Section E (Statistics 1)

Answer all the questions.

Answer these questions in a separate answer book, showing clearly the section chosen.

E1. Coloured cards are placed in three boxes as follows.

<table>
<thead>
<tr>
<th></th>
<th>Red</th>
<th>White</th>
<th>Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box 1</td>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Box 2</td>
<td>4</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Box 3</td>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

A box is selected at random and a card drawn at random from it is found to be red.

Find the probability that Box 1 was selected.

E2. Scientists observed that the mean number of alpha particles emitted from a radioactive source during time intervals of 7.5 seconds was 1. Assuming that the situation can be modelled by a Poisson distribution, obtain the probability that during a randomly selected interval of 7.5 seconds the number of alpha particles emitted was

(a) zero,
(b) four,
(c) greater than four.

What is the standard deviation of the number of alpha particles emitted during intervals of duration 7.5 seconds?

E3. In order to carry out a study of farms in a region, the farms were classified as large, medium and small. Local authority records revealed the following numbers of farms in each category.

<table>
<thead>
<tr>
<th>Category of farm</th>
<th>Number of farms in category</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large</td>
<td>30</td>
</tr>
<tr>
<td>Medium</td>
<td>320</td>
</tr>
<tr>
<td>Small</td>
<td>150</td>
</tr>
</tbody>
</table>

Explain how you would set about taking a stratified random sample of 50 farms in the region, specifying the number of farms in each category which you would sample.

State two advantages of stratified random sampling over simple random sampling.

[X106/701] Page fifteen
E4. A machine used for the automatic filling of breakfast cereal packs is known to deliver amounts of cereal with weights which are distributed with a standard deviation of 3 grams. When set for a production run, with packs labelled as 500 grams weight, the cereal manufacturer uses 505 grams as a target for the mean weight of the amounts. This is to ensure compliance with statutory regulations concerning under-filling of packs. During set-up, a trial run was carried out and a random sample of packs was found to have the following content weights.

509.9 507.7 506.7 507.5 506.7 506.0 506.5 505.2 510.5

Stating any assumption required, test the null hypothesis that the machine is operating "on target" using a two-tail test.

Explain why it would not be in the manufacturer's interest to simply carry out a one-tail test with alternative hypothesis that the mean amount is less than 505 grams.

5

E5. There is constant probability \( p \) that a computer "chip" produced at a factory is defective. State the distribution of \( X \), the number of defective chips, in random samples of size \( n \) and show that the sample proportion of defective chips has mean \( p \) and variance \( pq/n \).

The production manager estimates that, for the current manufacturing process, \( p = 0.20 \) and she wishes to monitor the defective rate using daily random samples of 80 chips.

Find the probability that, with the specified proportion, a sample contains between 9 and 23 defective chips (inclusive), justifying, and showing clearly, your method.

4

[END OF SECTION E]