X202/701

NATIONAL QUALIFICATIONS 2007

TUESDAY, 22 MAY
1.00 PM – 4.00 PM

APPLIED MATHEMATICS
ADVANCED HIGHER
Statistics

Read carefully

1. Calculators may be used in this paper.
2. Candidates should answer all questions.

    Section A assesses the Units Statistics 1 and 2
    Section B assesses the Unit Mathematics for Applied Mathematics

3. **Full credit will be given only where the solution contains appropriate working.**
4. A booklet of Statistical Formulae and Tables is supplied for all candidates.

Section A (Statistics 1 and 2)

Answer all the questions.

Marks

A1. In a major financial organisation, all junior executives, male (M) and female (F), are required to undertake an assessment in order to determine their suitability (S) for promotion to senior executive posts. An independent auditor determined that, for the methods used, $P(M|S) = 0{\cdot}9$ and that $P(F|S) = 0{\cdot}1$.

(*a*) Explain why it would be incorrect to claim sex discrimination on the basis of the inequality of these two probabilities. **1**

(*b*) Currently 75% of junior executives are deemed suitable for promotion and 20% of junior executives are female. Calculate both $P(S|M)$ and $P(S|F)$ and comment on the question of discrimination. **5**

A2. (*a*) Glass bottles manufactured for the food industry are monitored for quality by selecting a sample consisting of a randomly chosen bottle from the first 20 produced and every 20th bottle thereafter, as they come off the conveyor belt. Name this type of sampling. **1**

(*b*) The bottles are formed by the action of compressed air on gobs of molten glass within a moulding machine. There are 25 moulds in the machine and each bottle is stamped to identify the mould that it came from. Every hour, a sample consisting of four randomly chosen bottles from each of the 25 moulds is selected. Name this type of sampling. **1**

(*c*) Give either a potential advantage or potential disadvantage associated with each of the two methods and state, with a reason, which you would recommend. **3**

A3. A company operates a batch chemical production process with yield X kg, where $X \sim N(2250, 140^2)$. In order for the process to remain economically viable, the company wishes to raise the mean yield to at least 2400 kg. It has been suggested that the use of a new catalyst will lead to increased yield. An experiment is planned in which a random sample of 10 yields will be obtained and subsequently $H_0 : \mu = 2250$ will be tested, using a z-test at the 1% level of significance, against $H_1 : \mu > 2250$.

(*a*) State the critical region of the test in terms of z and show that it is equivalent to the sample mean satisfying the inequality $\bar{x} > 2353$. **3**

(*b*) With the new catalyst, $X \sim N(2400, 140^2)$. Calculate the probability that the test would fail to provide evidence of improved mean yield. **4**

A4. Passengers on a cruise ship were asked to assess, on a scale from 1 to 20 (where 1 represents none), the amount of motion experienced during their cruise. The median score from data for a large number of passengers was 12. After new stabilisers were fitted, the scores reported by a random sample of 15 passengers were:

9 11 12 11 8 14 11 7 10 12 13 9 11 8 9.

Perform a sign test in order to evaluate the evidence from the data that the new stabilisers have led to a reduction in the amount of motion experienced by passengers on the ship. **6**

A5. Blood pressure levels were measured in a sample of 19 diabetic men, aged 50 to 59 years. The sample mean systolic blood pressure was 146·6 mmHg and the sample standard deviation 17·9 mmHg.

(*a*) Stating two assumptions required, calculate a 95% confidence interval for the mean systolic blood pressure in the population of diabetic men aged 50 to 59 years. **5**

(*b*) Given that the mean systolic blood pressure in the population of men aged 50 to 59 years in good health is 137·4 mmHg, state what can be deduced from your answer in (*a*). **2**

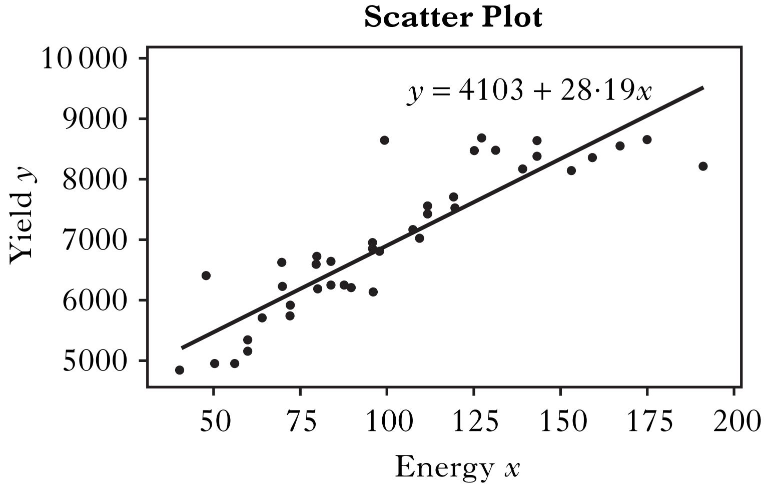
A6. A spinning frame in a textile factory is used to spin monofilament nylon yarn and has a large number of spinnerets. Yarn breakages at the spinnerets require spinning to be stopped in order to enable the operator to mend the breakages. The number of stoppages per hour, X, may be modelled by the Poisson distribution with mean 4.

