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Unit 2 : Sequences and Series - Lesson 9

Differentiation and Integration of Maclaurin Series

LI

- Obtain Maclaurin series using standard ones.

SC

- Standard Maclaurin series.
- Standard derivatives and integrals.

Power series can be differentiated and integrated term by term;
if $f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$, then :

$$f'(x) = a_1 + 2a_2 x + 3a_3 x^2 + \ldots$$

and

$$\int f(x)\,dx = a_0 x + \frac{a_1 x^2}{2} + \frac{a_2 x^3}{3} + \frac{a_3 x^4}{4} + \ldots$$

Example 1

Given that $\tan x = x + \dfrac{x^3}{3} + \dfrac{2x^5}{15} + \dfrac{17x^7}{315} + \ldots$,

find the Maclaurin series for $\sec^2(2x)$ up to x^4.

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \ldots$$

$$\therefore \qquad \tan(2x) = 2x + \frac{8x^3}{3} + \frac{64x^5}{15} + \frac{2\,176x^7}{315} + \ldots$$

Differentiating each side of the previous equation gives,

$$2\sec^2(2x) = 2 + 8x^2 + \frac{64x^4}{3} + \ldots$$

$$\Rightarrow \qquad \boxed{\sec^2(2x) = 1 + 4x^2 + \frac{32x^4}{3} + \ldots}$$

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Example 2

Using the Maclaurin series for $\ln(1 + x)$, obtain the Maclaurin series for $(1 + 3x)^{-1}$ up to x^3.

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$$

$$\therefore \quad \ln(1 + 3x) = 3x - \frac{9x^2}{2} + 9x^3 - \frac{81x^4}{4} + \ldots$$

Differentiating each side of the previous equation gives,

$$3(1 + 3x)^{-1} = 3 - 9x + 27x^2 - 81x^3 + \ldots$$

$$\Rightarrow \quad \boxed{(1 + 3x)^{-1} = 1 - 3x + 9x^2 - 27x^3 + \ldots}$$

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Example 3

Given that $\dfrac{1}{\sqrt{1 - x^2}} = 1 + \dfrac{x^2}{2} + \dfrac{3x^4}{8} + \dfrac{5x^6}{16} + \dots,$

find the Maclaurin series for $\sin^{-1} x$ up to x^5.

Recalling the standard integral,

$$\sin^{-1} x = \int \frac{1}{\sqrt{1 - x^2}}\, dx$$

integrating the given series gives,

$$\boxed{\sin^{-1} x = x + \frac{x^3}{6} + \frac{3x^5}{40} + \dots}$$

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Questions

1) Given that $\sec x = 1 + \dfrac{x^2}{2} + \dfrac{5x^4}{24} + \dfrac{61x^6}{720} + \ldots,$

find the Maclaurin series for $\sec(2x)\tan(2x)$ up to x^5.

2) Using the Maclaurin series for $\ln(1 + x)$, obtain the Maclaurin series for $(1 - 5x)^{-1}$ up to x^3.

3) Given that $\dfrac{1}{1 + x^2} = 1 - x^2 + x^4 - x^6 + \ldots,$

find the Maclaurin series for $\tan^{-1} x$ up to x^5.

Answers

1) $\sec(2x)\tan(2x) = 2x + \dfrac{20x^3}{3} + \dfrac{244x^5}{15} + \ldots$

2) $(1 - 5x)^{-1} = 1 + 5x + 25x^2 + 125x^3 + \ldots$

3) $\tan^{-1} x = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} + \ldots$