Polynomials - Lesson 5

Sketching Polynomials

LI

- Sketch polynomials given the polynomial equation.

SC

- Synthetic division.

To sketch a polynomial graph, we
need the following information :

- x - intercepts (put $y = 0$ then synthetic division to get roots).
- y - intercept (put $x = 0$).

(• Behaviour as $x \longrightarrow \pm \infty$.)

Behaviour as $x \longrightarrow \pm \infty$

$$f(x) = a x^3 + b x^2 + c x + d$$

For very large values of x (positive or negative), the x^3 term is dominant, so that,

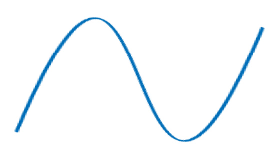
$$f(x) \approx a x^3$$

Case 1 $(a > 0)$:

As $x \longrightarrow + \infty$ (i. e. for large and positive x),
$f(x) \longrightarrow + \infty$ (i. e. y is large and positive).
positive x $(\text{positive})^3$ = positive

As $x \longrightarrow - \infty$ (i. e. for large and negative x),
$f(x) \longrightarrow - \infty$ (i. e. y is large and negative).
positive x $(\text{negative})^3$ = negative

Shape of graph for $a > 0$:

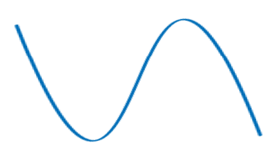


Case 2 $(a < 0)$:

As $x \longrightarrow + \infty$ (i. e. for large and positive x),
$f(x) \longrightarrow - \infty$ (i. e. y is large and negative).
negative x $(\text{positive})^3$ = negative

As $x \longrightarrow - \infty$ (i. e. for large and negative x),
$f(x) \longrightarrow + \infty$ (i. e. y is large and positive).
negative x $(\text{negative})^3$ = positive

Shape of graph for $a < 0$:



Example 1

Sketch the graph of $y = f(x)$, where
$f(x) = x^3 - 4x^2 + x + 6$.

- x - intercepts :

From Lesson 4, Example 1, the roots of f are :

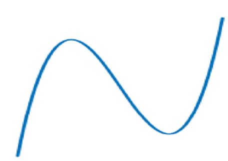
$$x = 3, \quad x = 2, \quad x = -1$$

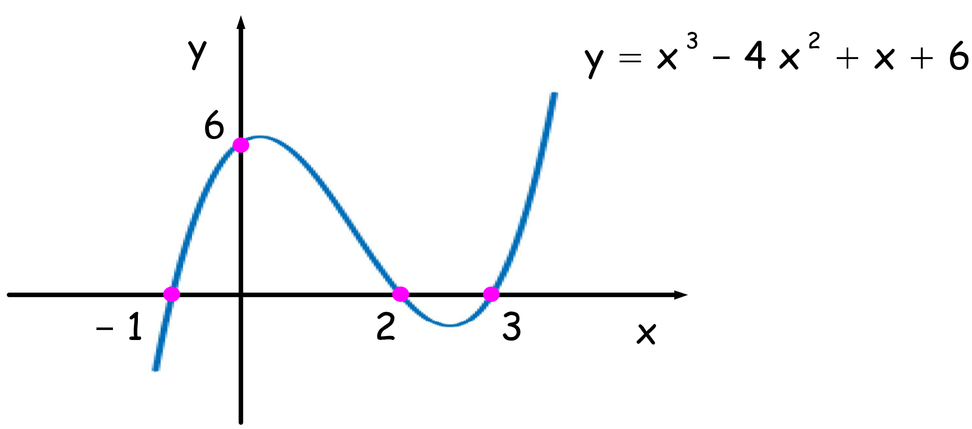
- y - intercept :

Putting $x = 0$ gives :

$$f(0) = (0)^3 - 4(0)^2 + (0) + 6$$

$$\Rightarrow \quad f(0) = 6$$

As $a = 1 > 0$, shape is : 



Example 2

Sketch the graph of $y = f(x)$, where
$f(x) = -x^3 + x^2 + 8x - 12$.

- x - intercepts :

Solve $f(x) = 0$.

$f(1) = -4$.

$f(-1) = -18$.

$f(2) = 0$.

So, $x = 2$ is a root of f.

	x^3	x^2	x^1	x^0
2	-1	1	8	-12
		-2	-2	12
	-1	-1	6	0

$\therefore \quad f(x) = (x - 2)(-x^2 - x + 6)$

$\Rightarrow \quad f(x) = (2 - x)(x^2 + x - 6)$

$\Rightarrow \quad f(x) = (2 - x)(x - 2)(x + 3)$

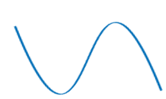
Roots are $x = 2$ and $x = -3$

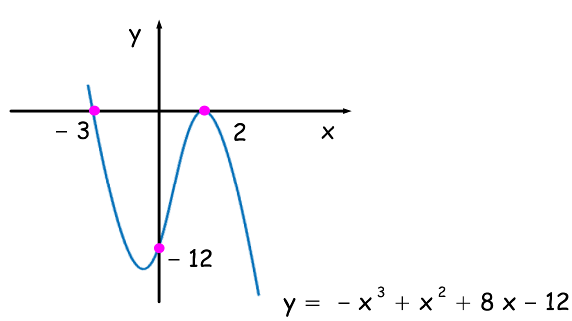
- y - intercept :

