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Linear and Parabolic Motion - Lesson 8

Friction

LI

- Know the meanings of Static, Limiting and Kinetic Friction.
- Draw FBDs for systems with friction.
- Solve problems involving friction.

SC

- Write equations for net force using FBDs.

Static Friction

Friction that exists when there is
no relative motion between objects

The diagram illustrates the definition of the coefficient of static friction, μ_s . It shows the equation $\mu_s = \frac{F_s}{N}$ with arrows pointing from descriptive text to each term: μ_s is labeled 'coefficient of static friction', F_s is labeled 'static frictional force', and N is labeled 'normal reaction force'. The word 'def' with an equals sign is placed above the fraction bar.

$$\mu_s \stackrel{\text{def}}{=} \frac{F_s}{N}$$

$$F_s \leq \mu_s N$$

Value of F_s required to just get the object moving (aka **Limiting Friction**) is :

$$(F_s)_{\text{max.}} = \mu_s N$$

Kinetic (aka Dynamic or Sliding) Friction

Friction that exists when there
is relative motion between objects

$$\mu_k \stackrel{\text{def}}{=} \frac{F_k}{N}$$

coefficient of
kinetic friction

kinetic frictional
force

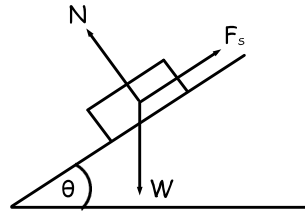
normal reaction
force

$$F_k = \mu_k N$$

Angle of Friction

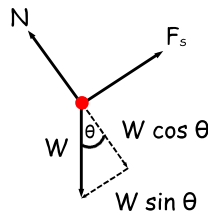
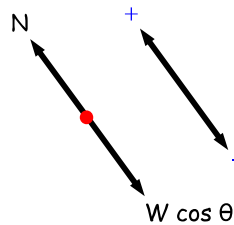
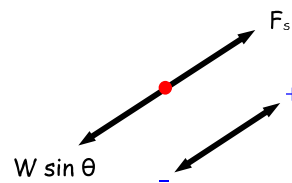
Object of mass m kg on rough slope (with coefficient of static friction μ_s ; angle of slope can change.

The problem is to find the angle required (aka Angle of Friction) to just overcome the static frictional force.



Angle of slope changes; tilt up (θ increases) so that object is on the verge of sliding down

For slope questions, it is simpler to resolve forces parallel to and perpendicular to the slope.

All Perpendicular ForcesAll Parallel Forces

The block is in equilibrium, so the net force perpendicular to the plane is 0 (and similar for parallel to the slope). Equating forces parallel and perpendicular to the slope gives, respectively,

$$F_s = mg \sin \theta$$

$$N = mg \cos \theta$$

Dividing the first equation by the second equation gives, remembering that at limiting friction, $F_s = \mu_s N$,

