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Linear and Parabolic Motion - Lesson 7

Newton's Laws, Free-Body Diagrams and Equilibrium

<u>LI</u>

- Know Newton's 3 Laws of Motion.

- Calculate missing tensions in static systems.

- Know and use Lami's Theorem for 3 forces.

<u>SC</u>

- Resolve forces in 2D.
- Draw a Free-Body Diagram (FBD) for an object.

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Newton's Laws

1st Law : Every object continues in its state of rest or uniform
                velocity unless acted upon by a non-zero force.

2nd Law : If a non-zero force acts upon an object, then
                it is equal to the rate of change of momentum :

$$\underline{F} \;=\; \frac{d}{dt}\,\underline{p} \;=\; \frac{d}{dt}\,(m\,\underline{v}) \;=\; m\,\frac{d}{dt}\,\underline{v} \;=\; m\,\underline{a}$$

3rd Law : For a force $\underline{F}$ acting from object A to object B,
                there is an equal and opposite force $-\underline{F}$ from B to A.

The 1st Law can be written in equation form in various ways :

$$\underline{F} \;=\; \underline{0} \qquad \longleftrightarrow \qquad \underline{a} \;=\; \underline{0} \qquad \longleftrightarrow \qquad \underline{v} \;=\; \text{constant vector}$$

If $\underline{F}_{AB}$ is the force from A acting on B and $\underline{F}_{BA}$ is the force
from B acting on A, then the 3rd Law can be written in
equation form as :

$$\underline{F}_{BA} \;=\; -\,\underline{F}_{AB}$$

The non-zero force referred to in Newton's Laws means the
net force (aka unbalanced force aka resultant of all forces)
acting on the object

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Free-Body Diagrams

A Free-Body Diagram (FBD) shows the forces acting on an object only; forces the object exerts on other objects are not shown.

FBD for Object at Rest on a Table

Upthrust

Upthrust = Weight

Weight

FBD for Aeroplane Flying at Constant Speed and Constant Height

Upthrust

Upthrust = Weight

Engine Force   Drag
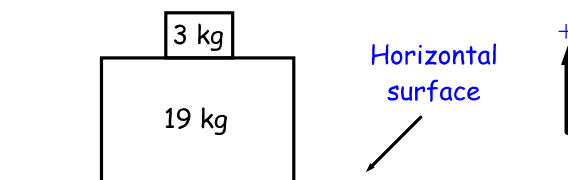
Engine Force = Drag

Weight

Equilibrium

In equilibrium, the net force is **0**.
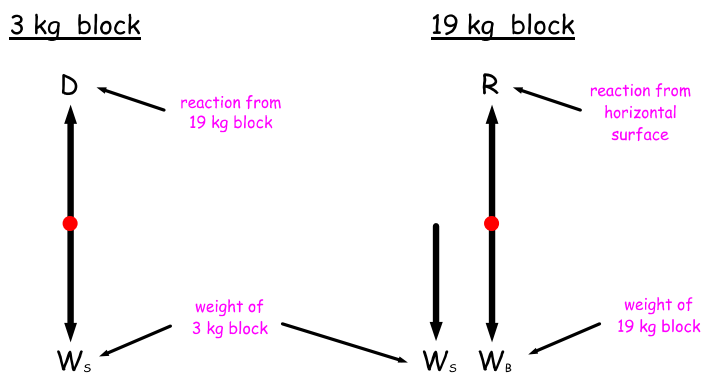
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<u>Example 1</u>

A mass of 19 kg rests on a horizontal surface and a 3 kg block rests on the 19 kg block. The whole system is in equilibrium.

Find the reaction between the masses and that between the larger mass and the horizontal surface in terms of g (the acceleration due to gravity).

3 kg

Horizontal surface

+

19 kg

Draw a FBD for each block :

<u>3 kg block</u>   <u>19 kg block</u>

D

reaction from 19 kg block

R

reaction from horizontal surface

weight of 3 kg block

weight of 19 kg block

W_s W_s W_B

Equilibrium for each block means that the net force on each block is $\underline{0}$:

$$D\,\underline{j} \;+\; W_s\,(-\,\underline{j}) \;=\; \underline{0} \qquad \Rightarrow \qquad D \;=\; W_s$$

$$R\,\underline{j} \;+\; W_s\,(-\,\underline{j}) \;+\; W_B\,(-\,\underline{j}) \;=\; \underline{0} \qquad \Rightarrow \qquad R \;=\; W_s \;+\; W_B$$

