Polynomials - Lesson 5

Sketching Polynomials

LT
- Sketch polynomials given the polynomial equation.

SC
- Synthetic division.
To sketch a polynomial graph, we need the following information:

- \( x \)-intercepts (put \( y = 0 \) then synthetic division to get roots).
- \( y \)-intercept (put \( x = 0 \)).

(• Behaviour as \( x \to \pm \infty \).)
Behaviour as $x \rightarrow \pm \infty$

$$f(x) = a x^3 + b x^2 + c x + d$$

For very large values of $x$ (positive or negative), the $x^3$ term is dominant, so that,

$$f(x) \approx a x^3$$

**Case 1 ($a > 0$):**

As $x \rightarrow +\infty$ (i.e. for large and positive $x$),

$$f(x) \rightarrow +\infty$$

(i.e. $y$ is large and positive).

**Shape of graph for $a > 0$:**

![Shape of graph for $a > 0$]

**Case 2 ($a < 0$):**

As $x \rightarrow -\infty$ (i.e. for large and negative $x$),

$$f(x) \rightarrow -\infty$$

(i.e. $y$ is large and negative).

**Shape of graph for $a < 0$:**

![Shape of graph for $a < 0$]
Example 1

Sketch the graph of \( y = f(x) \), where
\[
f(x) = x^3 - 4x^2 + x + 6.
\]

• **x-intercepts:**

From Lesson 4, Example 1, the roots of \( f \) are:

\[
x = 3, \quad x = 2, \quad x = -1
\]

• **y-intercept:**

Putting \( x = 0 \) gives:

\[
f(0) = (0)^3 - 4(0)^2 + (0) + 6
\]

\[
\Rightarrow f(0) = 6
\]

As \( a = 1 > 0 \), shape is:

As \( a = 1 > 0 \), shape is:
Example 2

Sketch the graph of \( y = f(x) \), where
\[
f(x) = -x^3 + x^2 + 8x - 12.
\]

- \( x \)-intercepts:
  
  Solve \( f(x) = 0 \).
  
  \[
f(1) = -4.
\]
  
  \[
f(-1) = -18.
\]
  
  \[
f(2) = 0.
\]

So, \( x = 2 \) is a root of \( f \).

\[
\begin{array}{c|cccc}
  & x^3 & x^2 & x^1 & x^0 \\
 2 & -1 & 1 & 8 & -12 \\
\end{array}
\]

\[
\begin{array}{c|cccc}
  & x^3 & x^2 & x^1 & x^0 \\
 2 & -2 & -2 & 12 & 0 \\
\end{array}
\]

\[
\begin{array}{c|cccc}
  & x^3 & x^2 & x^1 & x^0 \\
 2 & -1 & -1 & 6 & 0 \\
\end{array}
\]

\[
\therefore \ f(x) = (x - 2)(-x^2 - x + 6)
\]

\[
\Rightarrow \ f(x) = (2 - x)(x^2 + x - 6)
\]

\[
\Rightarrow \ f(x) = (2 - x)(x - 2)(x + 3)
\]

Roots are \( x = 2 \) and \( x = -3 \)

- \( y \)-intercept:

Putting \( x = 0 \) gives:

\[
f(0) = - (0)^3 + (0)^2 + 8(0) - 12
\]

\[
\Rightarrow \ f(0) = -12
\]

As \( a = -1 < 0 \), shape is:

\[
y = -x^3 + x^2 + 8x - 12
\]
Questions

Sketch these polynomials:

1) \( f(x) = x^3 + x^2 - 10x + 8. \)
2) \( p(x) = -x^3 + 7x - 6. \)
3) \( g(x) = x^3 - x^2 - 5x - 3. \)
4) \( d(x) = -2x^3 + 3x^2 + 2x. \)
5) \( v(x) = (x - 2)^3. \)
6) \( d(x) = x^3 + x^2 + x + 1. \)
Answers

1) \( f(x) = x^3 + x^2 - 10x + 8 \)

2) \( p(x) = -x^3 + 7x - 6 \)

3) \( g(x) = x^3 - x^2 - 5x - 3 \)

4) \( d(x) = -2x^3 + 3x^2 + 2x \)

5) \( v(x) = x^3 - 6x^2 + 12x - 8 \)

6) \( k(x) = x^3 + x^2 + x + 1 \)