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Solving Trigonometric Equations - Lesson 5

Solving Quadratic Trigonometric Equations Using Trigonometric Identities

LI

- Solve trigonometric equations of the form :

$$a \cos^2 x + b \sin x + c = 0$$

$$a \sin^2 x + b \cos x + c = 0$$

$$a \cos 2x + b \cos x + c = 0$$

$$a \cos 2x + b \sin x + c = 0$$

$$a \sin 2x + b \cos x + c = 0$$

$$a \sin 2x + b \sin x + c = 0$$

for various ranges of x (in degrees or radians).

SC

- Double Angle Formulae and $\sin^2 x + \cos^2 x = 1$.
- Factorising trinomials.
- Solve linear trig. equations.

Strategy

- Use a double angle formula or $\sin^2 x + \cos^2 x = 1$ to get the equation into one of the forms :

$$a \sin^2 x + b \sin x + c = 0$$

$$a \cos^2 x + b \cos x + c = 0$$

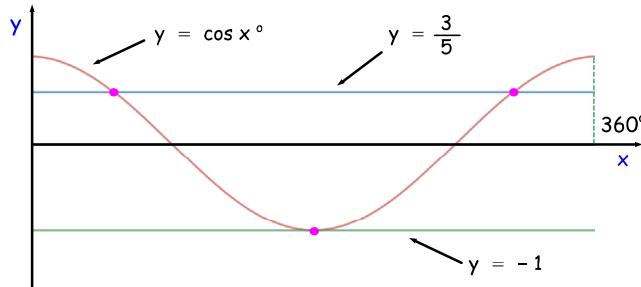
- Solve as per Lesson 4.

Example 1

Solve $5 \sin^2 x^\circ - 2 \cos x^\circ - 2 = 0$
 $(0^\circ \leq x^\circ \leq 360^\circ)$.

$$\begin{aligned} 5 \sin^2 x^\circ - 2 \cos x^\circ - 2 &= 0 \\ 5(1 - \cos^2 x^\circ) - 2 \cos x^\circ - 2 &= 0 \\ 5 - 5 \cos^2 x^\circ - 2 \cos x^\circ - 2 &= 0 \\ 5 \cos^2 x^\circ + 2 \cos x^\circ - 3 &= 0 \\ (5 \cos x^\circ - 3)(\cos x^\circ + 1) &= 0 \end{aligned}$$

$$\cos x^\circ = \frac{3}{5}, \cos x^\circ = -1$$

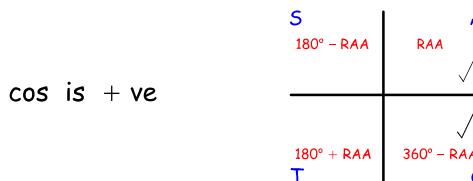


3 solutions expected

$$\cos x^\circ = \frac{3}{5} :$$

$$RAA = \cos^{-1}\left(\frac{3}{5}\right)$$

$$\Rightarrow RAA = 53.13\dots^\circ$$



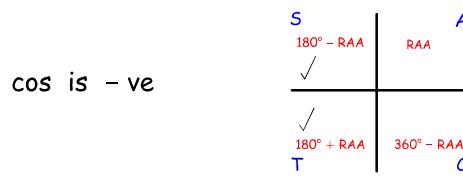
$$\therefore x^\circ = 53.13\dots^\circ, 360^\circ - 53.13\dots^\circ$$

$$\Rightarrow x^\circ = 53.13\dots^\circ, 306.86\dots^\circ$$

$$\cos x^\circ = -1 :$$

$$RAA = \cos^{-1}(1)$$

$$\Rightarrow RAA = 0^\circ$$



$$\therefore x^\circ = 180^\circ$$

$$\therefore x^\circ = 53.1^\circ, 180^\circ, 306.9^\circ \text{ (1 d.p.)}$$

Example 2

Solve $6 \cos^2 x^\circ + \sin x^\circ = 5$ ($0 \leq x \leq 360$).

