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Graphs of Related Functions - Lesson 6

Logarithmic Graphs

**LI**

- Know the Logarithmic Function and Logarithmic Graphs.
- Know how exponential and logarithmic graphs are connected.
- Sketch related graphs of logarithmic functions.

**SC**

- Graphs of related functions.
- Rules of logarithms.
The Logarithmic Graph

For any $a > 0$, the Logarithmic Function to Base $a$ is the function $y = \log_a x$

For any $a > 0$, the Logarithmic Graph to Base $a$ is the graph of the logarithmic function $y = \log_a x$
How the Exponential and Logarithmic Graphs are Related

The logarithmic graph is obtained by reflecting the graph of the corresponding (same base) exponential function in the line $y = x$.
Example 1

Sketch the graphs of $y = \log_3 x$ and $y = \log_{1/2} x$ on separate diagrams, indicating where each graph crosses the $y$-axis.

Also indicate the coordinates $(d, 1)$ for $y = \log_3 x$ and $(e, 1)$ for $y = \log_{1/2} x$.

The graph of $y = \log_{1/2} x$ is obtained by reflecting the graph of $y = (1/2)^x$ in the line $y = x$. 
Example 2

Shown below is the graph of \( y = \log_2 x \).

Sketch the graph of \( 2 \log_2 (x + 3) \).

The coordinates transform thus:

\[
(1, 0) \rightarrow (1 - 3, 0 \times 2) = (-2, 0) \\
(8, 3) \rightarrow (8 - 3, 3 \times 2) = (5, 6)
\]
Example 3

The diagram shows the graph of \( y = \log_b x \).

Use this to sketch the graphs of:

(a) \( y = \log_b x^2 \).
(b) \( y = \log_b (1/x) \).
(c) \( y = \log_b (9x) \).

(a) Using the rules of logarithms,

\[
y = \log_b x^2 = 2 \log_b x
\]

The coordinates transform thus:

\[
(1, 0) \rightarrow (1, 0 \times 2) = (1, 0)
\]
\[
(3, 1) \rightarrow (3, 1 \times 2) = (3, 2)
\]
(b) Using the rules of logarithms,
\[ y = \log \left( \frac{1}{x} \right) = \log x^{-1} = -\log x \]

The coordinates transform thus:

\[(1, 0) \rightarrow (1, 0 \times 1) = (1, 0)\]
\[(3, 1) \rightarrow (3, 1 \times 1) = (3, -1)\]

(c) Using the rules of logarithms,
\[ y = \log (9x) = \log 9 + \log x \]
\[= \log 3^2 + \log x \]
\[= 2 \log 3 + \log x \]
\[= 2 + \log x \]

The coordinates transform thus:

\[(1, 0) \rightarrow (1, 0 + 2) = (1, 2)\]
\[(3, 1) \rightarrow (3, 1 + 2) = (3, 3)\]
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