AH Mathematics

Applications of Algebra and Calculus

Practice Assessment 2

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Applications of Algebra and Calculus Assessment Standard 1.1

1 Expand $(3x + 2)^5$ using the Binomial Theorem and simplify as far as possible. **(3)**

2 Complex numbers are defined as follows : $z_1 = p + i$ and $z_2 = 5 - 2i$.

Express the following in the form $a + ib$:

a) $z_1 z_2$ **(1)**

b) $\frac{z_1}{z_2}$ **(2)**

Applications of Algebra and Calculus Assessment Standard 1.2

3 An arithmetic sequence is given by : 6, 20, 34, ...

Find :

a) the 40^{th} term of the sequence. **(2)**

b) the sum of the first 40 terms. **(2)**

4 A geometric sequence is given by : 12, 84, 588, ...

Find :

a) the 7^{th} term of the sequence. **(2)**

b) the sum of the first 7 terms. **(2)**

5 Find the first four terms of the Maclaurin series for $f(x) = e^{3x}$. **(3)**

Applications of Algebra and Calculus Assessment Standard 1.3

6 Evaluate $\sum_{k=1}^{12} (5k - 4)$. **(3)**

7 Use proof by induction to show that, $\forall\, n \geq 1,\ n \in \mathbb{N}$

$$\sum_{r=1}^{n} 8r = 4n(n+1)\ .$$ **(5)**

Applications of Algebra and Calculus Assessment Standard 1.4

8 $f(x) = \dfrac{x^2 + 3x - 14}{x - 3},\ x \in \mathbb{R} : x \neq 3$

For the graph $y = f(x)$:

a) Give the equation of the vertical asymptote. **(1)**

b) Show that there is a non-vertical asymptote and state the equation. **(2)**

9 Given that $f(x) = \cos(2x)$, sketch the graph of $|5f(x)|$,
where $0 \leq x \leq \pi$. **(2)**

10 Show that there is a point of inflexion on the graph of $y = 5x^3 - x$
at $x = 0$. **(3)**

Applications of Algebra and Calculus Assessment Standard 1.5

11 A car begins travelling from rest along a straight road. Its velocity,
$v(t)$ metres per second, is given by

$$v(t) = \frac{300t}{4t + 9}\ .$$

Find the acceleration of the car at 2 seconds. **(4)**

12 The area bounded by the curve $y = \sqrt{1 + \sin 3x}$ between $x = 0$

and $x = \dfrac{\pi}{3}$ is rotated 2π radians about the x – axis.

Determine the exact value of the volume of the solid formed. **(4)**