$$6 \cos^2 x^\circ + \sin x^\circ = 5$$

$$6 \cos^2 x^\circ + \sin x^\circ - 5 = 0$$

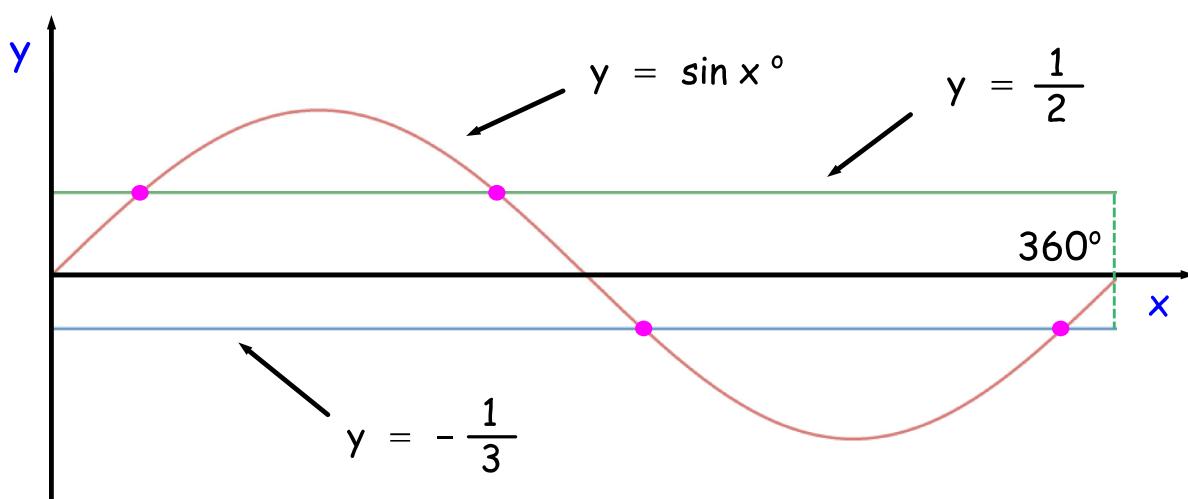
$$6(1 - \sin^2 x^\circ) + \sin x^\circ - 5 = 0$$

$$6 - 6 \sin^2 x^\circ + \sin x^\circ - 5 = 0$$

$$6 \sin^2 x^\circ - \sin x^\circ - 1 = 0$$

$$(3 \sin x^\circ + 1)(2 \sin x^\circ - 1) = 0$$

$$\sin x^\circ = -\frac{1}{3}, \sin x^\circ = \frac{1}{2}$$



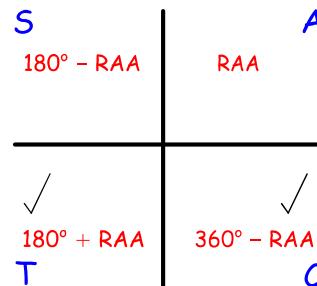
4 solutions expected

$$\sin x^\circ = -\frac{1}{3} :$$

$$RAA = \sin^{-1}\left(-\frac{1}{3}\right)$$

$$\Rightarrow RAA = 19.47\dots^\circ$$

\sin is - ve



$$\therefore x^\circ = 180^\circ + 19.47\dots^\circ, 360^\circ - 19.47\dots^\circ$$

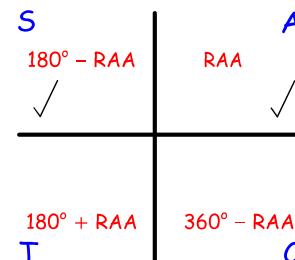
$$\Rightarrow x^\circ = 199.5^\circ, 340.5^\circ$$

$$\sin x^\circ = \frac{1}{2} :$$

$$RAA = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow RAA = 30^\circ$$

\sin is + ve



$$\therefore x^\circ = 30^\circ, 150^\circ$$

$$\therefore x^\circ = 30^\circ, 150^\circ, 199.5^\circ, 340.5^\circ \text{ (1 d.p.)}$$

Example 3 (non-calculator)

Solve $\cos 2x - 5 \sin x + 2 = 0$
 $(0 \leq x \leq 2\pi)$.

$$\cos 2x - 5 \sin x^{\circ} + 2 = 0$$

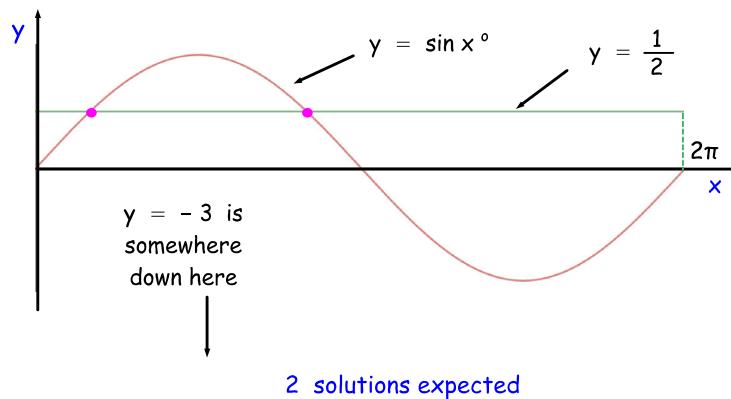
$$(1 - 2 \sin^2 x) - 5 \sin x^{\circ} + 2 = 0$$

$$1 - 2 \sin^2 x^{\circ} - 5 \sin x^{\circ} + 2 = 0$$

$$2 \sin^2 x^{\circ} + 5 \sin x^{\circ} - 3 = 0$$

$$(2 \sin x^{\circ} - 1)(\sin x^{\circ} + 3) = 0$$

$$\sin x^{\circ} = \frac{1}{2}, \sin x^{\circ} = -3$$

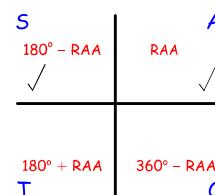


$$\sin x^{\circ} = \frac{1}{2} :$$

$$\text{RAA} = \sin^{-1} \left(\frac{1}{2} \right)$$

$$\Rightarrow \underline{\text{RAA} = \pi/6}$$

sin is + ve



$$\therefore \underline{x = \pi/6, 5\pi/6}$$

$$\sin x^{\circ} = -3 :$$

No solutions, as $y = \sin x$ doesn't meet $y = -3$
(min. value of $\sin x$ is -1).

