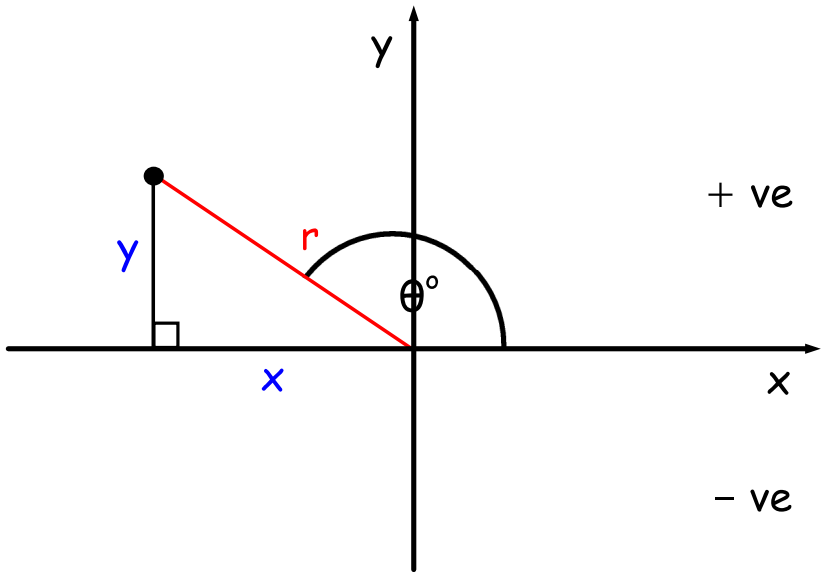
# Trigonometric Functions and Trigonometric Graphs (Amplitude and Periodicity)

LI

- Know the definitions of the basic trigonometric functions $y = \sin x^\circ$, $y = \cos x^\circ$ and $y = \tan x^\circ$.
- Find the amplitude and period of related trigonometric functions.

SC

- Know the basic trigonometric graphs $y = \sin x^\circ$, $y = \cos x^\circ$ and $y = \tan x^\circ$ between $0^\circ$ and $360^\circ$.




## The 3 Basic Trigonometric Functions

$$\sin\ \theta^\circ = \frac{y}{r}$$

$$\cos\ \theta^\circ = \frac{x}{r}$$

$$\tan\ \theta^\circ = \frac{y}{x}$$

The maximum value of a function is the biggest y - value

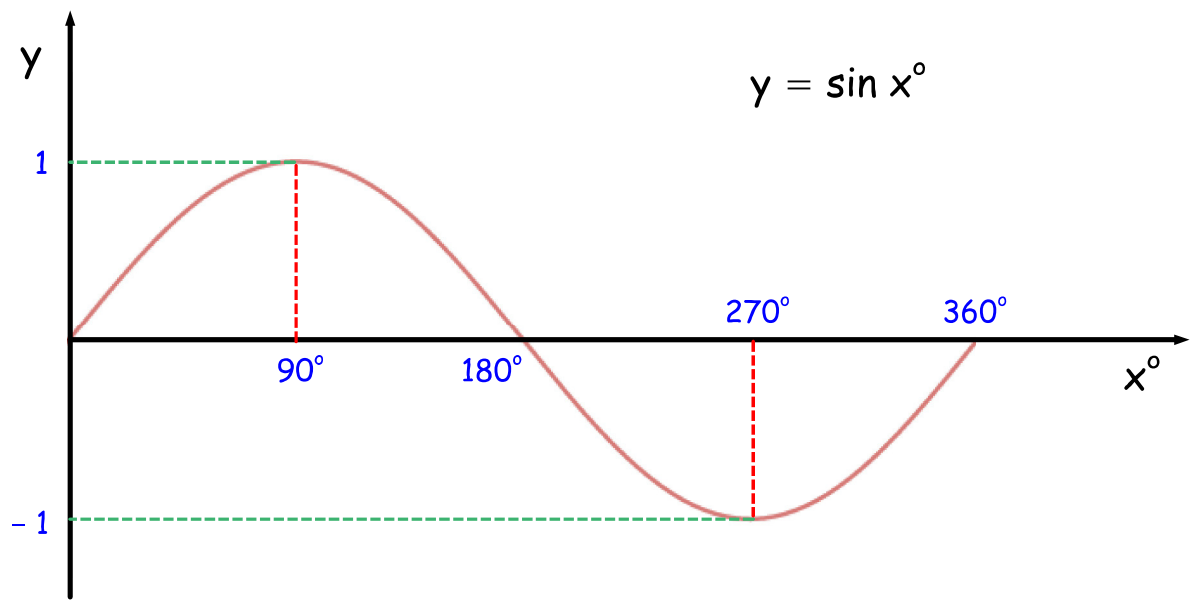
The minimum value of a function is the smallest y - value