The reaction between the masses has magnitude D; the first equation gives, using $W_s \;=\; 3\,g$,
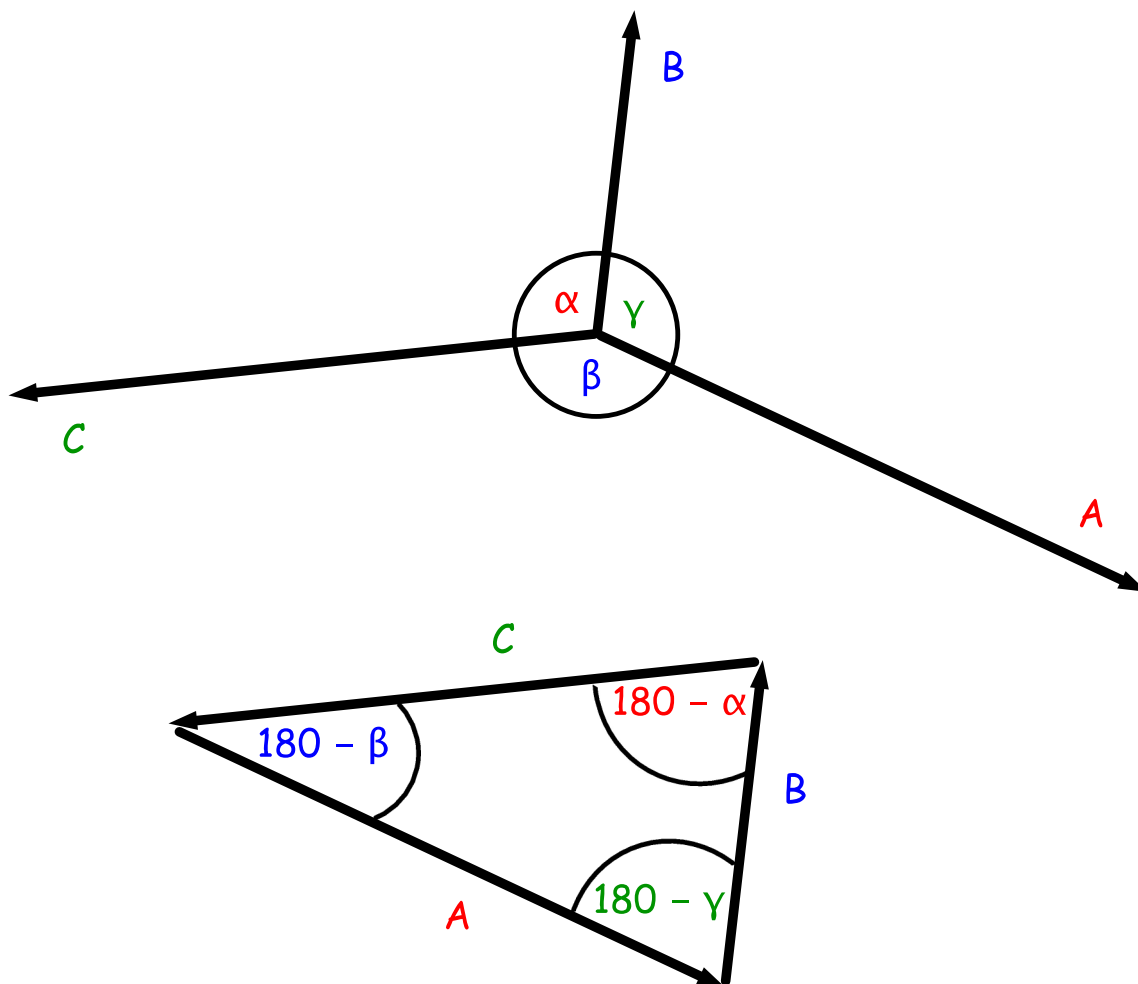
$$\boxed{D \;=\; 3\,g\,\text{N}}$$

The reaction between the the larger mass and the horizontal surface has magnitude R; the second equation gives, using $W_B \;=\; 19\,g$,

$$R \;=\; 3\,g \;+\; 19\,g$$

$$\Rightarrow \quad \boxed{R \;=\; 22\,g\,\text{N}}$$

<u>Lami's Theorem</u>

in the same   at the same   not on the
plane   point   same line

For 3 coplanar, concurrent and non-collinear forces
A, B, C, in equilibrium :

