

*Polynomials - Lesson 6*

## Polynomial Functions from Graphs

LI

- Find unknown coefficients in a polynomial.

SC

- Factor Theorem.
- Analysis of Coordinates.

General idea : when a graph  $y = f(x)$  crosses the  $x$ -axis at  $x = a$ ,  $(x - a)$  is a factor of  $f(x)$  :

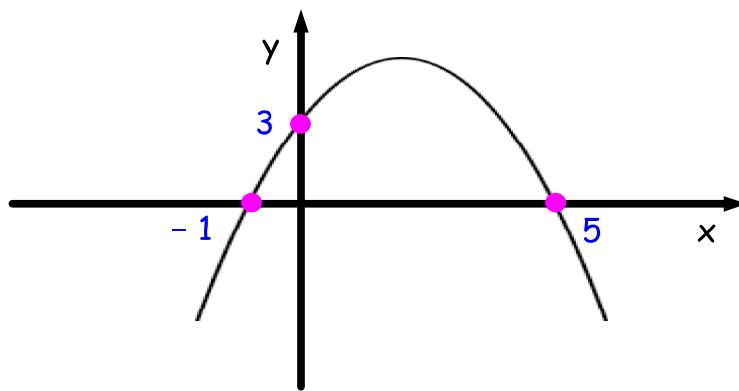
$$f(x) = (x - a) Q(x)$$

Quadratic with	Form of Quadratic
2 roots	$f(x) = k(x - a)(x - b)$
1 root	$f(x) = k(x - a)^2$

Cubic with	Form of Cubic
3 roots	$f(x) = k(x - a)(x - b)(x - c)$
2 roots	$f(x) = k(x - a)(x - b)^2$
1 root	$f(x) = k(x - a)^3$

Example 1

The following is the graph of a quadratic function  
 $y = f(x)$ .



Find its equation.

Two distinct roots, so function is of the form,

$$y = k(x - a)(x - b)$$

$x = -1$  and  $x = 5$  are roots, so,

$$y = k(x + 1)(x - 5)$$

Graph passes through  $(0, 3)$ , so,

$$3 = k(0 + 1)(0 - 5)$$

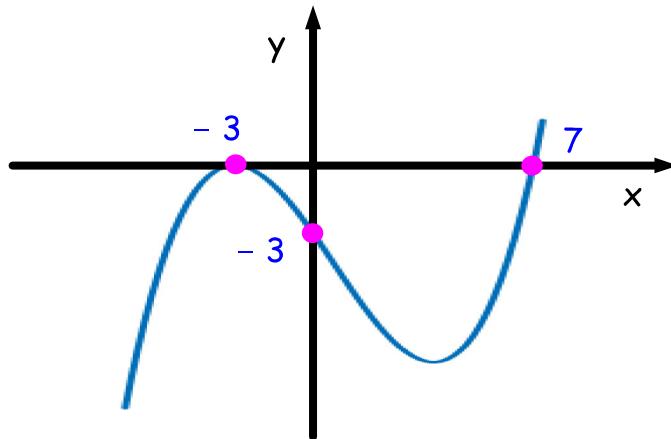
$$\Rightarrow 3 = -5k$$

$$\Rightarrow k = -\frac{3}{5}$$

$$\therefore \boxed{y = -\frac{3}{5}(x + 1)(x - 5)}$$

### Example 2

The following is the graph of a cubic function  
 $y = f(x)$ .



Find its equation.