Not all functions have a max. or min. value

The period of a function is how often it repeats

Not all functions are periodic

All trig. functions are periodic



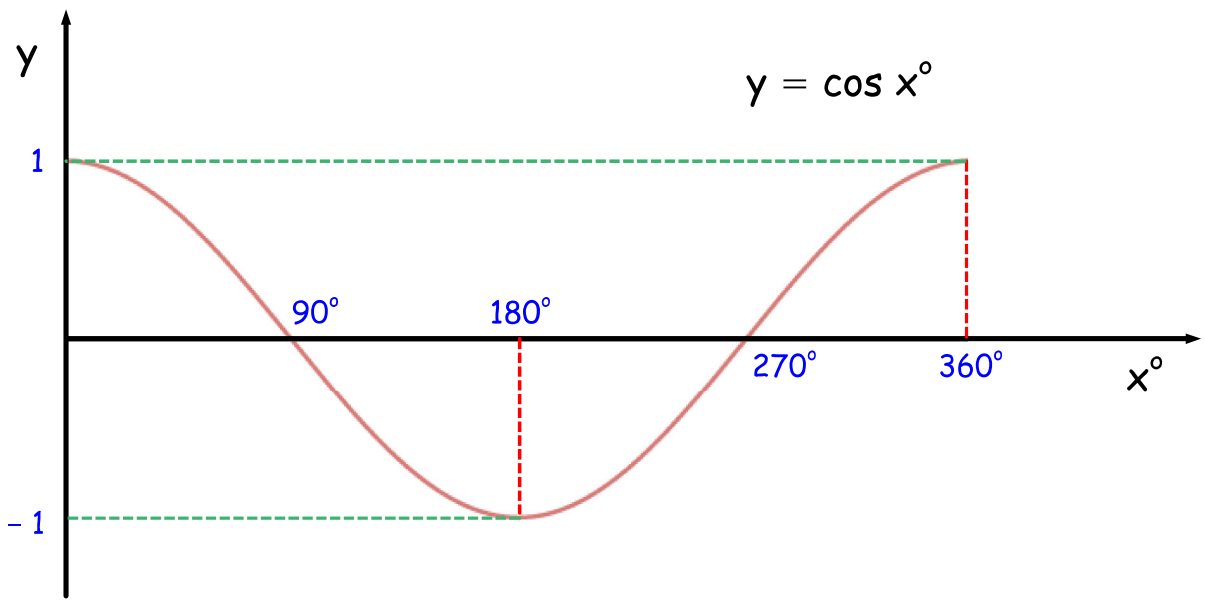

Properties of $y = \sin x^\circ$ :

Maximum Value $= 1$

Minimum Value $= -1$

Period $= 360^\circ$

Starts at 0



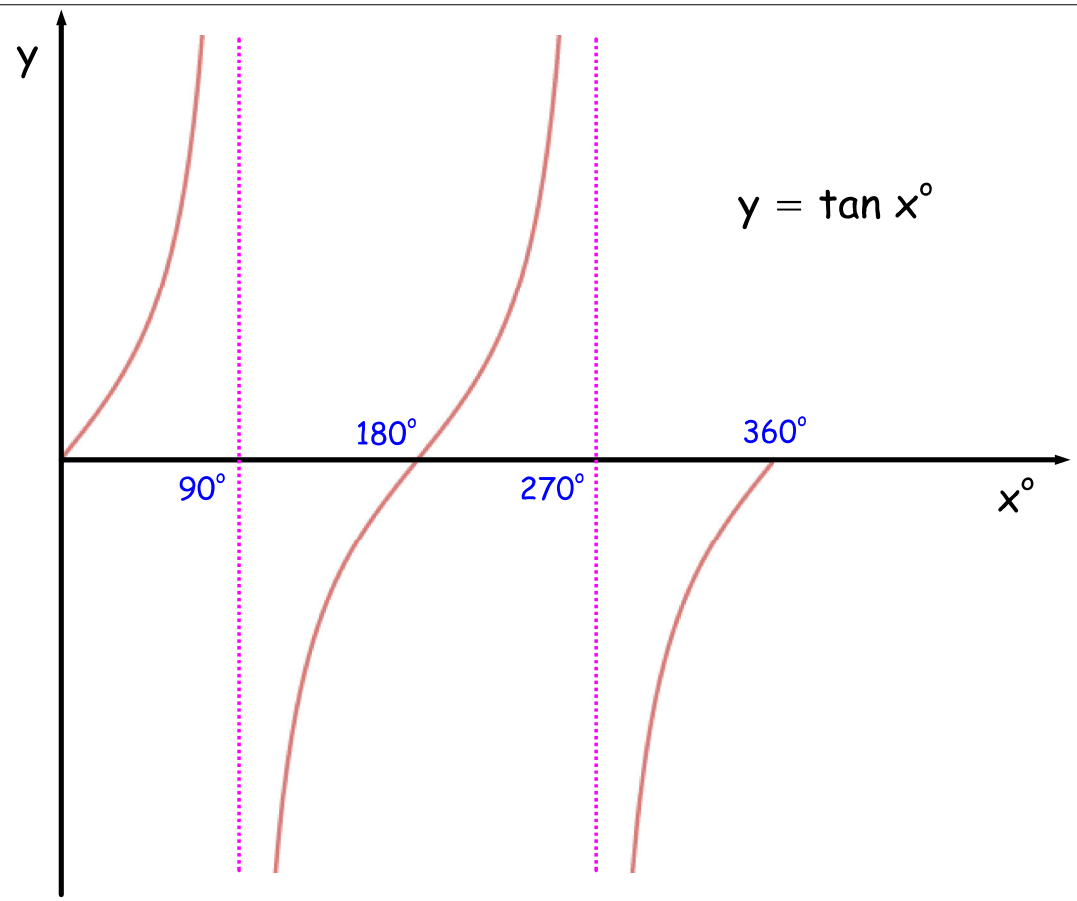

Properties of $y = \cos x^\circ$ :

Maximum Value = 1

Minimum Value = – 1

Period = 360°

Starts at 1




Properties of $y = \tan x^\circ$ :

No Maximum Value

No Minimum Value

Period $= 180^\circ$

Starts at 0

The vertical lines are not part of the graph; draw them as dotted

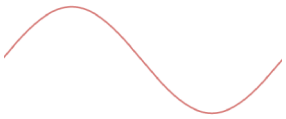
# Graphs of Related Functions

$y = A \sin bx^\circ$

Max. Value $= A$

Min. Value $= -A$

Period $= \dfrac{360^\circ}{b}$



$y = A \cos bx^\circ$

Max. Value $= A$

Min. Value $= -A$

Period $= \dfrac{360^\circ}{b}$

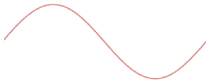
$y = A \tan bx^\circ$

No Max. Value

No Min. Value

Period $= \dfrac{180^\circ}{b}$

b tells us how many whole graphs

 ,  or 

we can fit in between $0^\circ$ and $360^\circ$.