(*a*) Obtain the probability that the number of stoppages in an hour is more than two standard deviations above the mean. **3**

(*b*) Obtain the smallest integer k such that $P(X > k) < 0{\cdot}001$. **2**

[Turn over

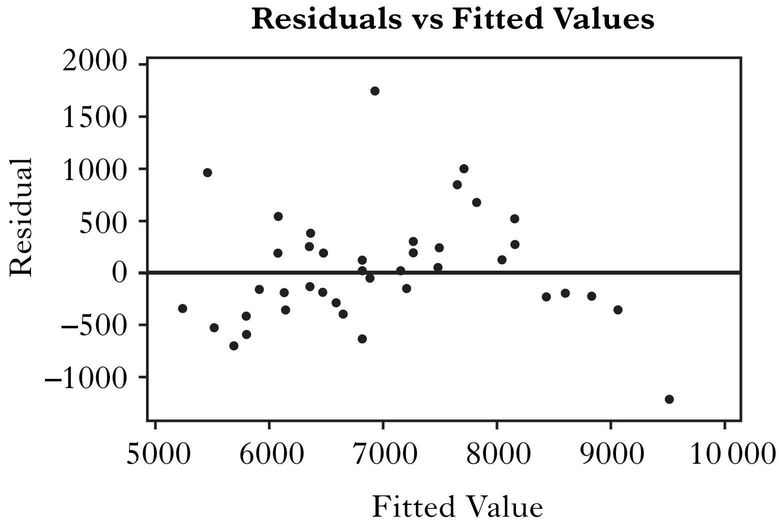
A7. Agricultural scientists carried out an investigation of milk yield (y) and energy intake (x) for a breed of cow. A random sample of 40 animals was used. An annotated scatter plot of the data with a fitted linear model is shown below.



(a) Given that, in the standard notation, S_{xx} is $53\,637{\cdot}186$ and SSR = 11 581 321, test the null hypothesis that the slope parameter in the linear model is zero, stating any assumptions required. **5**

(b) Given that, for one of the cows, the observed values x and y were 144 and 8640 respectively, calculate the corresponding fitted value and residual. **2**

(c) The plot of residuals against fitted values is shown below.



State the feature of this plot that suggested to the scientists that a quadratic model would be worth investigating. **1**

(d) A satisfactory residual plot was obtained for the quadratic model:

$$y = 2305 + 64{\cdot}80x - 0{\cdot}1644x^2.$$

Calculate the optimum energy intake, in terms of maximizing milk yield, for the breed of cow. **2**

Comment on your answer with reference to the scatter plot above. **1**

A8. Following a banquet attended by a large number of people, cases of food poisoning were reported and attributed by some guests to the seafood on the menu. A health agency contacted and interviewed a random sample of guests. The contingency table below was constructed from their responses.

	Well	*Ill with food poisoning*
Ate seafood	21	9
Did not eat seafood	38	4

(*a*) State the null and alternative hypotheses for a chi-squared test for association in the contingency table. **1**

(*b*) Write down estimates of the probability that a guest at the banquet ate seafood and of the probability that a guest was ill with food poisoning. **2**

(*c*) Explain how these estimates are used to calculate the expected frequency of guests in the sample who ate seafood and took ill with food poisoning. **2**

(*d*) Complete the test. **5**

A9. The random variable X has the continuous uniform (0, 1) distribution so that $E(X) = \frac{1}{2}$ and $V(X) = \frac{1}{12}$. The random variable $Y = (b - a)X + a$ has the continuous uniform (a, b) distribution.

(*a*) Show that the mean and variance of Y are $\frac{1}{2}(a + b)$ and $\frac{1}{12}(b - a)^2$. **3**

(*b*) An assembly operation for DVD players has three phases: Set-up; Build; and Test. The durations of the three phases are independent and have the continuous uniform (9,15), (84,96) and (15,21) distributions.

Show that the total time T to assemble a DVD player has mean 120 minutes and standard deviation 4·24 minutes. **5**

(*c*) Assuming that T is normally distributed, estimate the value of k such that $P(T \leq k) = 0{\cdot}9$. **3**

[*END OF SECTION A*]

[Turn over for Section B on *Page six*

Section B (Mathematics for Applied Mathematics)

Marks

Answer all the questions.

B1. Find the exact value of $\int_0^{\pi/6} x \sin 3x \, dx$. **5**

B2. Use the binomial theorem to expand $\left(x^3 - \frac{2}{x}\right)^4$ and simplify your answer. **4**

B3. A curve is defined parametrically by $x = \frac{t}{t^2+1}$, $y = \frac{t-1}{t^2+1}$.

Obtain $\frac{dy}{dx}$ as a function of t. **5**

B4. For the matrix $A = \begin{pmatrix} \lambda & 2 \\ 2 & \lambda - 3 \end{pmatrix}$, find the values of λ such that the matrix is singular. **3**

Write down the matrix A^{-1} when $\lambda = 3$. **1**

B5. Obtain the solution of the differential equation

$$x\frac{dy}{dx} - y = x^2 e^x$$

for which $y = 2$ when $x = 1$. **5**

B6. Express $\frac{8}{x(x+2)(x+4)}$ in partial fractions. **4**

Calculate the area under the curve

$$y = \frac{8}{x^3 + 6x^2 + 8x}$$

between $x = 1$ and $x = 2$. Express your answer in the form $\ln\frac{a}{b}$, where a and b are positive integers. **5**

[*END OF SECTION B*]

[*END OF QUESTION PAPER*]

[BLANK PAGE]