B

α   γ

β

C

A

C

180 − β   180 − α

B

A   180 − γ

Sine Rule gives,

$$\frac{A}{\sin(180 - \alpha)} = \frac{B}{\sin(180 - \beta)} = \frac{C}{\sin(180 - \gamma)}$$
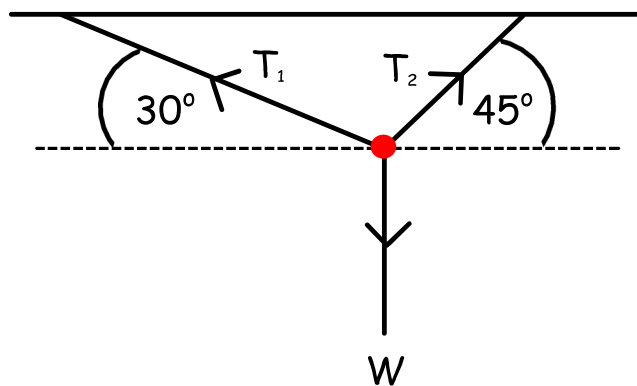
$$\Rightarrow \quad \frac{A}{\sin \alpha} = \frac{B}{\sin \beta} = \frac{C}{\sin \gamma}$$

(Lami's Theorem)

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<u>Example 2</u>

A mass of 5 kg is suspended in equilibrium by two light, inextensible strings which make angles of 30° and 45° with the horizontal.

Calculate the tension in each string.

As all 3 forces emanate from the same point (concurrent), act in the same plane ('x - y' plane; coplanar) and do not lie on the same straight line (non-collinear), Lami's Theorem can be applied.

To use Lami's Theorem, we need the angles between all the forces :
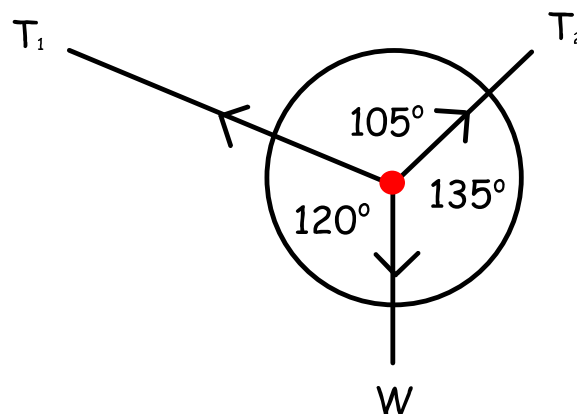
Lami's Theorem gives, remembering that $W = mg,$

$$\frac{W}{\sin 105°} = \frac{T_1}{\sin 135°} = \frac{T_2}{\sin 120°}$$

$$T_1 = \frac{5\,(9\,.\,8)\,\sin 135°}{\sin 105°}$$

$\Rightarrow$ $\boxed{T_1 = 35\,.\,9\,N\ (1\ d.p.)}$

$$T_2 = \frac{5\,(9\,.\,8)\,\sin 120°}{\sin 105°}$$

$\Rightarrow$ $\boxed{T_2 = 43\,.\,9\,N\ (1\ d.p.)}$

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This question can also be done by equating
horizontal and vertical components of all forces

T_1 T_2

$30°$ $45°$

W

T_1 T_2

$T_1 \sin 30°$ $T_2 \sin 45°$

$30°$ $45°$
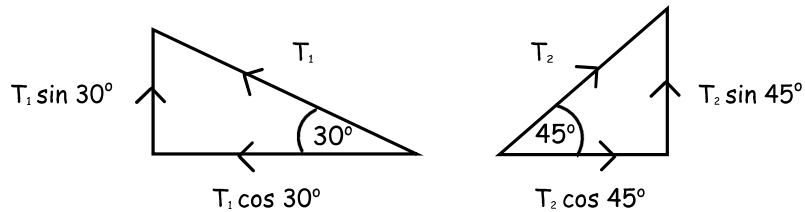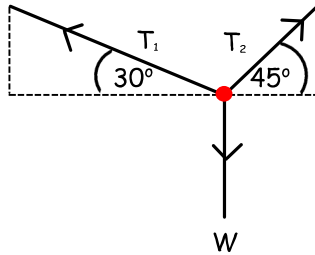
$T_1 \cos 30°$ $T_2 \cos 45°$