Two roots, so function is of the form,

$$y = k(x - a)(x - b)^2$$

$x = -3$  (repeated) and  $x = 7$  are roots, so,

$$y = k(x - 7)(x + 3)^2$$

Graph passes through  $(0, -3)$ , so,

$$-3 = k(0 - 7)(0 + 3)^2$$

$$\Rightarrow -3 = k(-7)(9)$$

$$\Rightarrow k = \frac{1}{21}$$

$$\therefore y = \frac{1}{21}(x - 7)(x + 3)^2$$

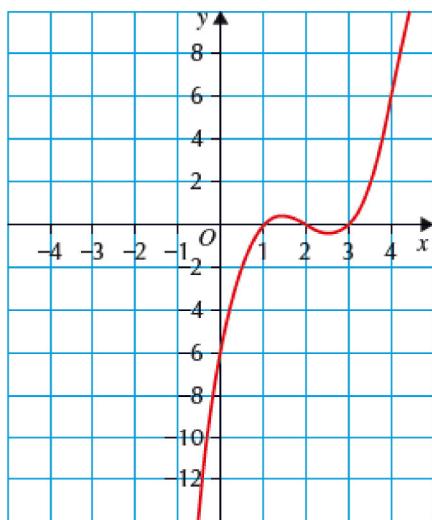
# CfE Higher Maths

pg. 159 - 161 Ex. 7H Q 1, 4

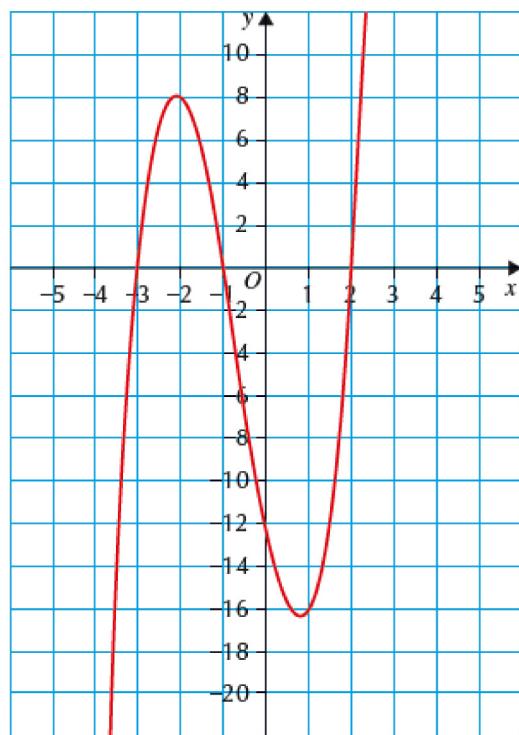