$$\frac{F_s}{N} = \tan \theta$$

$$\Rightarrow \frac{\mu_s N}{N} = \tan \theta$$

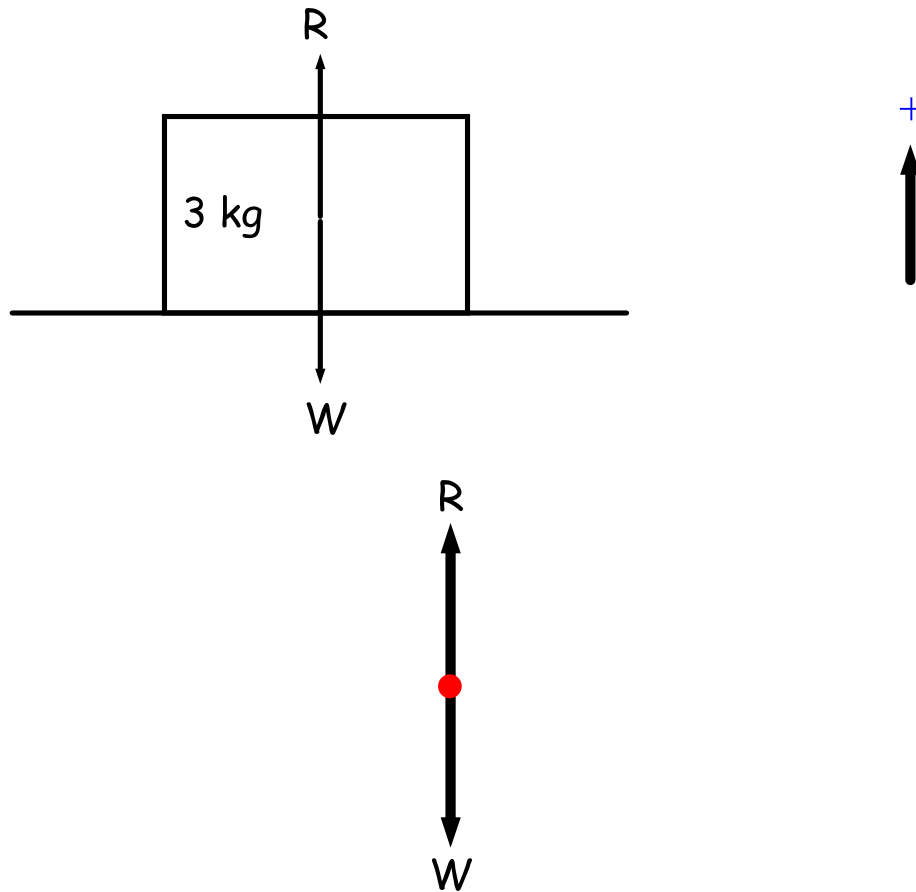
$$\Rightarrow \mu_s = \tan \theta$$

$$\Rightarrow \theta = \tan^{-1} \mu_s$$

(Angle of Friction)

Example 1

Calculate the maximum frictional force that can act when a block of mass 3 kg rests on a rough, horizontal surface, the coefficient of static friction between the surfaces being 0.4.



$$R = W = m g$$

$$(F_s)_{\max.} = \mu_s R$$

$$\therefore (F_s)_{\max.} = \mu_s m g$$

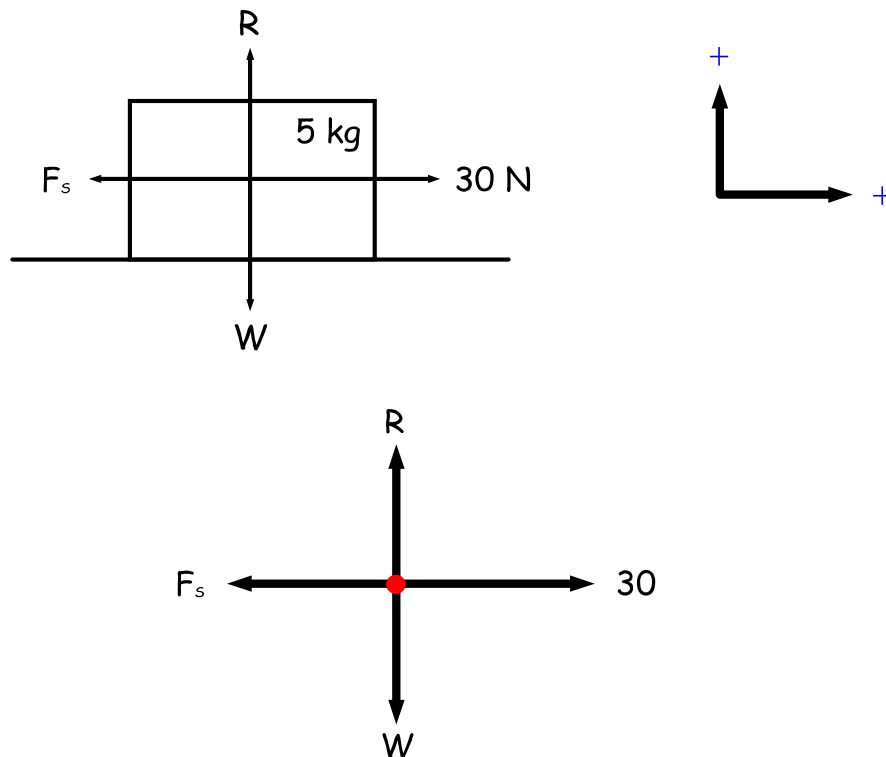
$$\Rightarrow (F_s)_{\max.} = 0.4 (3) (9.8)$$

$$\Rightarrow (F_s)_{\max.} = 11.76 \text{ N}$$

Example 2

A force of 30 N acts on a block of mass 5 kg resting on a rough, horizontal surface.