All Horizontal Forces   All Vertical Forces

$T_1 \cos 30°$ $T_2 \cos 45°$

$T_1 \sin 30°$ $T_2 \sin 45°$

W
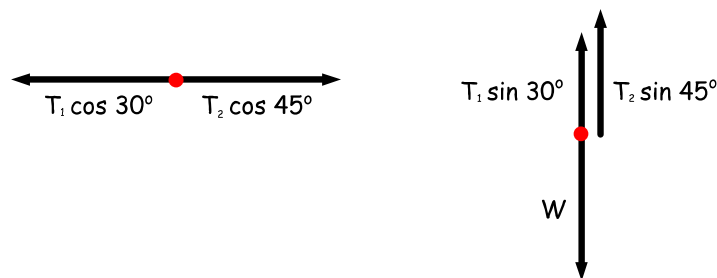
Equilibrium means all horizontal forces add up to give the zero
vector (and similarly for vertical). So, for horizontal,

$$T_1 \cos 30° \; = \; T_2 \cos 45°$$

$$\Rightarrow \qquad T_1 \; = \; \frac{\cos 45°}{\cos 30°} \; T_2$$

$$\Rightarrow \qquad \underline{T_1 \; = \; 0.816\,49\ldots T_2}$$

For vertical,

$$W \; = \; T_1 \sin 30° \; + \; T_2 \sin 45°$$

$$\Rightarrow \qquad 5\,(9.8) \; = \; 0.816\,49\ldots(\sin 30°)\,T_2 \; + \; \sin 45° \, T_2$$

$$\Rightarrow \qquad T_2 \; = \; \frac{49}{(0.408\,24\ldots \; + \; \sin 45°)}$$

$$\Rightarrow \qquad \boxed{T_2 \; = \; 43.9\,\text{N} \; (1\ \text{d.p.})}$$

$$T_1 \; = \; 0.816\,49\ldots T_2$$

$$\Rightarrow \qquad \boxed{T_1 \; = \; 35.9\,\text{N} \; (1\ \text{d.p.})}$$

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<u>Example 3</u>

Find the acceleration produced in a body of mass 2 kg subject to the forces $(14\,\underline{i} + 3\,\underline{j})\,N$ and $(-18\,\underline{i} + \underline{j})\,N$.

Also state the magnitude and direction of the acceleration.
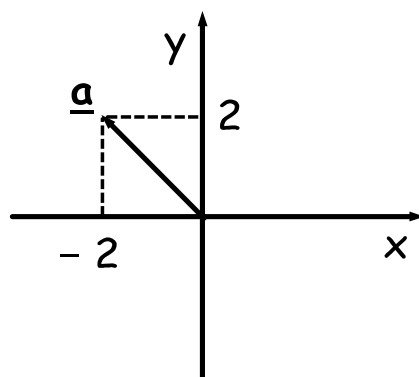
The net force acting on the body is,

$$\underline{F} = (14\,\underline{i} + 3\,\underline{j}) + (-18\,\underline{i} + \underline{j})$$

$\Rightarrow \qquad \underline{F} = -4\,\underline{i} + 4\,\underline{j}$

$\underline{F} = m\,\underline{a}$

$\therefore \qquad 2\,\underline{a} = -4\,\underline{i} + 4\,\underline{j}$

$\Rightarrow \qquad \boxed{\underline{a} = (-2\,\underline{i} + 2\,\underline{j})\,m\,s^{-2}}$

$\boxed{\text{Magnitude of acceleration : } \sqrt{8}\ m\,s^{-2}}$
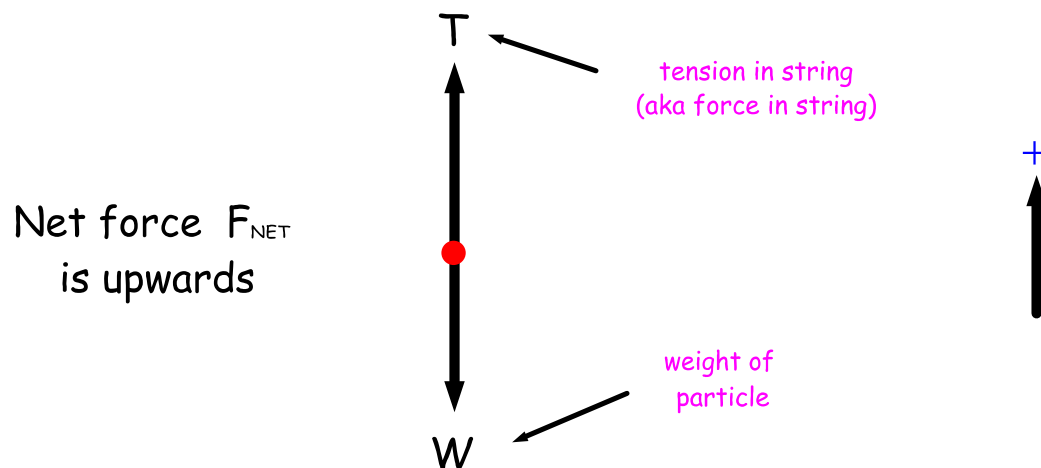
$\boxed{\text{Direction of acceleration : } 45^{\circ}\ W\ of\ N}$

<u>Example 4</u>

A particle of mass 200 g is attached to the lower end of a string that is positioned vertically.

