2008 Applied Mathematics

Advanced Higher – Statistics

Finalised Marking Instructions

General Marking Principles

These principles describe the approach taken when marking Advanced Higher Mathematics papers. For more detailed guidance please refer to the detailed Marking Instructions.

1 The main principle is to give credit for the skills demonstrated and the criteria met. Failure to have a correct method may not preclude a candidate gaining credit for their solution.

2 The answer to one part of a question, even if incorrect, can be accepted as a basis for subsequent dependent parts of the question.

3 The following are not penalised:

- working subsequent to a correct answer (unless it provides firm evidence that the requirements of the question have not been met)
- legitimate variation in numerical values/algebraic expressions.

4 Full credit will only be given where the solution contains appropriate working. Where the correct answer might be obtained by inspection or mentally, credit may be given.

5 Sometimes the method to be used in a particular question is explicitly stated; no credit will be given where a candidate obtains the correct answer by an alternative method.

6 Where the method to be used in a particular question is not explicitly stated in the question paper, full credit is available for an alternative valid method. (Some likely alternatives are included but these should not be assumed to be the only acceptable ones.)

In the detailed Marking Instructions which follow, marks are shown alongside the line for which they are awarded. There are two codes used, M and E. M indicates a method mark, so in question B6, M1 means a method mark for separating the variables. E is shorthand for error. For example, 2E1, means that a correct answer is awarded 2 marks but that 1 mark is deducted for each error.

Advanced Higher Applied Mathematics 2008

Section A − Statistics

A1. Let V denote 'bird infected with virus' and let + denote 'bird tests positive for virus'.

$\mathrm{P}(\bar{V} \mid +)$ is required. **1**

$$\mathrm{P}(\bar{V} \mid +) = \frac{\mathrm{P}(+ \cap \bar{V})}{\mathrm{P}(+)} = \frac{\mathrm{P}(\bar{V} \cap +)}{\mathrm{P}(+)} \qquad \mathbf{1}$$

$$= \frac{\mathrm{P}(\bar{V})\,\mathrm{P}(+ \mid \bar{V})}{\mathrm{P}(\bar{V})\,\mathrm{P}(+ \mid \bar{V}) + \mathrm{P}(V)\,\mathrm{P}(+ \mid V)}$$

$$= \frac{0{\cdot}9999 \times 0{\cdot}002}{0{\cdot}0001 \times 0{\cdot}999 + 0{\cdot}9999 \times 0{\cdot}002} \qquad \mathbf{1}$$

$$= \frac{0{\cdot}0019998}{0{\cdot}0020997} = 0{\cdot}952 \qquad \mathbf{1}$$

Other methods, e.g. tree diagram or Venn diagram, are acceptable.

A2. (a) A telephone number may be for a business. **1**

A household may not have a telephone. **1**

(b) A sampling frame is required i.e. a list of all households on the island. **1**

Now use random numbers to select a sample from the list. **1**

A3. (a)

$$X \sim N\left(53{\cdot}4,\ 4{\cdot}7^2\right)$$

$$\Rightarrow \mathrm{P}(X > 62) \qquad \mathbf{1}$$

$$= P\left(Z > \frac{62 - 53{\cdot}4}{4{\cdot}7}\right) \qquad \mathbf{1}$$

$$= 0{\cdot}0336 \qquad \mathbf{1}$$

(b)

$$z = 2{\cdot}33 \qquad \mathbf{1}$$

$$\Rightarrow \frac{62 - \mu}{4{\cdot}7} = 2{\cdot}33 \qquad \mathbf{1}$$

$$\Rightarrow \mu = 51{\cdot}05 \qquad \mathbf{1}$$

A4. (a)

$$b = \frac{S_{xy}}{S_{xx}} = \frac{0 \cdot 170456}{0 \cdot 144672} = 1 \cdot 1782$$

$$a = \bar{y} - b\bar{x} = 1 \cdot 7268 - 1 \cdot 1782 \times 0 \cdot 8916 = 0 \cdot 6763$$

