Experimental Data and Logarithmic Equations

**LI**
- Obtain equations for $y$ in terms of $x$ when $\log y$ is plotted against either (i) $x$ or (ii) $\log x$.

**SC**
- Logarithmic Rules.
When experiments are performed and graphs plotted, the equations are not always straight lines

However, when one or both variables are 'logged', the graphs are often straight lines

This allows us to deduce the actual equation linking $y$ to $x$

2 important situations can arise
Case 1: $y = kx^n$ (k is constant, i.e. a number)

$$y = kx^n$$

$$\therefore \log_w y = \log_w (kx^n)$$

$$\Rightarrow \log_w y = \log_w k + \log_w x^n$$

$$\Rightarrow \log_w y = \log_w k + n \log_w x$$

Let $Y = \log_w y$, $c = \log_w k$, $m = n$ and $X = \log_w x$.

Then,

$$Y = mX + c$$
Case 2: \( y = a b^x \) (\( a, b \) are constants)

\[
y = a b^x
\]

\[
\therefore \log_w y = \log_w (a b^x)
\]

\[
\Rightarrow \log_w y = \log_w a + \log_w b^x
\]

\[
\Rightarrow \log_w y = \log_w a + x \log_w b
\]

Let \( Y = \log_w y, c = \log_w a, X = x \) and \( m = \log_w b \).

Then,

\[
Y = mX + c
\]
Example 1

Results from an experiment are shown in this graph:

\[ \log_{10} y \]

\[ \log_{10} x \]

(2, 8.3)

0.3

Show that this graph represents a relationship of the form \( y = k \times^n \), stating the values of \( k \) and \( n \).

\[
\log_{10} y = m \log_{10} x + c
\]

\[ \therefore \log_{10} y = m \log_{10} x + c \log_{10} 10 \]

\[ \Rightarrow \log_{10} y = \log_{10} x^m + \log_{10} 10^c \]

\[ \Rightarrow \log_{10} y = \log_{10} (10^c x^m) \]

\[ \therefore y = k \times^n \quad (n = m, \ k = 10^c) \]

n = m

k = 10^c

\[ \therefore n = \frac{8.3 - 0.3}{2 - 0} \quad \therefore k = 10^{0.3} \]

\[ \Rightarrow n = 4 \quad \Rightarrow k = 1.995 \ldots \]

\[ \therefore k \approx 2 \]
Example 2

Show that the following graph represents a relationship of the form \( y = a \cdot b^x \), stating the values of \( a \) and \( b \).

\[
\log_{10} y = m \cdot x + c
\]

\[
\therefore \log_{10} y = m \cdot (\log_{10} 10) + c \log_{10} 10
\]

\[
\Rightarrow \log_{10} y = (\log_{10} 10^m \cdot x) + \log_{10} 10^c
\]

\[
\Rightarrow \log_{10} y = \log_{10} (10^c \cdot 10^m \cdot x)
\]

\[
\therefore y = a \cdot b^x \quad (a = 10^c, \ b = 10^m)
\]

\[
m = \frac{1.3 - 0.1}{0 - 3}
\]

\[
\Rightarrow m = -0.4
\]

\[
b = 10^m
\]

\[
\therefore b = 10^{-0.4}
\]

\[
\Rightarrow b = 0.398\ldots
\]

\[
\therefore b \approx 0.4
\]
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