## Questions

1 Determine an expression for  $f(x)$  from the graph of  $y = f(x)$ :

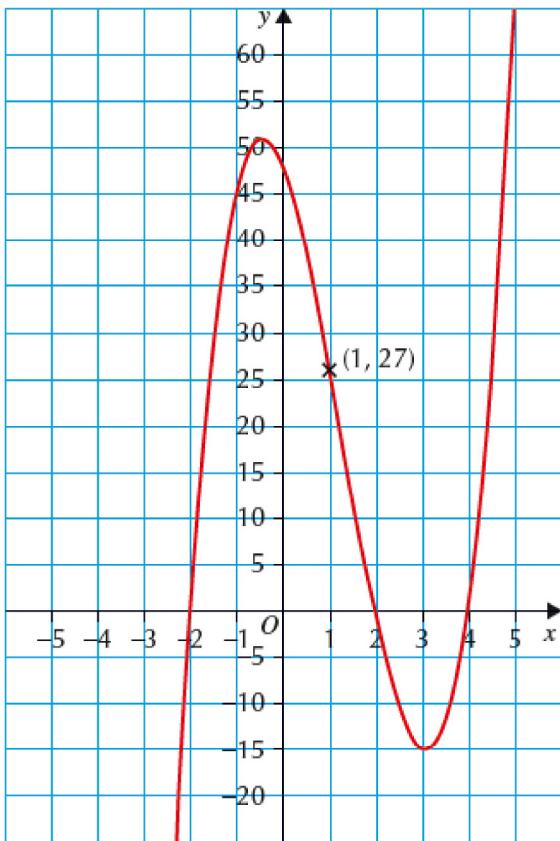
a



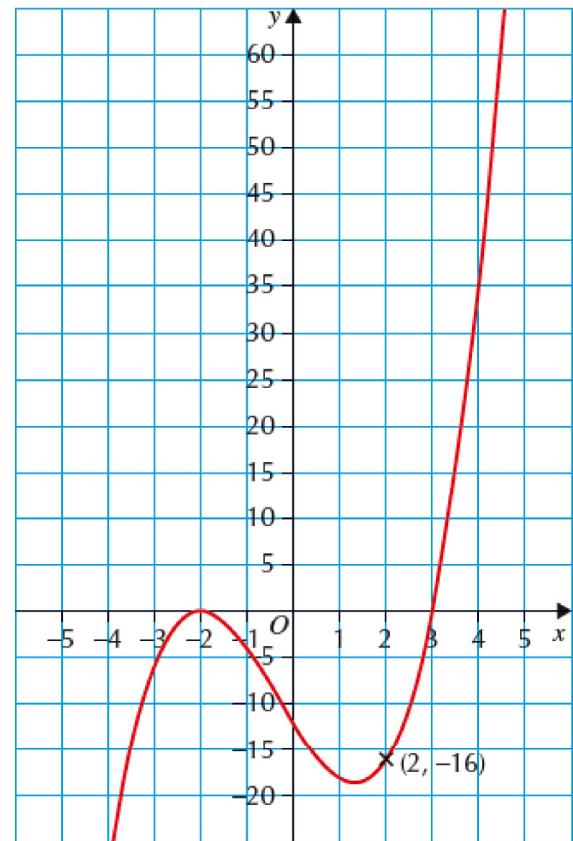
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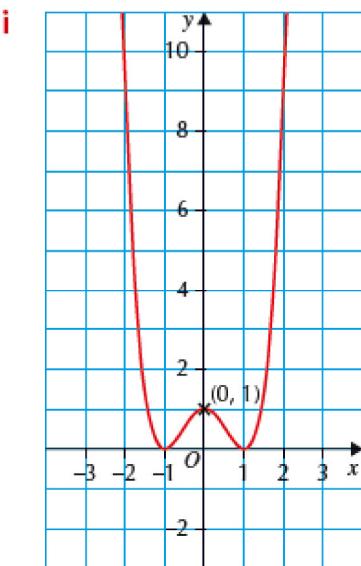
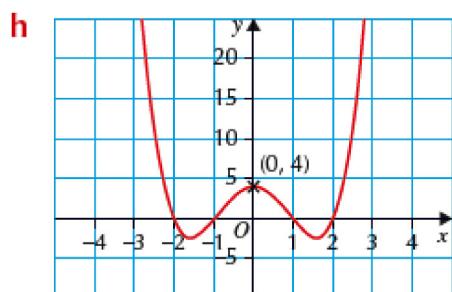
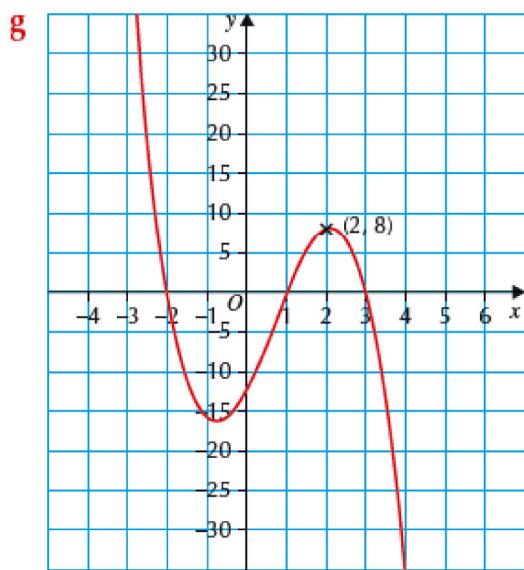
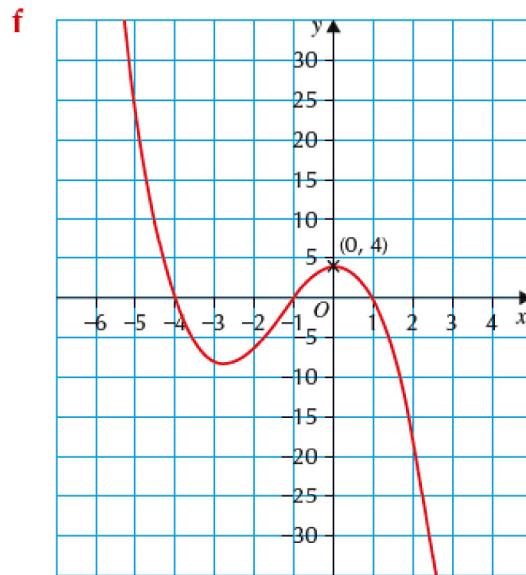
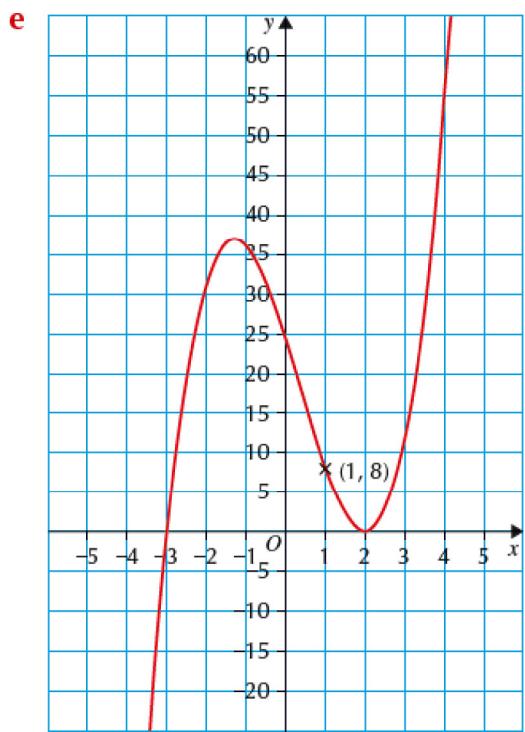


