

A function f is given **explicitly** (f is an **explicit function**) if the output value y is given in terms of the input value.

An explicit function is recognised when y is given as a function of x.

**Examples of Explicit Functions** 

y = sin x

 $y = x^3$ 

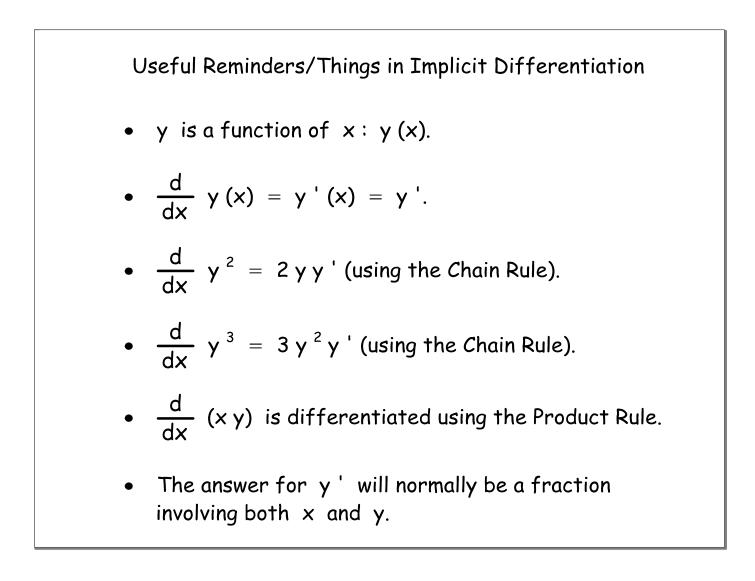
A function f is given **implicitly** (f is an **implicit function**) if the output value y is not given in terms of the input value.

An implicit function is usually identified when the variables x and y are mixed up in a higgledy-piggledy manner.

An implicitly defined function may or may not be solved for y.

**Examples of Implicit Functions** 

y - sin x = 0 x<sup>2</sup> + y<sup>2</sup> = 36 y sin y + x = 4 cos x



## Example 1

Find the gradient of the tangent at any point on the curve defined by  $x^4 - 3xy + y^3 = 13$ .  $x^{4} - 3xy + y^{3} = 13$  $4 x^{3} - 3 (1 \cdot y + x \cdot y') + 3 y^{2} y' = 0$ · .  $4 x^{3} - 3 y - 3 x y' + 3 y^{2} y' = 0$  $\Rightarrow$  $4x^{3} - 3y + y'(3y^{2} - 3x) = 0$  $\Rightarrow$  $y'(3y^2 - 3x) = 3y - 4x^3$  $\Rightarrow$  $y' = \frac{3y - 4x^3}{3y^2 - 3x}$  $\Rightarrow$ 

## Example 2

 $\Rightarrow$ 

 $\Rightarrow$ 

Find the equation of the tangent to the curve defined by  $x y + y^2 = 6$  at y = 1.

$$xy + y^{2} = 6$$
  

$$\therefore (1 \cdot y + xy') + 2yy' = 0$$
  

$$\Rightarrow y + xy' + 2yy' = 0$$
  

$$\Rightarrow y + y'(x + 2y) = 0$$
  

$$\Rightarrow y'(x + 2y) = -y$$

$$y' = -\frac{y}{x + 2y}$$

To find the gradient, we need an (x, y) coordinate. y = 1 in the equation  $xy + y^2 = 6$  gives x = 5.  $\therefore \qquad y'_{(5,1)} = -\frac{1}{5+2}$ 

$$y'_{(5,1)} = -1/7 = m$$

## When finding the 2<sup>nd</sup> derivative of an implicit function, it is better not to use the answer for the 1<sup>st</sup> derivative and then to differentiate that (because this involves the quotient rule)

Instead, it is better to use a line of working in obtaining the 1<sup>st</sup> derivative that has no fractional terms and differentiate that, rearrange and then solve for the 2<sup>nd</sup> derivative

Example 3  
Find 
$$\frac{dy}{dx}$$
 and  $\frac{d^2y}{dx^2}$  for the function defined  
implicitly by  $x^3 + 2xy = 7$ .  
Use the shorthand y' for  $\frac{dy}{dx}$  and y'' for  $\frac{d^2y}{dx^2}$ .  
 $x^3 + 2xy = 7$   
 $\therefore 3x^2 + 2(1.y + xy') = 0$   
 $\Rightarrow 3x^2 + 2y + 2xy' = 0$   
 $\Rightarrow 2xy' = -3x^2 - 2y$   
 $\Rightarrow 2xy' = -3x^2 - 2y$   
 $\Rightarrow y' = \frac{-(3x^2 + 2y)}{2x}$   
Differentiate  $\star$  wrt x to get (after some  
simplification):  
 $6x + 4y' + 2xy'' = 0$   
 $\therefore 6x - 4\left(\frac{3x^2 + 2y}{2x}\right) = -2xy''$   
 $\Rightarrow \frac{12x^2 - 12x^2 - 8y}{2x} = -2xy''$   
 $\Rightarrow \frac{-8y}{2x} = -2xy''$   
 $\Rightarrow \frac{-8y}{2x} = -2xy''$ 