The amplitude ($A$) is always a positive number and describes the height of the graph from the middle position :

$$A = \frac{\text{Maximum Value} - \text{Minimum Value}}{2}$$

## Example 1

State the maximum and minimum values, amplitude and period of $y = 2 \sin 4x^\circ$.

$$\boxed{\text{Max. value} = 2, \text{ Min. value} = -2}$$

$$A = \frac{\text{Maximum Value} - \text{Minimum Value}}{2}$$

$$A = \frac{2 - (-2)}{2}$$

$$\boxed{A = 2}$$

$$\text{Period} = 360^\circ \div 4$$

$$\boxed{\text{Period} = 90^\circ}$$

## Example 2

Find the period and maximum and minimum values of :

(a) $y = 3 \sin 2x^\circ$.

(b) $y = 6 \cos 6x^\circ$.

(c) $y = (5/3) \tan 360x^\circ$.

(a) $y = 3 \sin 2x^\circ$

$$\text{Per.} = 360^\circ \div 2$$

$$\boxed{\text{Per.} = 180^\circ}$$

$$\boxed{\text{Max.} = 3, \text{Min.} = -3}$$

(b) $y = 6 \cos 6x^\circ$

$$\text{Per.} = 360^\circ \div 6$$

$$\boxed{\text{Per.} = 60^\circ}$$

$$\boxed{\text{Max.} = 6, \text{Min.} = -6}$$

(c) $y = (5/3) \tan 360x^\circ$

$$\text{Per.} = 180^\circ \div 360$$

$$\boxed{\text{Per.} = (1/2)^\circ}$$

$$\boxed{\text{No max., No min.}}$$

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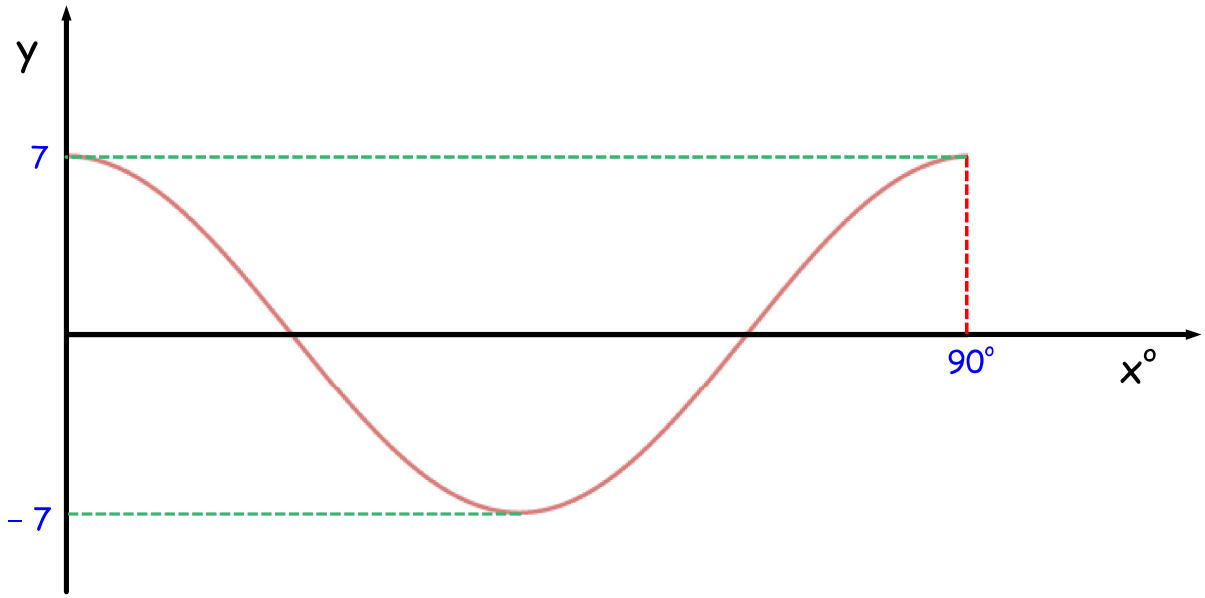
## Example 3

Find the equation of this graph :




$$y = 7 \cos 4x^\circ$$

Questions

State the max. and min. values (if they exist)
and the periodicity of these functions

(all letters apart from x and y are positive constants)

| | |
|---|---|
| 1) $y = 2 \sin 3x^\circ$ | 7) $y = 5 \sin (0.5)x^\circ$ |
| 2) $y = 4 \cos 4x^\circ$ | 8) $y = 347 \cos (1/4)x^\circ$ |
| 3) $y = 5 \tan 4x^\circ$ | 9) $y = k \tan (0.75)x^\circ$ |
| 4) $y = 17 \sin 30x^\circ$ | 10) $y = p \sin (0.6)x^\circ$ |
| 5) $y = 13.6 \cos 180x^\circ$ | 11) $y = r \cos (1/8)x^\circ$ |
| 6) $y = 5 \tan 90x^\circ$ | 12) $y = w \tan 720x^\circ$ |