Equation is $y = 0 \cdot 6763 + 1 \cdot 1782x$. **1**

$$SSR = S_{yy} - \frac{(S_{xy})^2}{S_{xx}} = 0 \cdot 221088 - \frac{0 \cdot 170456^2}{0 \cdot 144672} = 0 \cdot 02025$$

$$s^2 = \frac{SSR}{n - 2} = \frac{0 \cdot 02025}{48} \Rightarrow s = 0 \cdot 0205$$

$$t = \frac{b}{s / \sqrt{S_{xx}}} = \frac{1 \cdot 1782}{0 \cdot 0205 / \sqrt{0 \cdot 144672}} = 21 \cdot 86$$ **1**

The critical values of t with greater than 40 df for a two-tail test **1**
are approximately 3 (eg $2 \cdot 576$ at the 1%level) $<< 21 \cdot 86$, **1**
furnishing strong evidence that the slope parameter differs from zero. **1**
The evidence suggests that the model is useful for prediction. **1**

(b) Dieticians will be interested in making a prediction concerning the height of an individual patient with a given demi-span, rather than a prediction concerning the mean height of patients with a given demi-span. **1**

A5. Assume that the sample is random
(or differences are independent). **1**
H_0 : Median (After − Before) = 0
H_1 : Median (After − Before) > 0 **1**
$8 +; 1 -; 1 =$ **1**
Bin$(9, 0 \cdot 5)$ $P(B \leqslant 1) = \left(C_0^9 + C_1^9\right) 0 \cdot 5^9$ **1**
$= 0 \cdot 0195$. **1**

Since $0 \cdot 0195 < 0 \cdot 05$, we reject H_0 at the 5% level **1**
i.e. there is evidence that the therapy leads to increased self-esteem. **1**

A6.

Observed	17	56	27	
Expected	25	50	25	**1**

$$x^2 = \sum \frac{(O - E)^2}{E} = \frac{(17 - 25)^2}{25} + \frac{(56 - 50)^2}{50} + \frac{(27 - 25)^2}{25}$$

$$= 2 \cdot 56 + 0 \cdot 72 + 0 \cdot 16 = 3 \cdot 44$$ **1**

The critical value of chi-squared at the 5% level with 2 df **1**
is $5 \cdot 991 > 3 \cdot 44$ so we would accept H_0 **1**
i.e. there is no evidence against the stated ratios. **1**

A7. The variability is the same for both authors. **1**

$$\bar{x} = 23 \cdot 3, \ \sigma = 4 \cdot 6, \ n = 30.$$

A 95% confidence interval for the mean word length, μ, in the manuscript of unknown authorship is given by:

$$\bar{x} \pm z \frac{\sigma}{\sqrt{n}} \qquad \textbf{1}$$

$$= 23 \cdot 3 \pm 1 \cdot 96 \frac{4 \cdot 6}{\sqrt{30}} \qquad \textbf{1}$$

$$= 23 \cdot 3 \pm 1 \cdot 65 \quad \text{or} \quad [21 \cdot 65, \ 24 \cdot 95] \qquad \textbf{1}$$

Since 24·9 lies within the 95% confidence interval **1**

the data do not provide evidence against the work of unknown authorship being attributed to the medieval author. **1**

A8. (a) (0,2) or (1,1) or (2,0) **1**

For Poi(5.5), $P(0) = 0 \cdot 0041$, $P(1) = 0 \cdot 0225$, $P(2) = 0 \cdot 0618$ **1**

For Poi(3.5), $P(0) = 0 \cdot 0302$, $P(1) = 0 \cdot 1057$, $P(2) = 0 \cdot 1850$ **1**

Probability = $0 \cdot 0041 \times 0 \cdot 1850 + 0 \cdot 0225 \times 0 \cdot 1057 + 0 \cdot 0618 \times 0 \cdot 0302$ **1**

$= 0 \cdot 0050$ **1**

(b) T has mean $5 \cdot 5 + 3 \cdot 5 = 9$ **1**

and therefore variance 9. **1**

(c) For Poi(9), $P(T = 2) = 0 \cdot 0062 - 0 \cdot 0012 = 0 \cdot 0050$ **1**

(d) $P(T > k) < 0 \cdot 01 \Rightarrow P(T \leqslant k) > 0 \cdot 99$ **1**

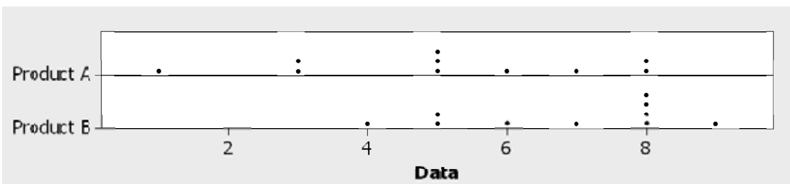
and for a Poi(9) the smallest k is 17. **1**