If the coefficient of static friction between the block and the surface is 0.6, determine whether or not the block will move.



The block will move if the 30 N force is greater than $(F_s)_{\max.}$. So, we first calculate $(F_s)_{\max.}$.

$$R = W = m g$$

$$(F_s)_{\max.} = \mu_s R$$

$$\therefore (F_s)_{\max.} = \mu_s m g$$

$$\Rightarrow (F_s)_{\max.} = 0.6 (5) (9.8)$$

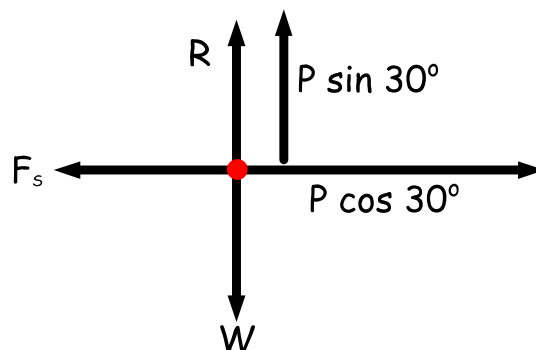
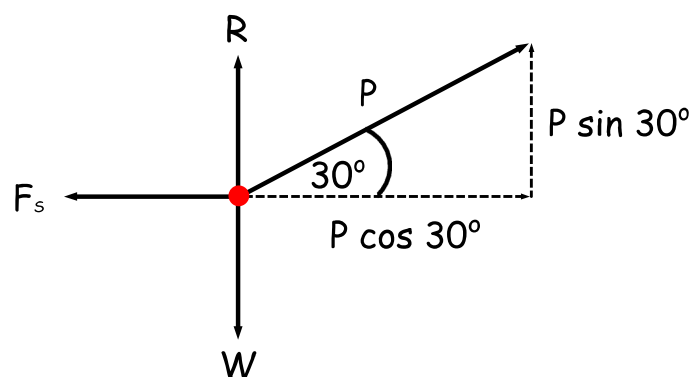
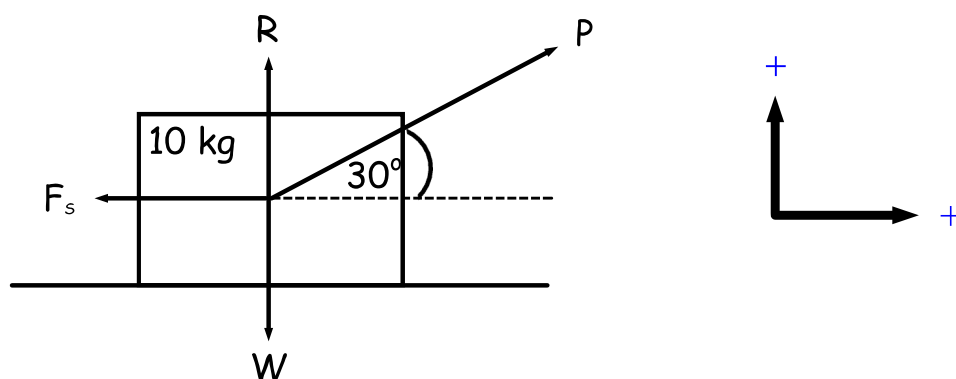
$$\Rightarrow \underline{(F_s)_{\max.} = 29.4 \text{ N}}$$

As $30 \text{ N} > (F_s)_{\max.} (= 29.4 \text{ N})$, the block will move.

