Read carefully

1. Calculators may be used in this paper.
2. Candidates should answer all questions.
3. Full credit will be given only where the solution contains appropriate working.
1. (a) Given \( f(x) = \cos^2 \theta e^{\tan \theta} \), \( -\frac{\pi}{2} < x < \frac{\pi}{2} \), obtain \( f'(x) \) and evaluate \( f'(\frac{\pi}{4}) \).
   
(b) Differentiate \( g(x) = \frac{\tan^{-1} 2x}{1 + 4x^2} \).

2. Obtain the binomial expansion of \((a^2 - 3)^4\).

3. A curve is defined by the equations
   \[ x = 5\cos \theta, \quad y = 5\sin \theta, \quad (0 \leq \theta < 2\pi). \]
   Use parametric differentiation to find \( \frac{dy}{dx} \) in terms of \( \theta \).

4. Given \( z = 1 + 2i \), express \( z^2(z + 3) \) in the form \( a + ib \).
   Hence, or otherwise, verify that \( 1 + 2i \) is a root of the equation
   \[ z^3 + 3z^2 - 5z + 25 = 0. \]
   Obtain the other roots of this equation.

5. Express \( \frac{1}{x^2 - x - 6} \) in partial fractions.
   Evaluate \( \int_0^1 \frac{1}{x^2 - x - 6}\,dx \).

6. Write down the \( 2 \times 2 \) matrix \( M_1 \) associated with an anti-clockwise rotation of \( \frac{\pi}{2} \) radians about the origin.
   Write down the matrix \( M_2 \) associated with reflection in the \( x \)-axis.
   Evaluate \( M_2 M_1 \) and describe geometrically the effect of the transformation represented by \( M_2 M_1 \).

7. Obtain the first three non-zero terms in the Maclaurin expansion of \( f(x) = e^x \sin x \).

8. Use the Euclidean algorithm to show that \((231, 17) = 1\) where \((a, b)\) denotes the highest common factor of \( a \) and \( b \).
   Hence find integers \( x \) and \( y \) such that \( 231x + 17y = 1 \).

9. Use the substitution \( x = (u - 1)^2 \) to obtain \( \int \frac{1}{(1 + \sqrt{u})^3}\,du \).

[X100/701]
10. Determine whether the function \( f(x) = x^4 \sin 2x \) is odd, even or neither. Justify your answer.  

11. A solid is formed by rotating the curve \( y = e^{-2x} \) between \( x = 0 \) and \( x = 1 \) through 360° about the \( x \)-axis. Calculate the volume of the solid that is formed.  

12. Prove by induction that \( \frac{d^n}{dx^n} (xe^x) = (x + n)e^x \) for all integers \( n \geq 1. \)  

13. The function \( f \) is defined by \( f(x) = \frac{x - 3}{x + 2}, \ x \neq -2, \) and the diagram shows part of its graph.

(a) Obtain algebraically the asymptotes of the graph of \( f. \)  
(b) Prove that \( f \) has no stationary values.  
(c) Does the graph of \( f \) have any points of inflexion? Justify your answer.  
(d) Sketch the graph of the inverse function, \( f^{-1}. \) State the asymptotes and domain of \( f^{-1}. \)  

14. (a) Find an equation of the plane \( \pi_1 \) containing the points \( A(1, 0, 3), \ B(0, 2, -1) \) and \( C(1, 1, 0). \) Calculate the size of the acute angle between \( \pi_1 \) and the plane \( \pi_2 \) with equation \( x + y - z = 0. \)  
(b) Find the point of intersection of plane \( \pi_2 \) and the line  
\[
\frac{x - 11}{4} = \frac{y - 15}{5} = \frac{z - 12}{2}.
\]  

[Turn over for Questions 15 and 16 on Page four]
15. (a) A mathematical biologist believes that the differential equation \( x \frac{dy}{dx} - 3y = x^4 \) models a process. Find the general solution of the differential equation.

Given that \( y = 2 \) when \( x = 1 \), find the particular solution, expressing \( y \) in terms of \( x \).

(b) The biologist subsequently decides that a better model is given by the equation \( y \frac{dy}{dx} - 3x = x^4 \).

Given that \( y = 2 \) when \( x = 1 \), obtain \( y \) in terms of \( x \).

16. (a) Obtain the sum of the series \( 8 + 11 + 14 + \ldots + 56 \).

(b) A geometric sequence of positive terms has first term 2, and the sum of the first three terms is 266. Calculate the common ratio.

(c) An arithmetic sequence, \( A \), has first term \( a \) and common difference 2, and a geometric sequence, \( B \), has first term \( a \) and common ratio 2. The first four terms of each sequence have the same sum. Obtain the value of \( a \).

Obtain the smallest value of \( n \) such that the sum to \( n \) terms for sequence \( B \) is more than \textbf{twice} the sum to \( n \) terms for sequence \( A \).

[END OF QUESTION PAPER]