A9. (a)

Product A	y	Product B
0	1	
	2	
00	3	
	4	0
000	5	00
0	6	0
0	7	0
00	8	0000
	9	0
	10	

(Back-to-back stem-and-leaf or dot-plot acceptable.)
Display
Labels
Advert B appears to be more effective

1,1,1



(b)

Rating	1	3	3	4	5	5	5	5	5	6
Product	A	A	A	B	A	A	A	B	B	A
Rank	1	2·5	2·5	4	7	7	7	7	7	10·5
Rating	6	7	7	8	8	8	8	8	8	9
Product	B	A	B	A	A	B	B	B	B	B
Rank	10.5	12.5	12.5	16.5	16.5	16.5	16.5	16.5	16.5	20

H_0 : Median A = Median B

H_1 : Median A < Median B. **1**

Rank sum for B ratings is $W = 127$. **1**

$\mathrm{E}(W) = \frac{1}{2}n(n + m + 1) = \frac{1}{2} \times 10 \times 21 = 105$

$\mathrm{V}(W) = \frac{1}{12}nm(n + m + 1) = \frac{1}{12} \times 100 \times 21 = 13{\cdot}23^2$ **1**

$\mathrm{P}(W \geqslant 127) = \mathrm{P}\left(Z \geqslant \dfrac{126.5 - 105}{13{\cdot}23}\right) = \mathrm{P}(Z \geqslant 1{\cdot}63$ **1**

$= 1 - \Phi(1{\cdot}63) = 1 - 0{\cdot}9484 = 0{\cdot}0516$ **1**

$0{\cdot}0516 > 0{\cdot}05$ so we accept H_0 at the 5% level **1**

i.e. there is no evidence of greater likelihood to purchase in the case of Product B. **1**

A10. (a) Taking $\bar{p} = 0{\cdot}6807$ for the first 60 days as p: **1**

Control Limits are $p \pm 3\sqrt{\frac{pq}{n}} = 0{\cdot}6807 \pm 3\sqrt{\frac{0{\cdot}6807 \times 0{\cdot}3193}{50}}$ **1**

$= 0.4829,\ 0{\cdot}8785$. **1**

(b) The fact that a number of points lie above the upper limit provides evidence that the proportion of units free from nonconformities has increased. **1**

(c) $p = 0{\cdot}8648$

$\Rightarrow \text{UCL} = 0{\cdot}8648 + 3\sqrt{\dfrac{0{\cdot}8648 \times 0{\cdot}1352}{50}} = 1{\cdot}01$ **1**

A proportion cannot exceed one. **1**

(d)

$p + 3\sqrt{\frac{pq}{n}} < 1$

$\Rightarrow 3\sqrt{\frac{pq}{n}} < q$ **1**

$\Rightarrow \frac{9pq}{n} < q^2$

$\Rightarrow n > 9\frac{p}{q}$ **1**

$\Rightarrow n > 9\dfrac{0{\cdot}8648}{0{\cdot}1352} = 57{\cdot}6$

Thus a sample size of at least 58 is required. **1**

Section B − Mathematics for Applied Mathematics

B1. (a) $AB = \begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} 4 & 6 \\ 0 & -3 \end{pmatrix} = \begin{pmatrix} 8 & 15 \\ 12 & 3 \end{pmatrix}$ **1**

(b)

$$4C + D = 4\begin{pmatrix} x & 2 \\ 0 & y \end{pmatrix} + \begin{pmatrix} 2 & 7 \\ 12 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 4x & 8 \\ 0 & 4y \end{pmatrix} + \begin{pmatrix} 2 & 7 \\ 12 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 4x + 2 & 15 \\ 12 & 4y - 1 \end{pmatrix}$$ **2E1**