Find the force in the string when the string raises the particle with an acceleration of $2.4 \, m \, s^{-2}$.

The FBD for the particle is :

Net force F_{NET}
    is upwards

T
tension in string
(aka force in string)

+

weight of
particle

W

$$F_{NET} = T - W$$

$$\therefore \quad 0.2\,(2.4) = T - 0.2\,(9.8)$$

$$\Rightarrow \qquad T = 0.48 + 1.96$$

$$\Rightarrow \qquad \underline{T = 2.44}$$

Force in string is 2.44 N upwards

<u>Example 5</u>

A particle of mass 8 kg moves with velocity
$(2t^2\,\underline{i} + 3\,\underline{j} - 6t\,\underline{k})\,m\,s^{-1}$.

If the start of its motion occurs at $t = 0$ seconds, find
the force vector acting on the particle 4 seconds after
motion commences.

$$\underline{a}\,(t) = \frac{d}{dt}\,\underline{v}\,(t)$$

$$\therefore \quad \underline{a}\,(t) = \frac{d}{dt}\,(2t^2\,\underline{i} + 3\,\underline{j} - 6t\,\underline{k})$$

$$\Rightarrow \quad \underline{\underline{a}\,(t) = 4t\,\underline{i} - 6\,\underline{k}}$$

$$\underline{F}\,(t) = m\,\underline{a}\,(t)$$

$$\therefore \quad \underline{F}\,(t) = 8\,(4t\,\underline{i} - 6\,\underline{k})$$

$$\Rightarrow \quad \underline{\underline{F}\,(t) = 32t\,\underline{i} - 48\,\underline{k}}$$

$$\therefore \quad \underline{F}\,(4) = 32\,(4)\,\underline{i} - 48\,\underline{k}$$

$$\Rightarrow \quad \boxed{\underline{F}\,(4) = (128\,\underline{i} - 48\,\underline{k})\,N}$$

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<u>Example 6</u>

A particle of mass 3 kg is acted upon by a force given by
(27 cos 3t **i** − 45 sin 5t **j** + 36 cos 4t **k**) N.

If the start of its motion occurs at t = 0 seconds, and the
particle has velocity − 6 **i** − 6 **j** when t = π/2 seconds,
find the velocity as a function of time.

$$\underline{F}\,(t) = m\,\underline{a}\,(t)$$

∴ $\underline{a}\,(t) = (1/3)\,\underline{F}\,(t)$

⇒ $\underline{a}\,(t) = (1/3)\,(27\cos 3t\ \underline{i} - 45\sin 5t\ \underline{j} + 36\cos 4t\ \underline{k})$

⇒ $\underline{a}\,(t) = 9\cos 3t\ \underline{i} - 15\sin 5t\ \underline{j} + 12\cos 4t\ \underline{k}$

$$\underline{v}\,(t) = \int \underline{a}\,(t)\ dt$$

∴ $\underline{v}\,(t) = \displaystyle\int (9\cos 3t\ \underline{i} - 15\sin 5t\ \underline{j} + 12\cos 4t\ \underline{k})\ dt$

⇒ $\underline{v}\,(t) = 3\sin 3t\ \underline{i} + 3\cos 5t\ \underline{j} + 3\sin 4t\ \underline{k} + \underline{C}$

$\underline{v}\,(\pi/2) = -6\,\underline{i} - 6\,\underline{j}$ gives,

$\underline{C} = (-6 - 3\,(-1))\,\underline{i} + (-6 - 3\,(0))\,\underline{j} + (0 - 0)\,\underline{k}$

⇒ $\underline{C} = -3\,\underline{i} - 6\,\underline{j}$

∴ $\boxed{\underline{v}\,(t) = (3\sin 3t - 3)\,\underline{i} + (3\cos 5t - 6)\,\underline{j} + 3\sin 4t\ \underline{k}}$

Blue Book

- pg. 44-45 Ex. 3 A Q 8 - 11, 16 - 19.
- pg. 48-49 Ex. 3 B Q 6 - 8.
- pg. 52-53 Ex. 3 C Q 5 - 8.
- pg. 86-87 Ex. 5 A Q 3(a), 4, 8.