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## Answers

| | |
|---|---|
| 1) $y = 2 \sin 3x^\circ$ Max. 2 Min. – 2 120° | 7) $y = 5 \sin (0.5)x^\circ$ Max. 5 Min. – 5 720° |
| 2) $y = 4 \cos 4x^\circ$ Max. 4 Min. – 4 90° | 8) $y = 347 \cos (1/4)x^\circ$ Max. 347 Min. – 347 1 440° |
| 3) $y = 5 \tan 4x^\circ$ No Max./Min. 45° | 9) $y = k \tan (0.75)x^\circ$ No Max./Min. 240° |
| 4) $y = 17 \sin 30x^\circ$ Max. 17 Min. – 17 12° | 10) $y = p \sin (0.6)x^\circ$ Max. p Min. – p 600° |
| 5) $y = 13.6 \cos 180x^\circ$ Max. 13.6 Min. – 13.6 2° | 11) $y = r \cos (1/8)x^\circ$ Max. r Min. – r 2 880° |
| 6) $y = 5 \tan 90x^\circ$ No Max./Min. 2° | 12) $y = w \tan 720x^\circ$ No Max./Min. 0.25° |

# Questions

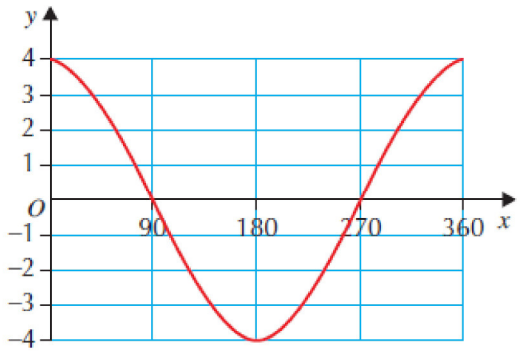
1) Sketch the graphs of the following for $0 \leqslant x \leqslant 360$.

a $y = 5\sin x°$ b $y = \frac{3}{4}\cos x°$ c $y = -4\sin x°$

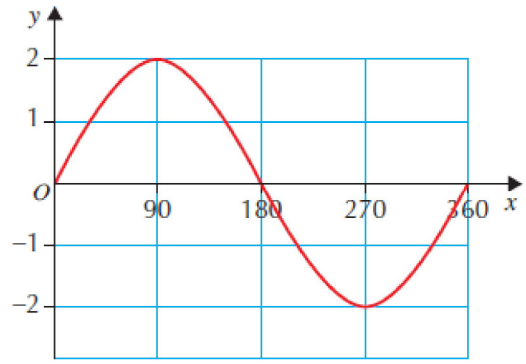
d $y = 3\cos x°$ e $y = \frac{1}{2}\sin x°$ f $y = -2\cos x°$

2) State the equations of each of the following graphs.