(c) $\begin{pmatrix} 8 & 15 \\ 12 & 3 \end{pmatrix} = \begin{pmatrix} 4x + 2 & 15 \\ 12 & 4y - 1 \end{pmatrix} \Rightarrow x = 1{\cdot}5; y = 1$ **1,1**

B2.

$$y = e^{2x}\cos x$$

$$\frac{dy}{dx} = 2e^{2x}\cos x + e^{2x}(-\sin x)$$ **1M, 2E1**

$$= 2e^{2x}\cos x - e^{2x}\sin x$$

B3.

$$\frac{4x - 3}{x(x^2 + 3)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 3}$$ **1M**

$$4x - 3 = A(x^2 + 3) + x(Bx + C)$$

$$= (A + B)x^2 + Cx + 3A$$

$$A = -1,\ B = 1,\ C = 4$$ **2E1**

$$\frac{4x - 3}{x(x^2 + 3)} = \frac{-1}{x} + \frac{x + 4}{x^2 + 3}$$ **1**

B4. (a)

$$\int \ln x\, dx = \int \ln x.1\, dx$$ **1**

$$= x\ln x - \int x.\frac{1}{x}dx$$

$$= x\ln x - x + c$$ **1**

(b) Volume of a solid of revolution $= \int \pi y^2 dx$ **1**

$$\text{Volume of goblet} = \pi\int_1^{10}\left(2\sqrt{\ln x}\right)^2 dx$$

$$= 4\pi\int_1^{10}\ln x\, dx$$ **1**

$$= 4\pi\left[x\ln x - x\right]_1^{10}$$ **1**

$$= 4\pi\left[(10\ln 10 - 10) - (0 - 1)\right]$$

$$= 4\pi\left[10\ln 10 - 9\right] \ (\approx 176{\cdot}25 \text{ is acceptable})$$ **1**

B5. (a)

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1) \qquad \mathbf{1}$$

$$\sum_{r=1}^{n}\left(6r^2 - r\right) = 6\sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r \qquad \mathbf{1}$$

$$= n(n+1)(2n+1) - \frac{1}{2}n(n+1) \qquad \mathbf{1}$$

$$= \tfrac{1}{2}n(n+1)(4n+1)$$

(b)

$$\sum_{r=5}^{10}\left(6r^2 - r\right) = \sum_{r=1}^{10}\left(6r^2 - r\right) - \sum_{r=1}^{4}\left(6r^2 - r\right) \qquad \mathbf{1}$$

$$= \tfrac{1}{2} \times 10 \times 11 \times 41 - \tfrac{1}{2} \times 4 \times 5 \times 17$$

$$= 2085 \qquad \mathbf{1}$$

B6. (a)

$$\int \frac{1}{T-22}\,dT = \int k\,dt \qquad \mathbf{M1,1}$$

$$\ln(T-22) = kt + c \qquad \mathbf{1}$$

$$T - 22 = e^{kt+c} \qquad \mathbf{1}$$

$$T = Ae^{kt} + 22$$

(b)

$$82 = Ae^{k\times 0} + 22 \Rightarrow A = 60 \qquad \mathbf{1}$$

$$62 = 60e^{k\times 5} + 22 \Rightarrow 40 = 60e^{5k} \Rightarrow e^{5k} = \frac{2}{3} \qquad \mathbf{1}$$

$$\Rightarrow \ln\frac{2}{3} = 5k \Rightarrow k = \frac{1}{5}\ln\frac{2}{3} \qquad \mathbf{1}$$

$$T = 60e^{\frac{1}{5}\left(\ln\frac{2}{3}\right)t} + 22 \qquad \left(T = 60e^{-0\cdot08t} + 22 \text{ is acceptable}\right) \qquad \mathbf{1}$$

$$T = 60e^{2\ln\frac{2}{3}} + 22$$

$$= 60e^{\ln\frac{4}{9}} + 22$$

$$= 60 \times \tfrac{4}{9} + 22 = 48\tfrac{2}{3} \qquad \left(\approx 48\cdot7 \text{ is acceptable}\right) \qquad \mathbf{1}$$

[END OF MARKING INSTRUCTIONS]