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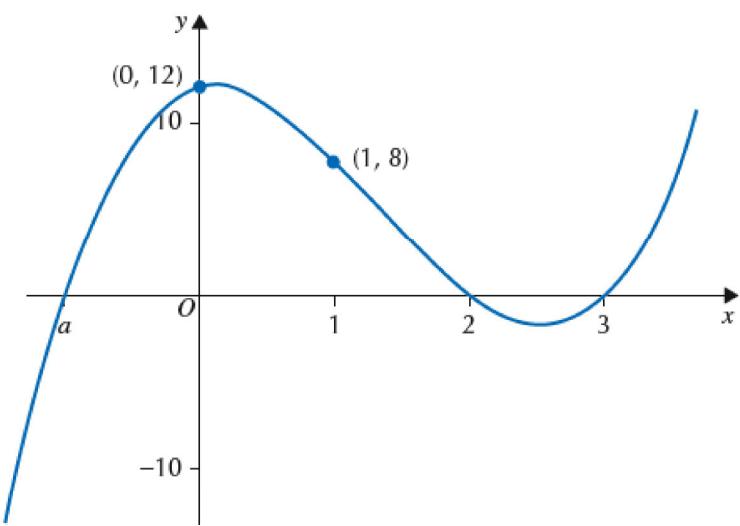


d





- 4 Determine an expression for  $f(x)$  from the graph of  $y = f(x)$  and calculate the value of  $a$ .



## Answers

- 1   a    $f(x) = x^3 - 6x^2 + 11x - 6$   
b    $f(x) = 2x^3 + 4x^2 - 10x - 12$   
c    $f(x) = 3x^3 - 12x^2 - 12x + 48$   
d    $f(x) = x^3 + x^2 - 8x - 12$   
e    $f(x) = 2x^3 - 2x^2 - 16x + 24$   
f    $f(x) = -x^3 - 4x^2 + x + 4$   
g    $f(x) = -2x^3 + 4x^2 + 10x - 12$   
h    $f(x) = x^4 - 5x^2 + 4$   
i    $f(x) = x^4 - 2x^2 + 1$
- 4    $f(x) = 2x^3 - 8x^2 + 2x + 12$   
 $a = -1$