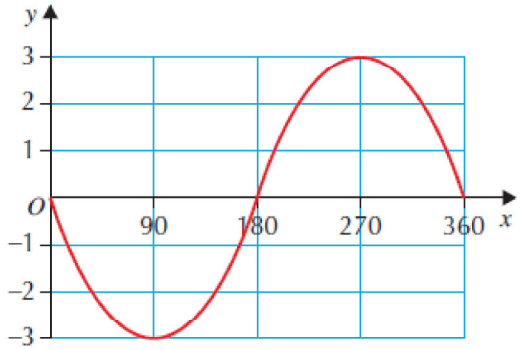
a



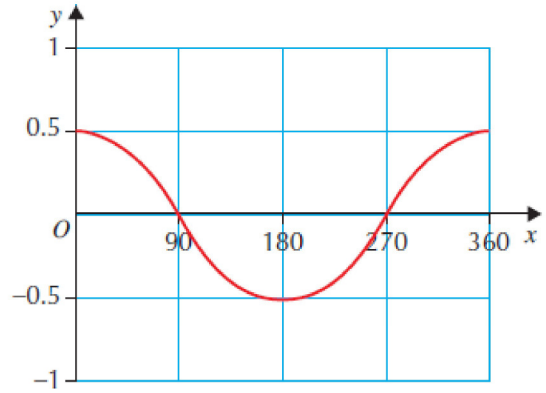

b



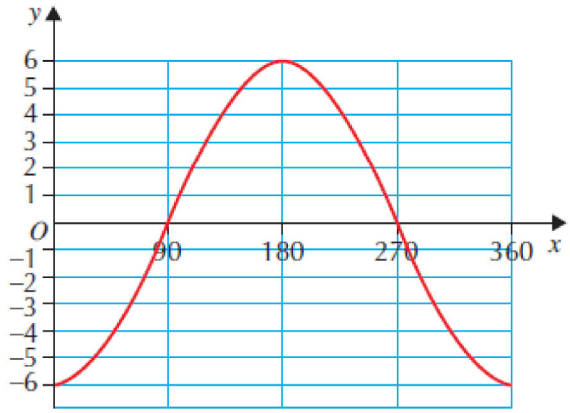

c



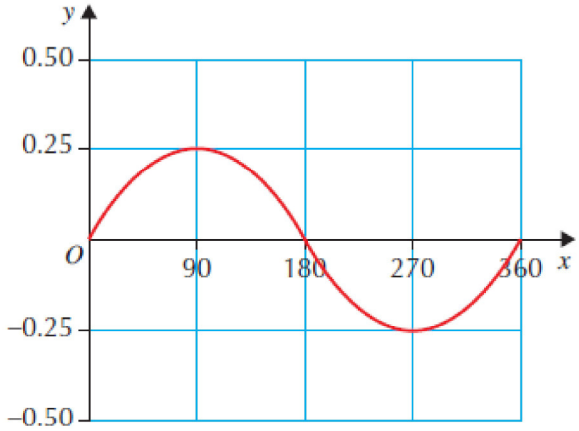

d




e




f



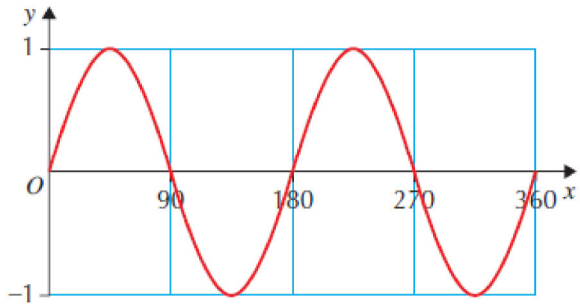
3) Sketch the graphs of the following for $0 \leqslant x \leqslant 360$.

**a** $y = \cos 3x°$ **b** $y = \cos 4x°$ **c** $y = \sin 5x°$

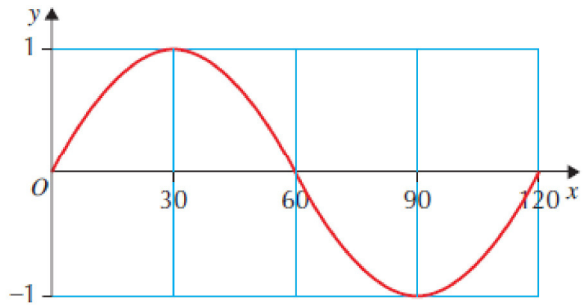
**d** $y = \sin 2x°$ **e** $y = \cos \frac{1}{4}x°$ **f** $y = \sin \frac{1}{2}x°$

4) State the equations of each of the following.

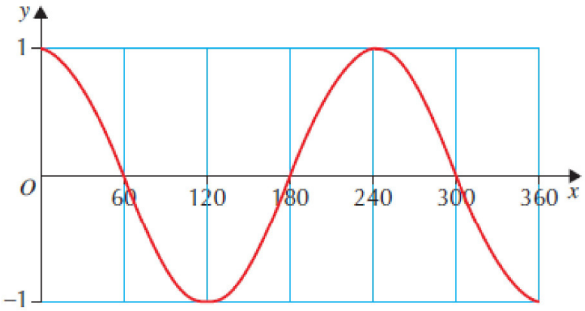
**a**



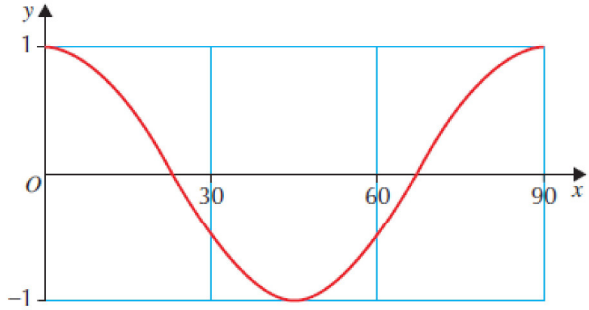

**b**



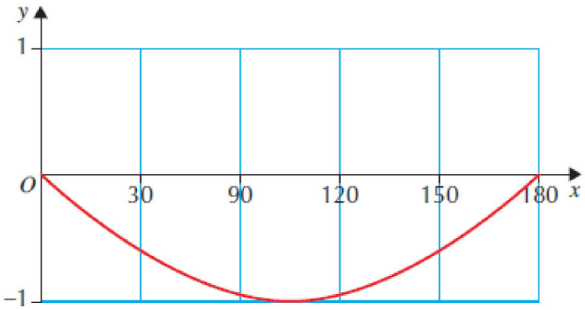

**c**



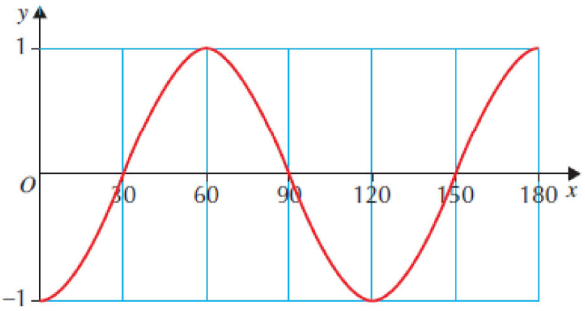

**d**




**e**




**f**



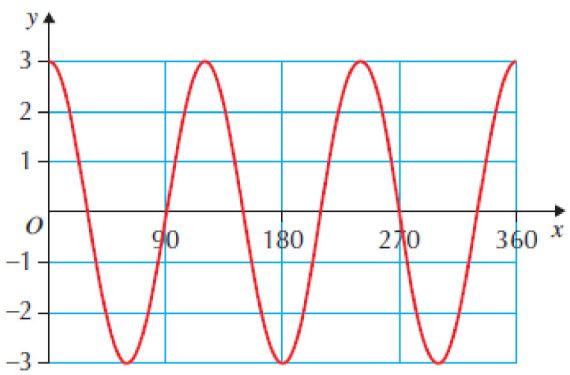
5) Sketch the graphs of the following for $0 \leqslant x \leqslant 360$.

