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Exponentials and Logarithms - Lesson 5

Experimental Data and Logarithmic Equations

LI

- Obtain equations for y in terms of x when log y is plotted against either (i) x or (ii) log x.

SC

- Logarithmic Rules.

Jan 10-12:11

When experiments are performed and graphs plotted, the equations are not always straight lines

However, when one or both variables are 'logged', the graphs are often straight lines

This allows us to deduce the actual equation linking y to x

2 important situations can arise

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Case 1 : $y = k x^n$ (k is constant, i.e. a number)

$$y = k x^n$$

$$\therefore \quad \log_w y = \log_w (k x^n)$$

$$\Rightarrow \quad \log_w y = \log_w k + \log_w x^n$$

$$\Rightarrow \quad \log_w y = \log_w k + n \log_w x$$

Let $Y = \log_w y$, $c = \log_w k$, $m = n$ and $X = \log_w x$.

Then,

$$\boxed{Y = mX + c}$$

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Case 2 : $y = a\,b^{x}$ (a, b are constants)

$$y = a\,b^{x}$$

$$\therefore \quad \log_w y = \log_w (a\,b^{x})$$

$$\Rightarrow \quad \log_w y = \log_w a + \log_w b^{x}$$

$$\Rightarrow \quad \log_w y = \log_w a + x \log_w b$$

Let $Y = \log_w y$, $c = \log_w a$, $X = x$ and $m = \log_w b$.

Then,

$$\boxed{Y = mX + c}$$

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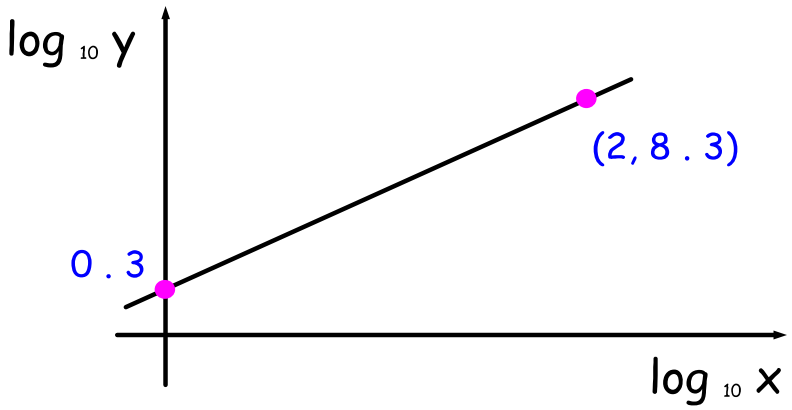
Example 1

Results from an experiment are shown in this graph :



Show that this graph represents a relationship of the form $y = k x^n$, stating the values of k and n.

$$\log_{10} y = m \log_{10} x + c$$

$$\therefore \quad \log_{10} y = m \log_{10} x + c \log_{10} 10$$

$$\Rightarrow \quad \log_{10} y = \log_{10} x^m + \log_{10} 10^c$$

$$\Rightarrow \quad \log_{10} y = \log_{10} (10^c x^m)$$

$$\therefore \quad \boxed{y = k x^n \ (n = m, k = 10^c)}$$

$$n = m$$

$$\therefore \quad n = \frac{8.3 - 0.3}{2 - 0}$$

$$\Rightarrow \quad \boxed{n = 4}$$

$$k = 10^c$$

$$\therefore \quad k = 10^{0.3}$$

$$\Rightarrow \quad k = 1.995\ldots$$

$$\therefore \quad \boxed{k \approx 2}$$

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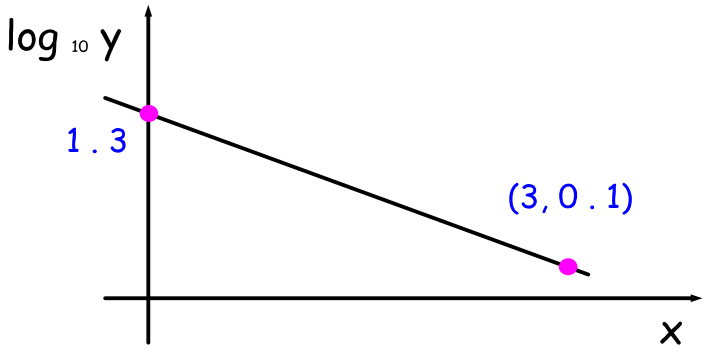
Example 2

Show that the following graph represents a relationship of the form $y = a b^x$, stating the values of a and b.



