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Force, Energy and Periodic Motion - Lesson 2

Centripetal Motion and Friction

LI

- Solve problems involving Centripetal Force and Friction.

SC

- Equations of circular motion.
- Equations of friction.

Definition :

A **centripetal force** is a force that tends to make a body follow a curved path.

Centripetal force is usually applied to circular motion.

Centripetal force is not a special new type of force; it's just like 'm a', the net force, but applied to circular motion.

Using Newton's 2nd Law $\underline{\mathbf{F}} = m\,\underline{\mathbf{a}}$ together with the acceleration of an object in circular motion $\underline{\mathbf{a}} = -\omega^2 r\,\underline{\mathbf{e}}_r$, and denoting the centripetal force by $\underline{\mathbf{F}}_c$, we have,

$$\boxed{\underline{\mathbf{F}}_c = -m\,\omega^2 r\,\underline{\mathbf{e}}_r = -\frac{m v^2}{r}\,\underline{\mathbf{e}}_r}$$

In magnitude form,

$$\boxed{F_c = m\,\omega^2 r = \frac{m v^2}{r}}$$

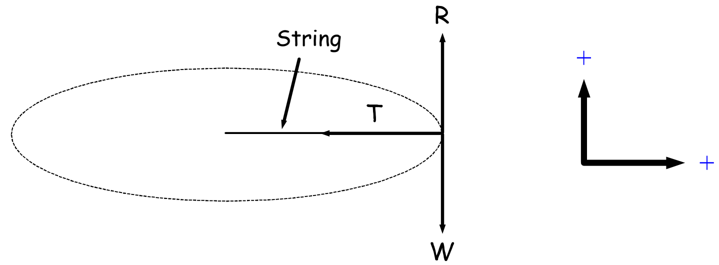
Example 1

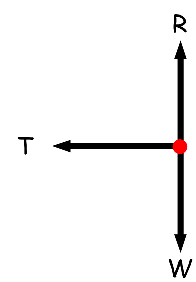
A particle of mass 300 g is attached to one end of a light, inextensible string of length 40 cm, the other end of the string being fixed at O on a smooth horizontal surface.

If the particle moves in a circle, find the tension in the string when the particle has :

(a) speed $2\sqrt{2}$ m s^{-1}.
(b) angular speed 5 rad s^{-1}.

'Smooth' means no friction.





Equating vertically gives $R = mg$; correct, but not particularly useful for calculating the tension (which is horizontal).

The net centripetal force is provided by the tension; so,

$$-T = -F_c = -m\omega^2 r = -\frac{mv^2}{r}$$

(a) We have,

$$T = \frac{mv^2}{r}$$

$$\therefore \quad T = \frac{(0.3)(2\sqrt{2})^2}{0.4}$$

$$\Rightarrow \quad \boxed{T = 6\text{ N}}$$

(b) We have,

$$T = mr\omega^2$$

$$\therefore \quad T = (0.3)(0.4)(5)^2$$

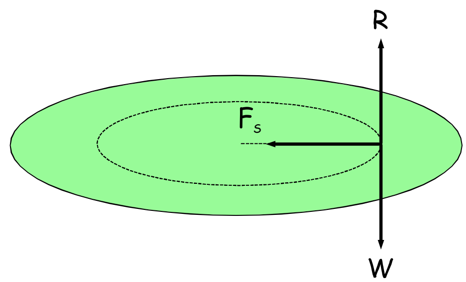
$$\Rightarrow \quad \boxed{T = 3\text{ N}}$$

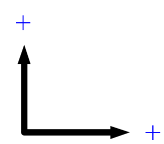
Example 2

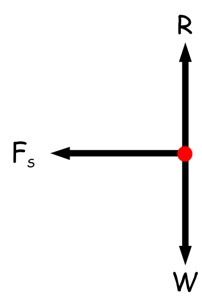
A tiny box is placed on the surface of a horizontal disc at a point 5 cm from the centre of the disc. The box is on the point of slipping when the disc rotates at 1 . 4 rev s^{-1}.

Show that the coefficient of friction between the box and the surface of the disc is $\pi^2/25$.

The box will tend to slide away from the 5 cm mark; hence, there is a static friction force acting inwards to prevent this.







Equating vertically gives $R = W = m\,g$. As the box is on the point of slipping (outwards), we also have $(F_s)_{max.} = \mu_s R$. Hence,

$$R = W = m\,g$$

$$(F_s)_{max.} = \mu_s R$$

$$\therefore \quad \underline{(F_s)_{max.} = \mu_s m\,g}$$

The net (centripetal) force is supplied by static friction,

$$F_c = (F_s)_{max.} = \mu_s m\,g$$

$$\therefore \quad \mu_s m\,g = m\,r\,\omega^2$$

$$\Rightarrow \quad \underline{\mu_s = r\,\omega^2/g}$$

We have, in SI units, $r = 5/100$ m $= 1/20$ m, $g = 98/10$ m s^{-2} $= 49/5$ m s^{-2} and $\omega = (14/10)\,(2\pi)$ rad $s^{-1} = 14\pi/5$ rad s^{-1}.

$$\therefore \quad \mu_s = (1/20)\,(196\,\pi^2/25)/(49/5)$$

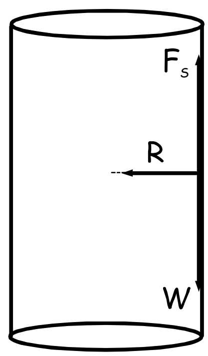
$$\Rightarrow \quad \boxed{\mu_s = \pi^2/25}$$

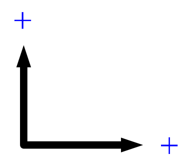
Example 3

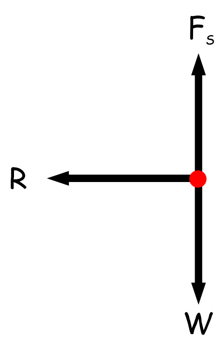
A hollow circular cylinder of radius r rotates at ω about its axis of symmetry (which is vertical). A body of mass m rotates, without slipping, on the inner surface of the cylinder.

Given that the body is on the point of slipping down the cylinder, show that $g = \mu_s r \omega^2$.

The body is prevented from slipping down due to static friction (upwards). The centripetal force is provided by the reaction from the interior cylinder surface.







Equating vertically gives,

$$\mu_s R = (F_s)_{max.} = W = m g$$

$$\Rightarrow \quad \underline{g = \mu_s R/m}$$

The net centripetal force is provided by the reaction force. So,

$$R = F_c = m r \omega^2$$

$$\therefore \quad g = \mu_s (m r \omega^2)/m$$

$$\Rightarrow \quad \boxed{g = \mu_s r \omega^2}$$

Blue Book

- pg. 313-315 Ex. 13 B Q 4 - 12, 18, 21.