**a** $y = 3\sin 2x°$ **b** $y = 5\cos 3x°$ **c** $y = 2\cos \frac{1}{2}x°$ **d** $y = 2\sin 4x°$

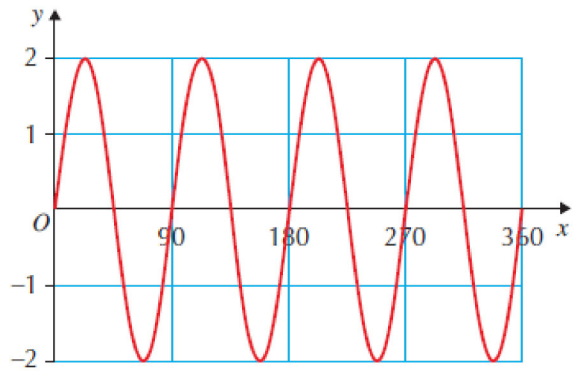
**e** $y = -2\cos 3x°$ **f** $y = -3\sin 2x°$ **g** $y = 2\cos 5x°$ **h** $y = 3\sin \frac{1}{2}x°$

6) State the equations of each of the following graphs.

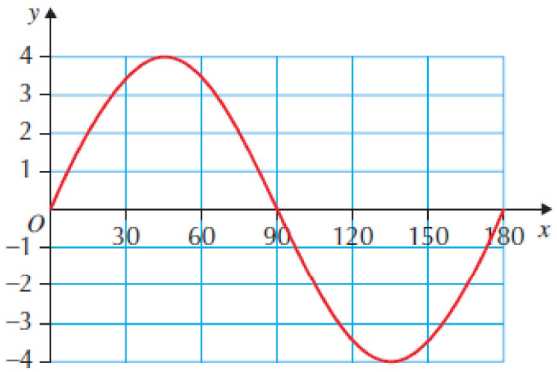
**a**



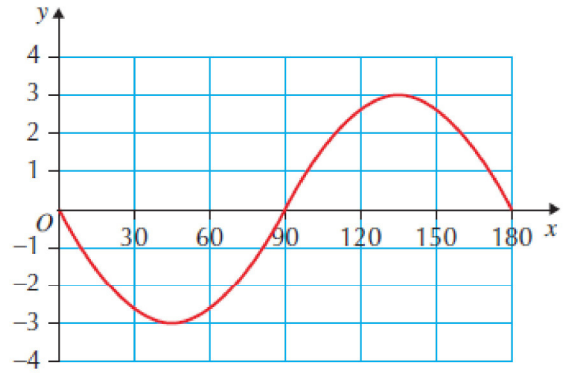

**b**



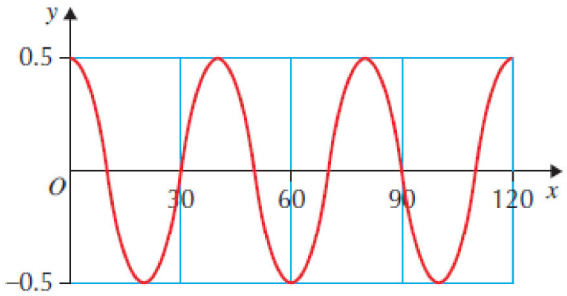

**c**



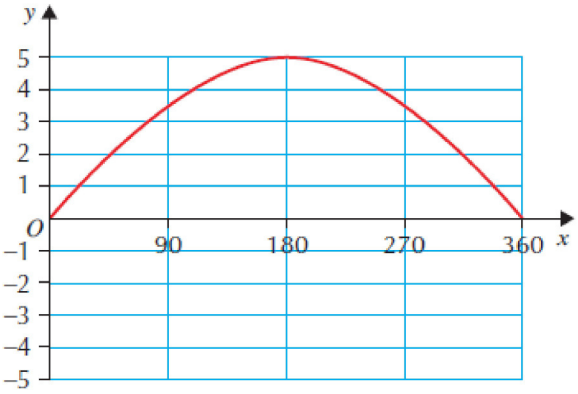

**d**




**e**

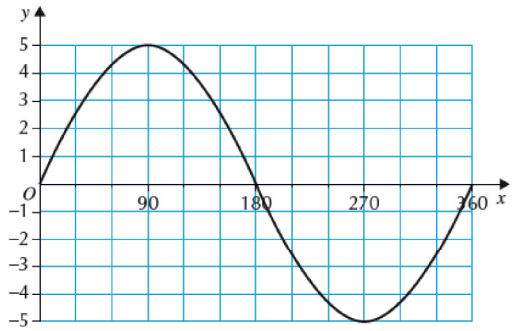



**f**



## Answers

1) (a)

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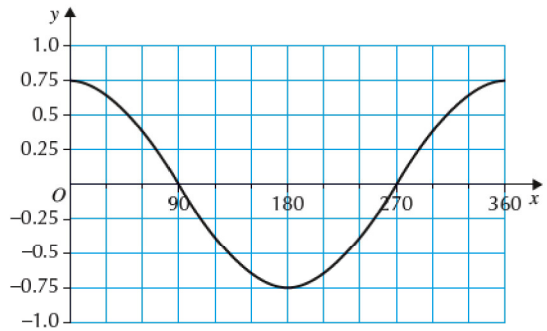


