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*Vectors - Lesson 5*

# The Scalar Product, Perpendicular Vectors and Angles between Vectors

LI

- Know how to calculate the Scalar Product of 2 vectors.

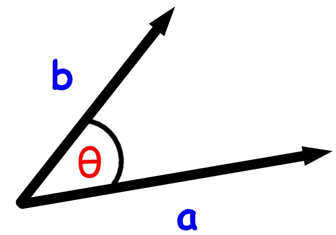
SC

- Arithmetic.
- 2 forms of the Scalar Product Formula.

# Scalar Product

The Scalar Product (aka Dot Product) of vectors **a** and **b** is :

$$\boxed{\begin{gathered}\mathbf{a} \bullet \mathbf{b} = |\mathbf{a}|\,|\mathbf{b}| \cos\theta \\ 0^\circ \le \theta \le 180^\circ\end{gathered}}$$




Vectors **a** and **b** must be pointing out from the same point

## Component Form of the Scalar Product

If $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ then :

$$\boxed{\mathbf{a} \bullet \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3}$$

component form of scalar product

## Important facts :

$\mathbf{i} \bullet \mathbf{i} = 1$ $\mathbf{i} \bullet \mathbf{j} = 0$

$\mathbf{j} \bullet \mathbf{j} = 1$ $\mathbf{j} \bullet \mathbf{k} = 0$

$\mathbf{k} \bullet \mathbf{k} = 1$ $\mathbf{k} \bullet \mathbf{i} = 0$

Perpendicular vectors

$\Updownarrow$

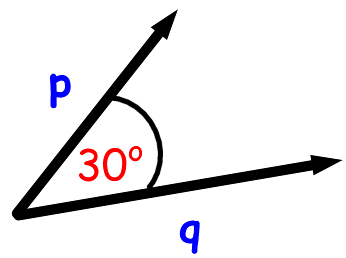
Angle between them is $90°$

$\Updownarrow$

Their scalar product is $0$

Example 1

Calculate $\mathbf{p} \bullet \mathbf{q}$ for the following :




$|\mathbf{p}| = 4$

$|\mathbf{q}| = 2$

$$\mathbf{p} \bullet \mathbf{q} = |\mathbf{p}|\,|\mathbf{q}| \cos \theta$$

$$\Rightarrow \quad \mathbf{p} \bullet \mathbf{q} = 4 \,.\, 2 \,.\, \cos 30^{\circ}$$

$$\Rightarrow \quad \mathbf{p} \bullet \mathbf{q} = \frac{8\sqrt{3}}{2}$$

$$\Rightarrow \quad \boxed{\mathbf{p} \bullet \mathbf{q} = 4\sqrt{3}}$$

Example 2

$A$, $B$ and $C$ are the points with coordinates $(3, 2, 1)$, $(-6, 0, 2)$ and $(-2, -1, 7)$ respectively.

Calculate the value of $\overrightarrow{AB} \bullet \overrightarrow{AC}$.

$$\overrightarrow{AB} = \begin{pmatrix} -9 \\ -2 \\ 1 \end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix} -5 \\ -3 \\ 6 \end{pmatrix}$$

$$\begin{aligned} \overrightarrow{AB} \bullet \overrightarrow{AC} &= (-9 \times -5) + (-2 \times -3) + (1 \times 6) \\ &= 45 + 6 + 6 \\ &= \boxed{57} \end{aligned}$$

Example 3

Calculate the angle between the vectors
$\mathbf{u} = 3\,\mathbf{i} + 2\,\mathbf{j} - 5\,\mathbf{k}$ and $\mathbf{v} = 4\,\mathbf{i} + \mathbf{j} + 3\,\mathbf{k}$.

$$\mathbf{u} \bullet \mathbf{v} = (3 \times 4) + (2 \times 1) + (-5 \times 3)$$

$$\Rightarrow \quad \mathbf{u} \bullet \mathbf{v} = 12 + 2 - 15$$

$$\Rightarrow \quad \underline{\mathbf{u} \bullet \mathbf{v} = -1}$$

The magnitude of $\mathbf{u}$ is,

$$|\,\mathbf{u}\,| = \sqrt{3^2 + 2^2 + (-5)^2}$$

$$\Rightarrow \quad |\,\mathbf{u}\,| = \sqrt{9 + 4 + 25}$$

$$\Rightarrow \quad \underline{|\,\mathbf{u}\,| = \sqrt{38}}$$

The magnitude of $\mathbf{v}$ is,

$$|\,\mathbf{v}\,| = \sqrt{4^2 + 1^2 + 3^2}$$

$$\Rightarrow \quad |\,\mathbf{v}\,| = \sqrt{16 + 1 + 9}$$

$$\Rightarrow \quad \underline{|\,\mathbf{v}\,| = \sqrt{26}}$$

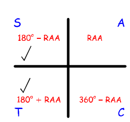
$$\mathbf{u} \bullet \mathbf{v} = |\,\mathbf{u}\,|\,|\,\mathbf{v}\,| \cos\theta$$

$$\therefore \quad -1 = \sqrt{38}\,\sqrt{26}\,\cos\theta$$

$$\Rightarrow \quad \cos\theta = -\frac{1}{\sqrt{38}\,\sqrt{26}}$$

$$\therefore \quad RA = \cos^{-1}\left(\frac{1}{\sqrt{38}\,\sqrt{26}}\right)$$

$$\therefore \quad RA = 88\,.\,2\ldots^{\circ}$$




$$\therefore \quad \theta = 180^{\circ} - 88\,.\,2^{\circ},\ 180^{\circ} + 88\,.\,2^{\circ}$$

$$\Rightarrow \quad \underline{\theta = 91\,.\,8^{\circ},\ 268\,.\,2^{\circ}}$$

As the angle between two vectors is always between $0^{\circ}$ and $180^{\circ}$, $268\,.\,2^{\circ}$ cannot be correct.

$$\boxed{\theta = 91\,.\,8^{\circ}}$$

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Example 4

Show that the vectors in Example 3 are not perpendicular.

As $\mathbf{u} \bullet \mathbf{v} = -1 \neq 0$, $\mathbf{u}$ and $\mathbf{v}$ are not perpendicular

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## Example 5

Given that $k \leq 0$, find $k$ so that the vectors

$$\mathbf{a} = \begin{pmatrix} 2k \\ -1 \\ 3 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} k \\ k \\ -2 \end{pmatrix}$$

are perpendicular.

As $\mathbf{a}$ and $\mathbf{b}$ are perpendicular,

$$\mathbf{a} \bullet \mathbf{b} = 0$$

$$\therefore \quad (2k \times k) + (-1 \times k) + (3 \times -2) = 0$$

$$\Rightarrow \quad 2k^2 - k - 6 = 0$$

$$\Rightarrow \quad (2k + 3)(k - 2) = 0$$

$$\therefore \quad \underline{k = -3/2, 2}$$

$$\boxed{\text{As } k \leq 0, k = -3/2}$$

CfE Higher Maths

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