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Unit 2 : Sequences and Series - Lesson 8

Combinations and Compositions of Maclaurin Series

LI

- Find more complicated Maclaurin series using standard ones.

SC

- Standard Maclaurin series.

Example 1

Find the Maclaurin series for $\cos(2x)$ up to x^6 .

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$\therefore \cos(2x) = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \dots$$

$$\Rightarrow \cos(2x) = 1 - \frac{4x^2}{2} + \frac{16x^4}{24} - \frac{64x^6}{720} + \dots$$

$$\Rightarrow \boxed{\cos(2x) = 1 - 2x^2 + \frac{2x^4}{3} - \frac{4x^6}{45} + \dots}$$

Example 2

Find the Maclaurin expansion for e^{3x+x^2} up to x^3 .

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\therefore e^{3x} = 1 + (3x) + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!} + \dots$$

$$\Rightarrow e^{3x} = 1 + 3x + \frac{9x^2}{2} + \frac{9x^3}{2} + \dots$$

$$\therefore e^{x^2} = 1 + x^2 + \frac{(x^2)^2}{2!} + \frac{(x^2)^3}{3!} + \dots$$

$$\Rightarrow e^{x^2} = 1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6} + \dots$$

$$e^{3x+x^2} = e^{3x} \cdot e^{x^2}$$

$$\therefore e^{3x+x^2} = \left(1 + 3x + \frac{9x^2}{2} + \frac{9x^3}{2} + \dots \right)$$

$$\times \left(1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6} + \dots \right)$$

$$\Rightarrow e^{3x+x^2} = \left(1 + 3x + \frac{9x^2}{2} + \frac{9x^3}{2} + \dots \right)$$

$$+ \left(x^2 + 3x^3 + \dots \right)$$

$$\Rightarrow e^{3x+x^2} = 1 + 3x + \frac{11x^2}{2} + \frac{15x^3}{2} + \dots$$

Example 3

Find the Maclaurin series for $\ln(1 - x^2)$ up to x^4 .

$$\ln(1 - x^2) = \ln[(1 - x)(1 + x)] = \ln(1 - x) + \ln(1 + x)$$

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

$$\therefore \ln(1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots$$

$$\Rightarrow \boxed{\ln(1 - x^2) = -x^2 - \frac{x^4}{2} - \dots}$$

Example 4

Find the Maclaurin expansion for $e^x \sin x$ up to x^5 .

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$\therefore e^x \sin x = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \right)$$

$$\times \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \right)$$

$$\Rightarrow e^x \sin x = \left(x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!} + \frac{x^5}{4!} + \dots \right)$$

$$+ \left(-\frac{x^3}{3!} - \frac{x^4}{3!} - \frac{x^5}{2!3!} - \dots \right)$$

$$+ \left(\frac{x^5}{5!} - \dots \right)$$

$$\Rightarrow e^x \sin x = x + x^2 + \frac{x^3}{3} - \frac{x^5}{30} + \dots$$

Example 5

Find the Maclaurin expansion for $\ln(2 + x)$ up to x^3 .

$$\ln(2 + x) = \ln(2(1 + (x/2))) = \ln 2 + \ln(1 + (x/2))$$

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$$

$$\therefore \ln(1 + (x/2)) = \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{24} - \dots$$

$$\Rightarrow \boxed{\ln(2 + x) = \ln 2 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{24} - \dots}$$

Questions

Find these Maclaurin expansions :

1) $\sin(3x)$, up to x^5 .

2) e^{4x+x^2} , up to x^3 .

3) $\ln(3+x)$, up to x^4 .

4) $\ln(9-x^2)$, up to x^4 .

5) $e^{-x} \cos(4x)$, up to x^3 .

Answers

$$1) \sin(3x) = 3x - \frac{9x^3}{2} + \frac{81x^5}{40} - \dots$$

$$2) e^{4x+x^2} = 1 + 4x + 9x^2 + \frac{44x^3}{3} + \dots$$

$$3) \ln(3+x) = \ln 3 + \frac{x}{3} - \frac{x^2}{18} + \frac{x^3}{81} - \frac{x^4}{324} + \dots$$

$$4) \ln(9-x^2) = \ln 9 - \frac{x^2}{9} - \frac{x^4}{162} - \dots$$

$$5) e^{-x} \cos(4x) = 1 - x - \frac{15x^2}{2} + \frac{47x^3}{6} + \dots$$