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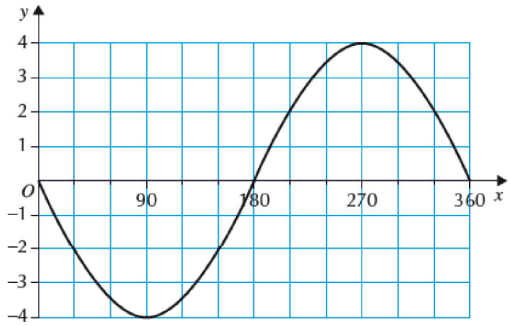


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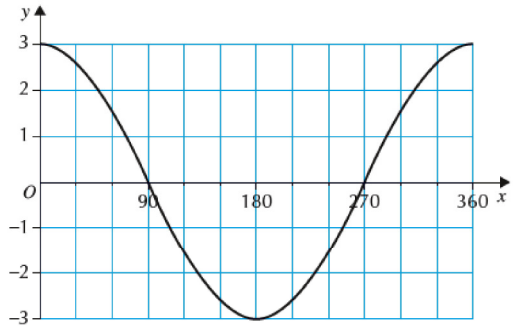


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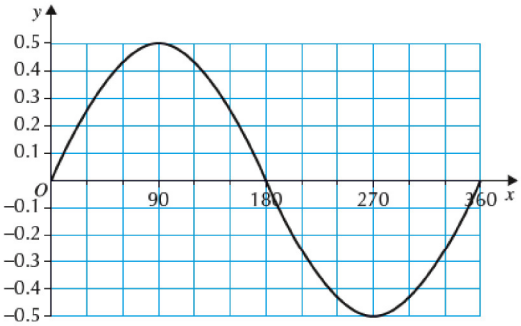


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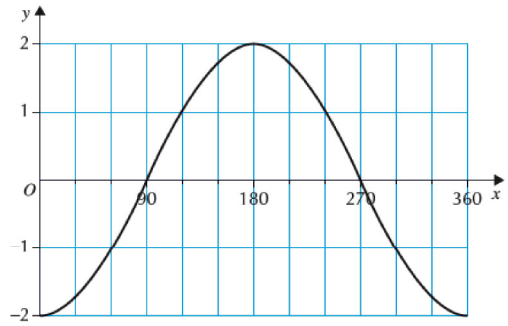
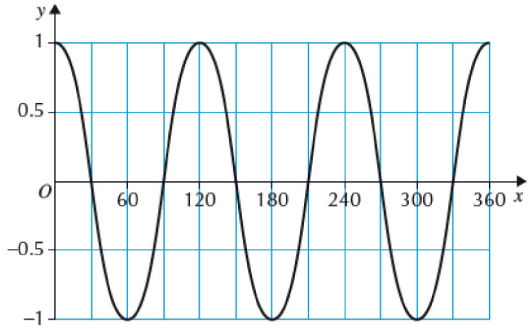



2) (a) $y = 4 \cos x^\circ$ (b) $y = 2 \sin x^\circ$ (c) $y = -3 \sin x^\circ$

(d) $y = (1/2) \cos x^\circ$ (e) $y = -6 \cos x^\circ$ (f) $y = (1/4) \sin x^\circ$

3) (a)

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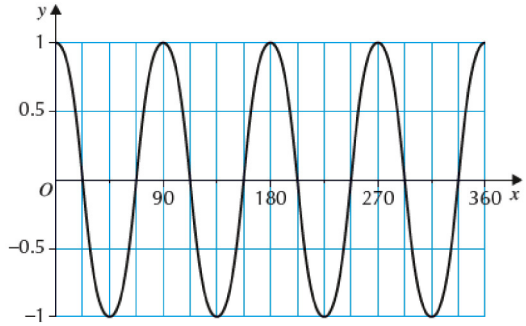


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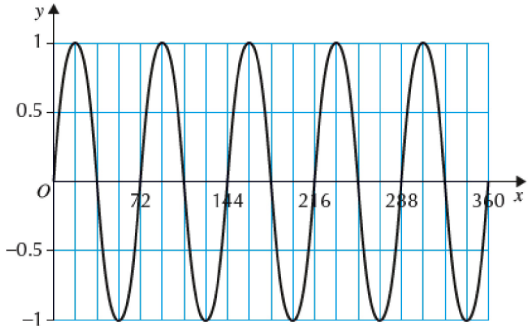


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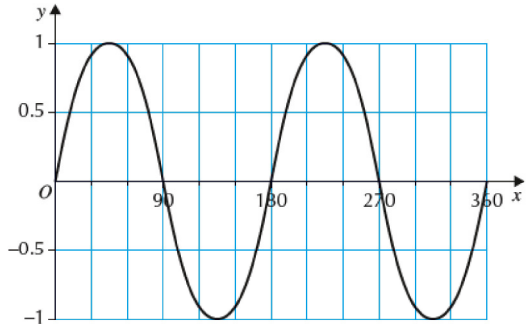


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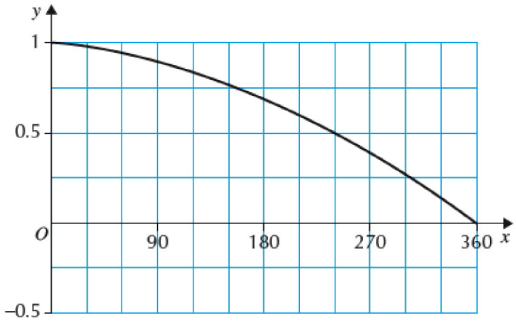