Example 3

A 10 kg trunk lies on a rough, horizontal floor, with the coefficient of static friction being $\sqrt{3}/4$.

Show that the magnitude of the force P necessary so that the trunk is on the verge of moving horizontally if P is applied at 30° to the horizontal, is given by $P = 4g$ newtons.



Limiting friction, so forces horizontally are balanced (and similarly for vertical forces). Hence,

$$F_s = \mu_s R = P \cos 30^\circ$$

$$W = R + P \sin 30^\circ \Rightarrow \underline{R = W - P \sin 30^\circ}$$

$$\therefore P \cos 30^\circ = \mu_s R$$

$$\Rightarrow P \cos 30^\circ = \mu_s (W - P \sin 30^\circ)$$

$$\Rightarrow P \cos 30^\circ = \mu_s m g - \mu_s P \sin 30^\circ$$

$$\Rightarrow \mu_s m g = P \cos 30^\circ + \mu_s P \sin 30^\circ$$

$$\Rightarrow P = \frac{\mu_s m g}{(\cos 30^\circ + \mu_s \sin 30^\circ)}$$

$$\therefore P = \frac{(\sqrt{3}/4)(10)g}{((\sqrt{3}/2) + \sqrt{3}/4(1/2))}$$

Multiplying top and bottom by $8/\sqrt{3}$ gives,

$$P = \frac{20g}{(4 + 1)}$$

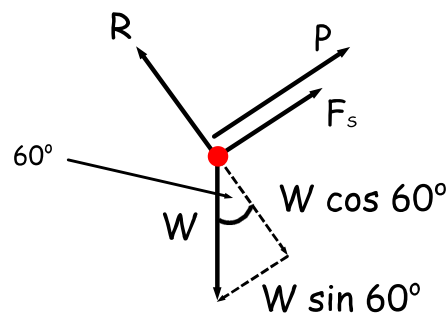
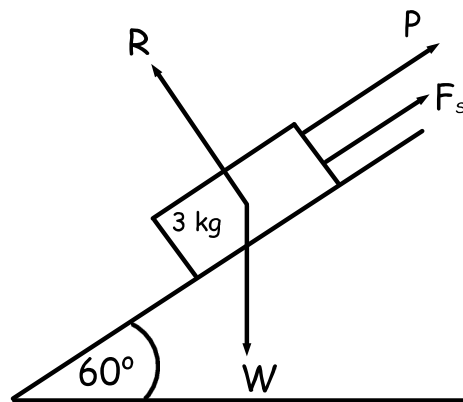
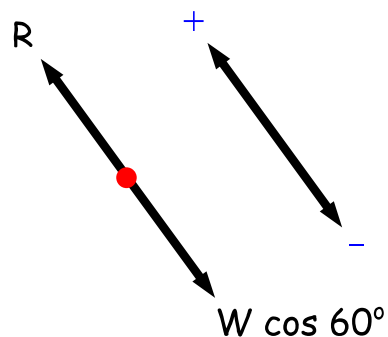
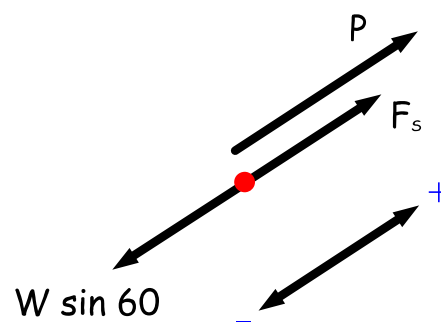
\Rightarrow

$$P = 4g \text{ newtons}$$

Example 4

A mass of 3 kg rests on a rough plane inclined at 60° to the horizontal, the coefficient of static friction being $\sqrt{3}/5$.

Find (in terms of g) the force P acting parallel to the plane which must be applied to the mass to just prevent motion down the plane.