$$\therefore \boxed{x = \pi/6, 5\pi/6}$$

Example 4

Solve $2 \sin 2x + 3 \sin x^{\circ} = 0$ ($0 \leq x \leq 2\pi$).

Give all answers to 3 significant figures.

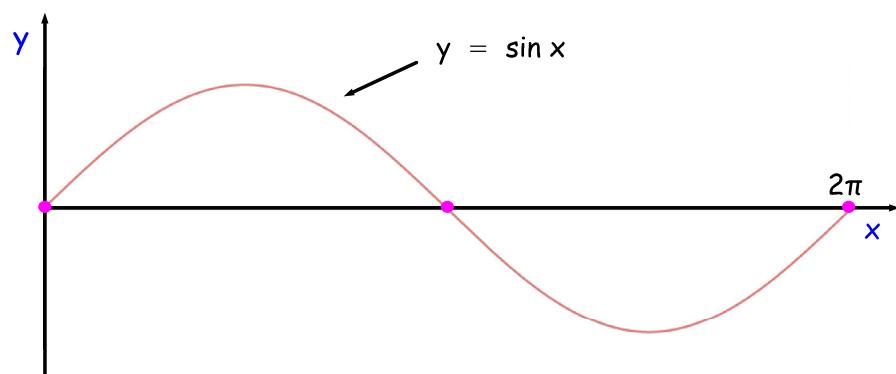
$$2 \sin 2x + 3 \sin x = 0$$

$$2(2 \sin x \cos x) + 3 \sin x = 0$$

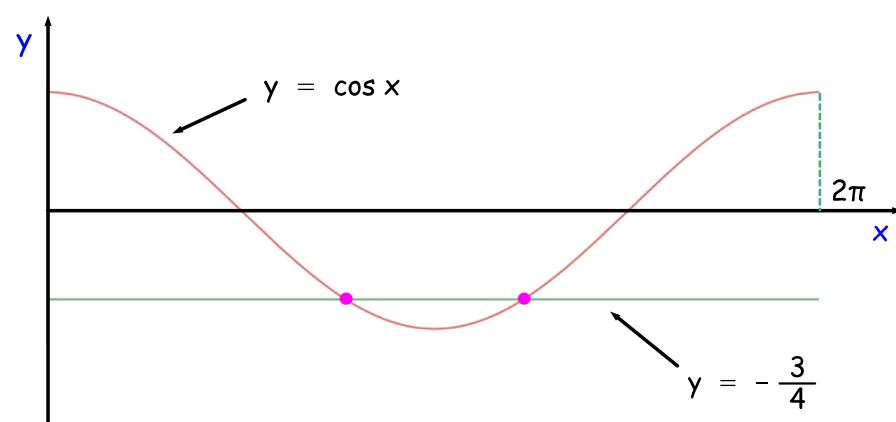
$$4 \sin x \cos x + 3 \sin x = 0$$

$$\sin x (4 \cos x + 3) = 0$$

$$\sin x = 0 \quad , \cos x = -\frac{3}{4}$$



3 solutions expected



2 solutions expected

$$\underline{\sin x = 0 :}$$

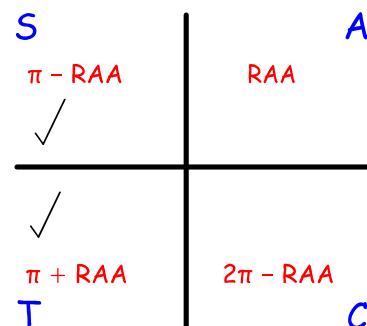
From the graph of $y = \sin x$, there are 3 solutions :

$$\begin{aligned} x &= 0, \pi, 2\pi \\ \Rightarrow x &= 0, 3.141\dots, 6.283\dots \end{aligned}$$

$$\underline{\cos x^o = -\frac{3}{4} :}$$

$$\begin{aligned} RAA &= \cos^{-1}\left(-\frac{3}{4}\right) \\ \Rightarrow RAA &= 0.722\dots \end{aligned}$$

\cos is -ve



$$\therefore x = \pi - 0.722\dots, \pi + 0.722\dots$$

$$\Rightarrow x = 2.418\dots, 3.864\dots$$

$$\therefore \boxed{x = 0, 2.418\dots, 3.864\dots}$$

CfE Higher Maths

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pg. 193 Ex. 8G All Q