Putting $x = 0$ gives :

$$f(0) = -(0)^3 + (0)^2 + 8(0) - 12$$

$\Rightarrow \quad f(0) = -12$

As $a = -1 < 0$, shape is : 



Questions

Sketch these polynomials :

1) $f(x) = x^3 + x^2 - 10x + 8.$

2) $p(x) = -x^3 + 7x - 6.$

3) $g(x) = x^3 - x^2 - 5x - 3.$

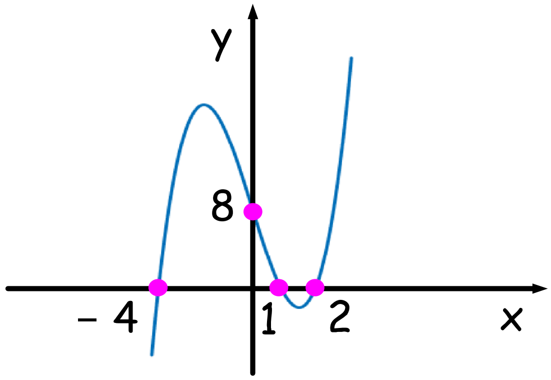
4) $d(x) = -2x^3 + 3x^2 + 2x.$

5) $v(x) = (x - 2)^3.$

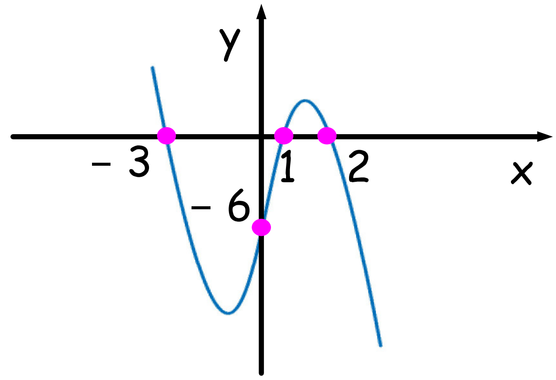
6) $d(x) = x^3 + x^2 + x + 1.$

Answers

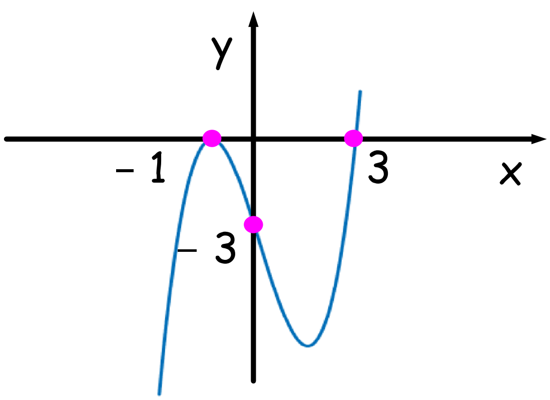
1) $f(x) = x^3 + x^2 - 10x + 8$



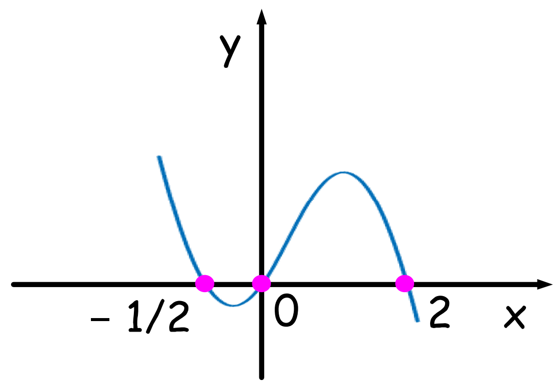
2) $p(x) = -x^3 + 7x - 6$



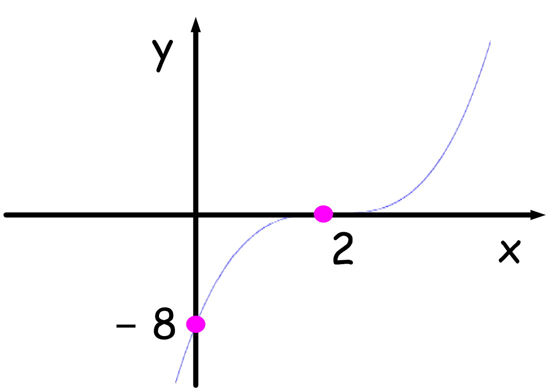
3) $g(x) = x^3 - x^2 - 5x - 3$



4) $d(x) = -2x^3 + 3x^2 + 2x$



5) $v(x) = x^3 - 6x^2 + 12x - 8$



6) $k(x) = x^3 + x^2 + x + 1$