$$\log_{10} y = m x + c$$

$$\therefore \quad \log_{10} y = m x (\log_{10} 10) + c \log_{10} 10$$

$$\Rightarrow \quad \log_{10} y = (\log_{10} 10^{m x}) + \log_{10} 10^{c}$$

$$\Rightarrow \quad \log_{10} y = \log_{10} (10^{c} 10^{m x})$$

$$\therefore \quad \boxed{y = a b^{x} \ (a = 10^{c}, b = 10^{m})}$$

$$m = \frac{1 . 3 - 0 . 1}{0 - 3}$$

$$\Rightarrow \quad m = - 0 . 4$$

$$b = 10^{m}$$

$$\therefore \quad b = 10^{- 0 . 4}$$

$$\Rightarrow \quad b = 0 . 398 \ldots$$

$$\therefore \quad \boxed{b \approx 0 . 4}$$

$$a = 10^{c}$$

$$\therefore \quad a = 10^{1 . 3}$$

$$\Rightarrow \quad a = 19 . 952 \ldots$$

$$\therefore \quad \boxed{a \approx 20}$$

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CfE Higher Maths

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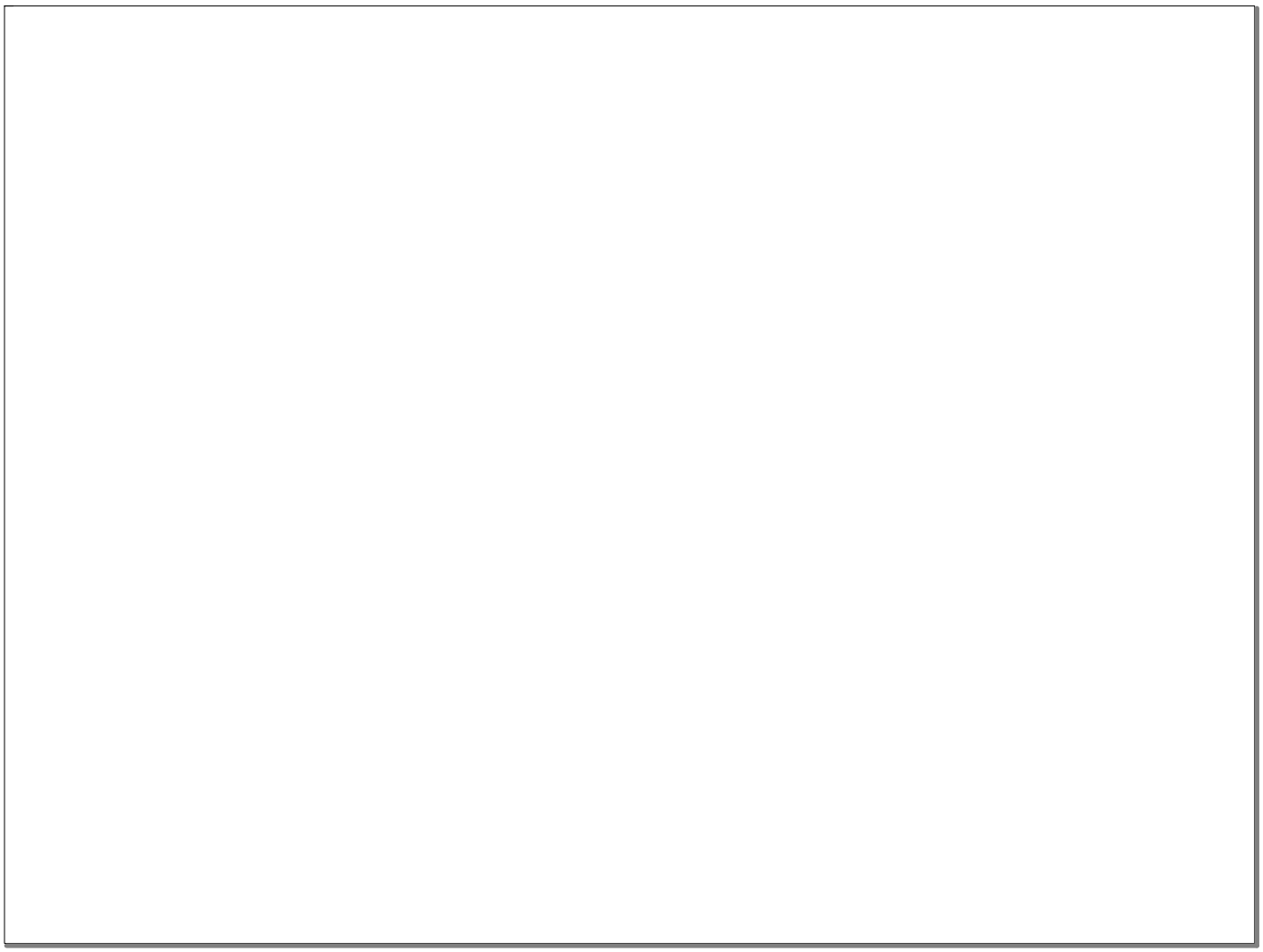

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