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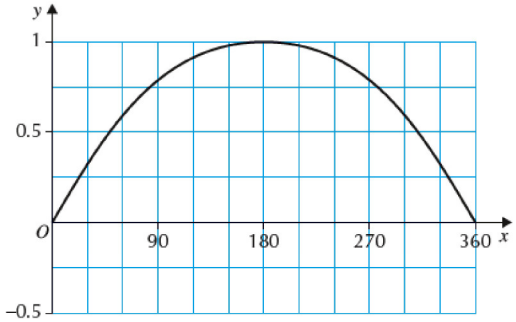
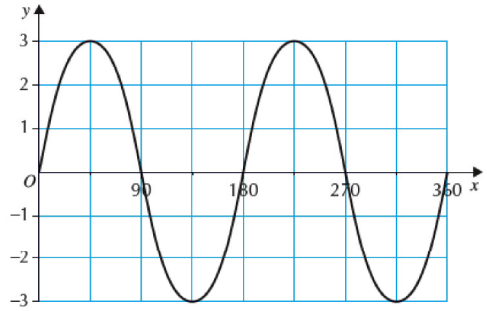



4) (a) $y = \sin 2x^\circ$ (b) $y = \sin 3x^\circ$ (c) $y = \cos (3/2)x^\circ$

(d) $y = \cos 4x^\circ$ (e) $y = -\sin x^\circ$ (f) $y = -\cos 3x^\circ$

5) (a)

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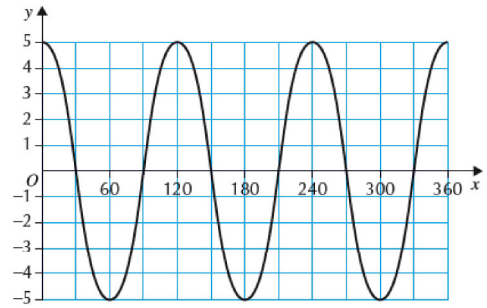


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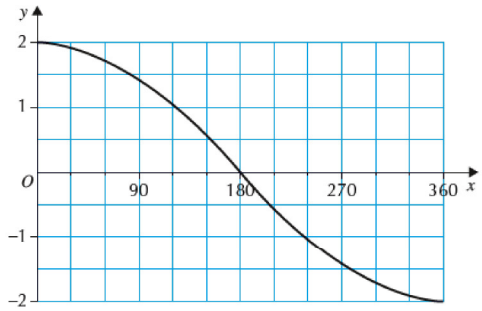


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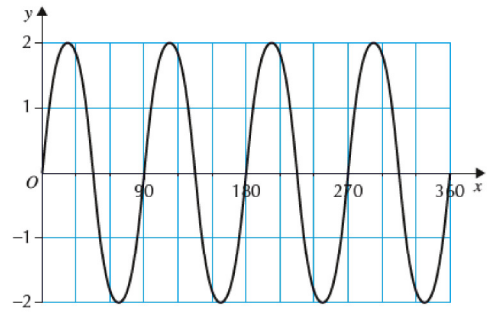


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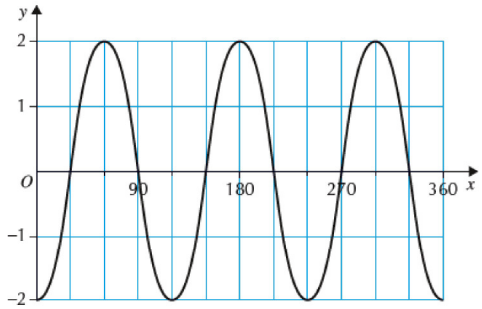


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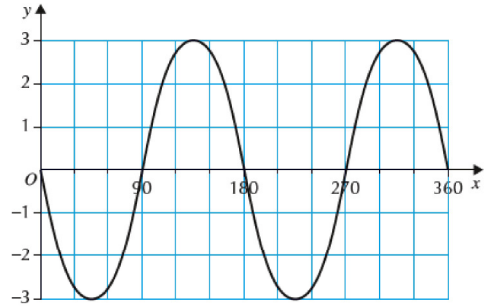


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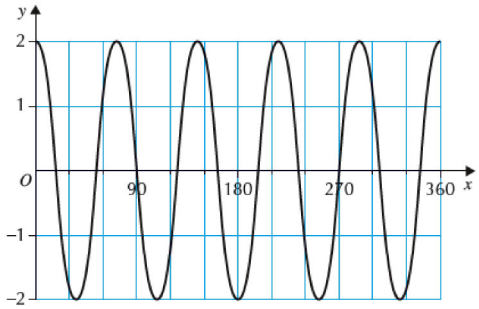
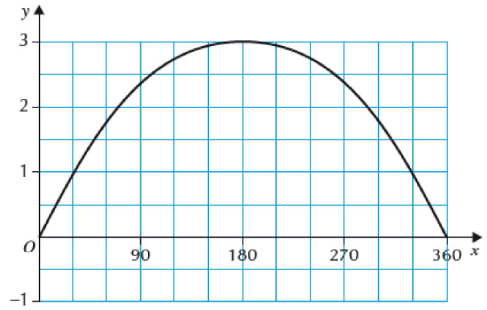



(h)




6) (a) $y = 3\cos 3x^\circ$ (b) $y = 2\sin 4x^\circ$ (c) $y = 4\sin 2x^\circ$

(d) $y = -3\sin 2x^\circ$ (e) $y = (1/2)\cos 9x^\circ$ (f) $y = 5\sin (1/2)x^\circ$

Feb 28-07:49