All Perpendicular ForcesAll Parallel Forces

To just prevent motion down the plane, a state of limiting friction exists. Hence,

$$R = W \cos 60^\circ$$

$$P + F_s = W \sin 60^\circ$$

$$\therefore P + \mu_s R = mg \sin 60^\circ$$

$$\Rightarrow P = mg \sin 60^\circ - \mu_s R$$

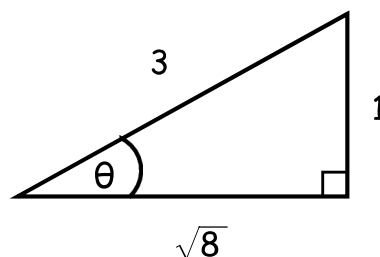
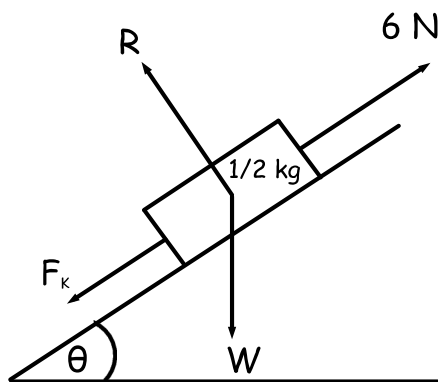
$$\Rightarrow P = m g \sin 60^\circ - \mu_s m g \cos 60^\circ$$

$$\therefore P = (3 g/2) ((\sqrt{3} - (\sqrt{3}/5))$$

$$\Rightarrow P = \frac{6 g \sqrt{3}}{5} \text{ N}$$

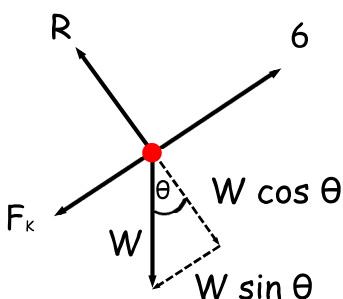
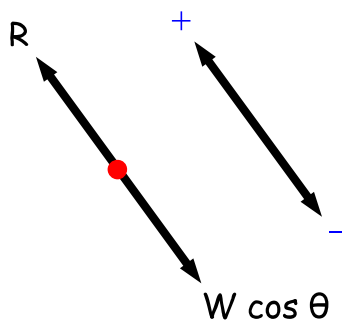
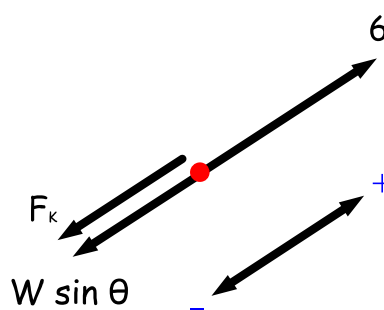
Example 5

Calculate the acceleration of a 0.5 kg mass up a rough plane inclined at θ to the horizontal (where $\sin \theta = 1/3$), when the mass is given a force of 6 N up the plane (i. e. parallel to the plane); the coefficient of kinetic friction is $1/\sqrt{2}$.



$$\sin \theta = 1/3$$

$$\cos \theta = \sqrt{8}/3$$

All Perpendicular ForcesAll Parallel Forces

Net force F_{NET}
is up the slope

$$R = W \cos \theta$$

$$F_{\text{NET}} = 6 - F_k - W \sin \theta$$

$$\therefore F_{\text{NET}} = 6 - \mu_k R - W \sin \theta$$

$$\Rightarrow F_{\text{NET}} = 6 - \mu_k W \cos \theta - W \sin \theta$$

$$\Rightarrow F_{\text{NET}} = 6 - \mu_k m g \cos \theta - m g \sin \theta$$

$$\Rightarrow F_{\text{NET}} = 6 - m g (\mu_k \cos \theta + \sin \theta)$$

$$\therefore (1/2) a = 6 - (1/2) (9.8) ((1/\sqrt{2}) (\sqrt{8/3}) + (1/3))$$

$$\Rightarrow (1/2) a = 1.1$$

$$\Rightarrow a = 2.2 \text{ m s}^{-2}$$

Blue Book

- pg. 110-114 Ex. 6 A Q 1 - 18, 22.
- pg. 118-121 Ex. 6 B Q 2 - 8, 11, 14, 21.